Long Memory Processes ($1/f^{\alpha}$ Type) in Human Coordination

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Consider the task of synchronizing the movement of one's limb to a periodic environmental signal. Using rescaled range analysis and the spectral maximum likelihood estimator, we establish that the experimental errors of human synchronization exhibit $1/f^{\alpha}$ type long memory where α is about 0.5, and that the underlying stochastic process can be modeled by fractional Gaussian noise. In addition, we provide a preliminary model of this phenomenon using stochastic delayed differential equations. [S0031-9007(97)04698-X]

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The wide occurrence of $1/f^{\alpha}$ type of long memory (long range correlated) processes in electrical systems and in solid state devices has long posed a challenging problem for physics [1]. A number of mechanisms, ranging from the superposition of many independent relaxation processes [2] to self-organized criticality [3], are proffered to explain this phenomenon. In this Letter we report the manifestation of long memory processes in a human sensorimotor coordination experiment in which a subject synchronizes his finger tapping with an external periodic stimulus. Using an array of diagnostic tools including rescaled range analysis and the spectral maximum likelihood estimator, we show that the error time series, defined as the time between a predetermined point in the tapping cycle and the onset of the stimulus, exhibits long memory of $1/f^{\alpha}$ type, and can be modeled as fractional Gaussian noise [4]. The average value of α is found to be about 0.5. This result adds the present human sensorimotor coordination system to a growing list of biological examples in which one observes long range correlated random fluctuations [5–7]. In addition, we report our attempt at modeling the experimental findings using stochastic delay differential equations. Our motivation is to present a unifying mechanism for a diverse set of long memory processes, observed under a variety of sensorimotor conditions [7], by incorporating both the inevitable occurrence of noise (white) in the nervous system and delay feedback networks involved in controlling the motor output.

Experiment and data collection.—Five right-handed male subjects ranging in age from 25 to 35 took part in the synchronization experiment. Seated in a sound attenuated chamber, each subject was instructed to cyclically press his index finger against a computer key in synchrony with a periodic series of auditory beeps, delivered through a headphone. Two frequency conditions, $F_1 = 2$ Hz ($T_1 = 500$ ms) and $F_2 = 1.25$ Hz ($T_2 = 800$ ms), were studied. These frequencies were chosen such that the subject was able to perform the required tapping motion continuously [8,9].

Each experimental session consisted of the subject performing 1200 continuous taps for a given frequency. A computer program was used to register the time of a specific point in the tapping cycle in millisecond resolution. The data collected were the interresponse intervals (IRIs) I_i, and the synchronization or tapping errors e_i. As defined in Fig. 1, e_i is the time between the computer recorded response time R_i and the metronome onset S_i, i.e., $e_i = R_i - S_i$, and I_i is the time between successive tapping responses, i.e., $I_i = R_{i+1} - R_i$. From the figure it is clear that I_i and e_i are not independent variables, in particular,

$$\begin{cases} I_i = T + e_{i+1} - e_i \,, \\ e_i = e_0 + \sum_{k=1}^{i} (I_{k-1} - T) \,. \end{cases} \tag{1}$$

This relation has important consequences in the sequel for delineating which of the two time series is the more fundamental in the present experiment.

Results of data analysis.—Twenty time series, each consisting of 1200 points, are collected from the five subjects, each performing two sessions for a given frequency condition. Each time series is indexed by the order of responses. For some trials, due mainly to a loss of concentration, subjects missed a metronome stimulus or tapped 100 ms or more earlier than the onset of stimulus. We refer to these phenomena as "glitches" to contrast them with the much smaller synchronization errors we treat here. A careful examination reveals that this kind of glitch occurred fewer than 5 out of 1200 responses and does not appear to affect the long term characteristics we seek to quantify.

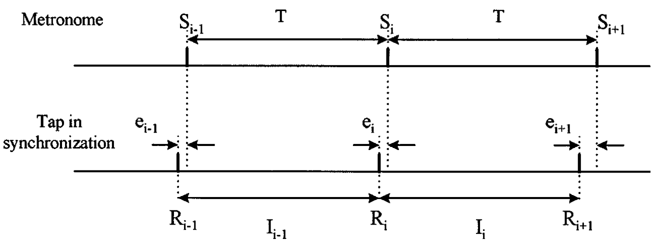

FIG. 1. Definition for the synchronization error e_i and interresponse interval I_i.

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Figure 2 shows a typical example of an error time series and the corresponding IRIs for $F_1 = 2$ Hz. Both time series appear to be stationary. In addition, the distribution of the error variable e_i, shown as a histogram in the inset of Fig. 2(a), is well fit by a Gaussian with mean -16.9 ms and standard deviation 20.3 ms. A chi-square test confirms the assertion that e_i is Gaussian distributed. From Eq. (1) the IRI variable I_i is also a Gaussian random variable [see inset in Fig. 2(b)].

Initial hints about the long memory character of the time series in Fig. 2(a) are provided by computing its spectral density using 1024 points after discarding the first 50 points to eliminate transient. The result, plotted on a log-log scale in Fig. 3(a), roughly follows a straight line, suggesting that the spectral density $S(f)$ scales with frequency f as a power law, $S(f) \sim f^{-\alpha}$, where $\alpha \approx 0.54$. From the Wiener-Khinchin theorem this implies that the autocorrelation function $C(k)$ of the original error time series e_i decays with the time lag k also as a power law

$$C(k) \sim k^{-\beta}, \tag{2}$$

where $\beta = 1 - \alpha \approx 0.46$. Recall that a long memory process is mathematically defined as a process whose autocorrelation function $C(k)$ sums to infinity [10],

$$\sum_{k=0}^{k=\infty} C(k) = \infty. \tag{3}$$

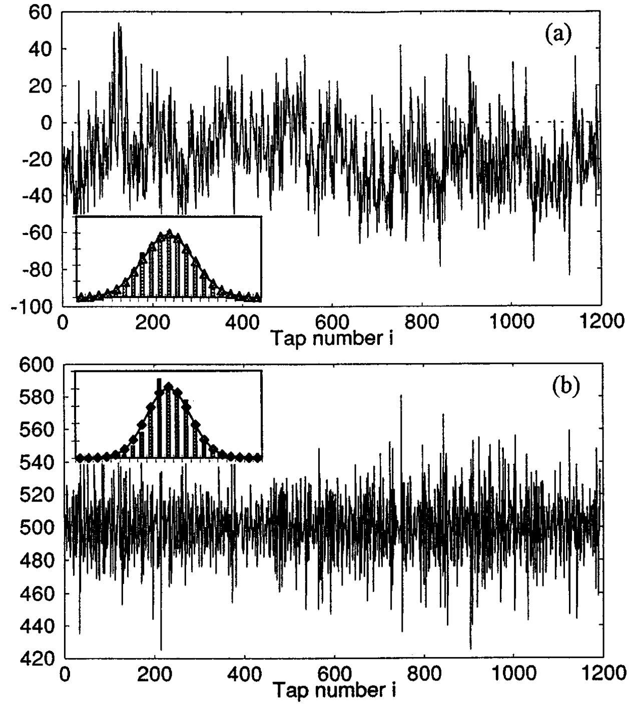

FIG. 2. Example of an error time series (a) and the corresponding interresponse intervals (b) (in ms). Histograms and their Gaussian fits for the time series are shown in the insets. Notice that most synchronization errors and their average are negative, meaning that on the average the subject tapped before the beep.

The autocorrelation function in Eq. (2), with $0 < \beta \approx 0.46 < 1$, meets this definition. This establishes the error time series in Fig. 2(a) as coming from a long memory process, specifically, a fractional Gaussian noise process. Similar results are obtained for all twenty error time series from the experiment. Also the average spectral density for the ten error time series from each frequency condition is observed to obey a power law with slope close to $1/2$ [11].

Another index for long memory processes is the Hurst exponent H. It relates to α through [10]

$$H = (1 + \alpha)/2. \tag{4}$$

A direct way to estimate the value of H is the trend-corrected rescaled range analysis [5] originally used by Hurst to analyze yearly minimums of the Nile River [12]. Let $\overline{e}_i = e_i - \overline{e}$ where $\overline{e}$ denotes the sample mean of the given error time series. Consider the cumulative sum $L(n,s) = \sum_{i=1}^{i=s} \overline{e}_{n+i}$, where $L(n,s)$ can be regarded as the position of a random walk after s steps. Define the trend-corrected range of the random walk as $R(n,s) = \max\{L(n,p) - pL(n,s)/s, 1 \le p \le s\} - \min\{L(n,p) - pL(n,s)/s, 1 \le p \le s\}$. Let $S^2(n,s)$ denote the sample variance of the data set $\{\overline{e}_{n+i}\}_{i=1}^{i=s}$. If the average rescaled statistic $Q(s) = \langle R(n,s)/S(n,s)\rangle_n$ scales with s as a power law for large s, $Q(s) \sim s^H$, then H is the Hurst exponent. One can show that, if the autocorrelation function $C(k)$ sums to a finite number, then generally $H = 1/2$, corresponding to the case of short term memory. If Eq. (3) holds, then $1/2 < H < 1$, and the time series is said to have long persistent memory.

Figure 3(b) shows the log-log plot of $Q(s)$ versus s for the error time series shown in Fig. 2(a). A straight line fit to the data gives $H = 0.79$ which is consistent with $H = 0.77$ obtained from Fig. 3(a) and Eq. (4). Applying the same rescaled range analysis to all the error time series, we summarize the calculated Hurst exponents in Table I (top row for each trial). Averaging all the relevant entries in the table gives $\overline{H} = 0.723 \pm 0.071$ which is significantly greater than $H = 1/2$.

The fractional Gaussian noise characteristic established for the error time series further enables us to apply a

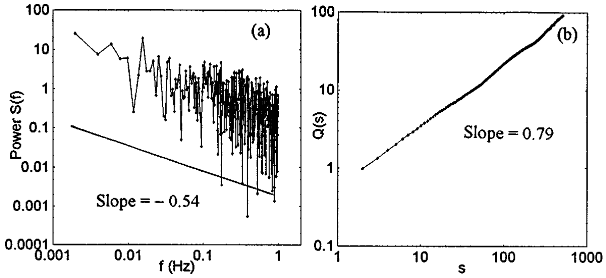

FIG. 3. (a) Spectral density of the error time series in Fig. 2(a). We have converted the unit of frequency from 1/beat to Hz (see Gilden *et al.* in [7]). (b) Log-log plot of averaged R/S value $Q(s)$ against window size s for the time series in Fig. 2(a).

TABLE I. Hurst exponents for error time series. For each trial, the first row gives H calculated based on R/S analysis, the second row gives MLE results (standard deviations are 0.024).

Stimulus period	Subject A	Subject B	Subject C	Subject D	Subject E
500 ms	0.74	0.77	0.76	0.71	0.71
trial *A*	0.895	0.69	0.73	0.80	0.845
500 ms	0.79	0.72	0.68	0.80	0.76
trial *B*	0.835	0.845	0.83	0.85	0.83
800 ms	0.66	0.75	0.58	0.71	0.70
trial *A*	0.79	0.74	0.555	0.815	0.685
800 ms	0.81	0.70	0.71	0.80	0.69
trial *B*	0.755	0.78	0.64	0.775	0.665

more systematic statistical method—the frequency domain maximum likelihood estimation (MLE) [10]—to the calculation of the Hurst exponent. The result is displayed as the lower number in each box in Table I. The overall average gives $\overline{H} = 0.77 \pm 0.085$. Note that most of the MLE values are slightly larger than the corresponding H values from the R/S method. This is consistent with the observation that the rescaled range analysis tends to underestimate the Hurst exponent when $H > 0.75$ [13].

At first blush, the IRI I_i appears to be another natural variable for characterizing the synchronization process. However, if the same analysis is applied to the I_i time series from the present experiment, without realizing the relation in Eq. (1), one can easily arrive at erroneous conclusions. As an illustration of what to expect from such an analysis we show in Figs. 4(a) and 4(b) the spectral density and the rescaled range plot for the time series in Fig. 2(b). The positive slope of the power spectrum is a reflection of rapid local fluctuations in the time series, indicating its origin as a differenced time series [see Eq. (1)], and the value of the slope relates to α through the known formula $2 - \alpha$. The R/S plot also shows the characteristic shape of a differenced time series which is not linear on log-log scale. In fact, the local slope of the R/S plot in this case approaches zero as the time scale increases [14]. Clearly, these results are not informative in terms of revealing the underlying long memory. In this regard we propose the notion of a *fundamental time series* to capture the role played by the error time series [15]. We expect this notion to be of importance in experimental situations where several interrelated dynamical variables all appear to provide an adequate description of the problem.

Modeling and discussion.—A common feature of physical and biological systems that exhibit $1/f^{\alpha}$ type long memory is that they all have many interacting components. Observation of $1/f^{\alpha}$ behavior has been seen as an indicator of complex self-organized dynamics. In view of the diverse set of problems in which they arise, it is generally accepted that no single mechanism can explain all long memory processes.

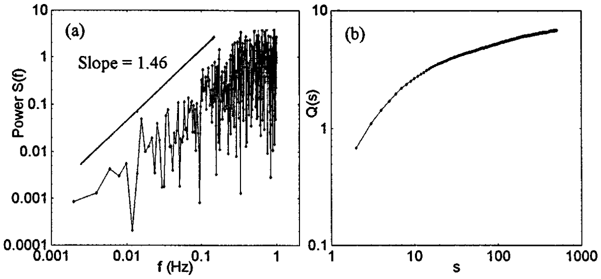

FIG. 4. The power spectrum (a) and the R/S plot (b) for the IRI time series in Fig. 2(b). The parabolically shaped curve in (b) should not be construed as giving multiple values of H (see text).

In this Letter we focus our attention on aspects of sensorimotor coordination as embodied by the synchronization experiment. From a modeling point of view two points are salient. First, the nervous system is a massively parallel processing unit which possesses intrinsic spontaneous activities in addition to the control of the synchronization task. To a first approximation we may consider those unrelated processes as a white noise background. Second, the actual control of the synchronization task is not entirely centralized. Sensory feedback may play an important role for the production of correct behavior. Transmission delays through the neural circuitry imply that a proper mathematical description of the synchronization problem should be a stochastic delay differential equation [16]. Specifically, we hypothesize that the nervous system controls the behavior by comparing the input periodic stimulus represented by $\sin(\omega t)$ with a delayed version of the actual movement $x(t - \tau)$, where x is normalized finger position and τ denotes sensory delay. [We comment that the delay feedback assumption is essential in reproducing the observed phase advance in Fig. 2(a).] Below we present a model which is modified from a previous model of bimanual rhythmic coordination [17],

$$\ddot{x} + \dot{x}^3 - \dot{x} + \dot{x}x^2 + x = \beta[\sin(\omega t) - x(t - \tau)]^2 + \sqrt{Q}\xi_t\,, \tag{5}$$

where β gives the strength of the coupling, ξ_t represents Gaussian white noise of unit variance, and Q is noise amplitude. When $\beta = 0$ the equation has a natural period of 2π. If we choose the period of the stimulus around this value the output of the oscillator can be phase locked to the input (synchronized). [Note that we make no effort to conform the driving frequency here to that used in the experiment, although it can be done by rescaling the variables in Eq. (5). This is partly because Eq. (5), although based on detailed experimental observations [18], is a phenomenological model, not a model derived from first principles.] Because of the stochastic term ξ_t, the phase difference or synchronization error here is obtained by computing the peak distance between the input and the output signals. The error time series obtained this way looks very similar to that from the experiment. For $Q = 0.0004$, $\beta = 0.4$, $\tau = 0.4$, $\omega = 2\pi \times 0.14 = 0.88$, using the numerical method in [19] to obtain a time series of 1024 phase errors, the Hurst exponent estimated from MLE is about 0.75 (0.72 from R/S). For $Q = 0$, the period of oscillation for Eq. (5) is about $T_s = 7.143$. For the experiment with $T_1 = 500$ ms, the same τ/T_s ratio yields a sensory delay of $500 \times (0.4/7.143) \approx 28$ ms, which is actually a reasonable value for the real nervous system.

The present modeling effort is an attempt to establish that random noise and sensory delay in the nervous system are the origins of observed long memory processes in sensorimotor experiments. It is known that certain stochastic partial differential equations display long spatial memory [10]. Recent work shows that delay differential equations and partial differential equations are intimately related [20]. This provides the hope that certain delay differential equations may have long temporal memory. Future effort will be directed toward making these considerations more concrete [11].

In conclusion, the analysis presented in this Letter shows strongly that the error time series associated with the synchronization task has long term memory. In addition, one can model the underlying stochastic processes as fractional Gaussian noises. Previous work has not explored sufficiently long time series to establish this effect (e.g., [8]). We further postulate that this phenomenon may be attributed to the inevitable occurrence of noise and sensory delay in the nervous system. The functional significance of long memory in human coordination may be related to the flexibility of switches from one pattern to another [11,21]. More investigations are underway to explore these possibilities.

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[1] P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981); M. B. Weissman, *ibid.* **60**, 537 (1988).

[2] E. W. Montroll and M. K. Shlesinger, Proc. Natl. Acad. Sci. U.S.A. **79**, 3380 (1982); C. W. J. Granger, J. Econometrics **14**, 227 (1980).

[3] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

[4] B. B. Mandelbrot and J. W. Van Ness, SIAM Rev. **10**, 422 (1968); B. B. Mandelbrot and J. R. Wallis, Water Resour. Res. **3**, 967 (1969).

[5] J. B. Bassingthwaighte, L. S. Liebovitch, and B. J. West, *Fractal Physiology* (Oxford University Press, New York, 1994).

[6] M. Kobayashi and T. Musha, IEEE Trans. Biomed. Eng. **29**, 465 (1982); F. Grüneis, M. Nakao, and M. Yamamoto, Biol. Cybernet. **62**, 407 (1990); D. T. Kaplan and M. Talajic, Chaos **1**, 251 (1991); R. F. Voss, Phys. Rev. Lett. **68**, 3805 (1992); C.-K. Peng *et al.*, Nature (London) **356**, 168 (1992); M. Ding, B. Tuller, and J. A. S. Kelso, Chaos **5**, 70 (1995); R. G. Turcott and M. C. Teich, Ann. Biomed. Eng. **24**, 269 (1996).

[7] A. Schenkel, J. Zhang, and Y. C. Zhang, Fractals **1**, 47 (1993); J. J. Collins and C. J. De Luca, Chaos **5**, 57 (1995); J. M. Hausdorff *et al.*, J. Appl. Phys. **80**, 1448 (1996); D. L. Gilden, T. Thornton, and M. W. Mallon, Science **267**, 1837 (1995).

[8] H. Woodrow, J. Exp. Psych. **XV**, 357 (1932); D. Hary and G. P. Moore, Hum. Neurobiol. **4**, 73 (1985); Biol. Cybern. **56**, 305 (1987).

[9] D. Engström, J. A. S. Kelso, and T. Holroyd, Hum. Mov. Sci. **15**, 809 (1996).

[10] J. Beran, *Statistics for Long-Memory Processes* (Chapman & Hall, New York, 1994); G. Samorodnitsky and M. S. Taqqu, *Stable and Non-Gaussian Random Processes* (Chapman & Hall, New York, 1994).

[11] Y. Chen, M. Ding, and J. A. S. Kelso (to be published).

[12] H. E. Hurst, Trans. Am. Soc. Civ. Eng. **116**, 770 (1951); H. E. Hurst, R. P. Black, and Y. M. Simaiki, *Long-term Storage: An Experimental Study* (Constable, London, 1965).

[13] H. E. Scepers, J. van Beek, and J. B. Bassingthwaighte, IEEE Eng. Med. Biol. Mag. **57** (1992).

[14] Y. Chen, M. Ding, L. Qian, and J. A. S. Kelso (to be published).

[15] This is understood in two ways. First, the error time series manifests a fundamental random process, namely, a fractional Gaussian noise process, with a well-defined Hurst exponent. Second, the partial sums of the error time series yield a random walk (fractional Brownian motion), while the partial sums of the IRIs give back the stationary error time series.

[16] For modeling using delay differential equations, see A. Longtin and J. G. Milton, J. Math. Biol. **51**, 605 (1989); A. Beuter *et al.*, Bull. Math. Biol. **55**, 525 (1993).

[17] H. Haken, J. A. S. Kelso, and H. Bunz, Biol. Cybern. **51**, 347 (1985).

[18] B. Kay, J. A. S. Kelso, E. Saltzman, and G. Schöner, J. Exp. Psychol. Hum. Percept. Perf. **13**, 178 (1987).

[19] R. Mannella and V. Palleschi, Phys. Rev. A **40**, 3381 (1989).

[20] G. Giacomelli and A. Politi, Phys. Rev. Lett. **76**, 2686 (1996).

[21] G. Schöner, H. Haken, and J. A. S. Kelso, Biol. Cybern. **53**, 442 (1986).