Magnetization Instability in a Two-Dimensional System

A. J. Daneshvar, C. J. B. Ford, M. Y. Simmons, A. V. Khaetskii, A. R. Hamilton, M. Pepper, and D. A. Ritchie
Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge, CB3 0HE, United Kingdom
(Received 8 July 1997)

The spin magnetization of a system of fermions confined to two dimensions, and subject to a strong magnetic field, has been predicted to exhibit a first-order phase transition as a function of its Zeeman energy. We present strong experimental evidence that such a paramagnetic to ferromagnetic phase transition occurs at Landau level filling $\nu = 4$ in the two-dimensional hole gas. We demonstrate that our data are not explained by perturbative effects such as Landau level anticrossing, and show that exchange coefficients measured at odd filling factors are consistent with the predicted size of the gap at $\nu = 4$. [S0031-9007(97)04681-4]

PACS numbers: 73.20.Dx, 71.18.+y, 71.70.–d

The ground state of a two-dimensional system (2DS) of fermions exhibiting the integral quantum Hall effect is determined by the interplay between the contributions to the total energy from the strong magnetic field and the interparticle interactions. As the single-particle energies are varied, the system can undergo a phase transition to a spin-polarized state driven by the Coulomb exchange interaction.

In the quantum Hall effect, the single-particle energy spectrum of an ideal, zero-thickness, 2DS is quantized into highly degenerate Landau levels by the component of magnetic field $B_\perp$, perpendicular to the plane of confinement. The energy gaps at odd filling factors ν, where ν is the ratio of the density to the degeneracy of a spin-split Landau level, have been shown to be increased above the Zeeman energy E_Z by the exchange interaction [1,2] which arises from the Pauli exclusion principle. This prevents parallel-spin fermions from approaching one another, thus reducing their Coulomb interaction energy compared with that of opposite-spin fermions.

Figures 1(a) and 1(b) show the dependence of the noninteracting Landau levels on the total magnetic field B for a system at constant even and odd filling factors, respectively. At $\nu =$ odd the two Landau levels that approach each other are empty, whereas at even filling factors one is unoccupied but the other is full. This difference is crucial, because in the latter case, the system has the opportunity to lower its energy of interaction by promoting all of the particles in the highest occupied Landau level into the nearest unoccupied Landau level of opposite spin. This process dramatically lowers the Coulomb interaction energy of the system, by the exchange mechanism, because it increases the total number of particles of the same spin. The noninteracting energy cost of this transfer may be tuned using the Zeeman energy until it is outweighed by the reduction in the total interaction energy. The system then undergoes a first-order paramagnetic-ferromagnetic phase transition (in which the magnetization increases discontinuously) *before* the energy gap to single-particle excitations reaches zero [3,4]. This effect preempts a spin-density wave instability which might be expected from consideration of the magnetoexciton [3,5].

The variation of the Zeeman energy at fixed ν may be realized experimentally by tilting the sample with respect to the magnetic field, because the cyclotron energy $\hbar\omega_C$ depends on $B_\perp$, while the spin splitting depends on all components of B. Despite clear theoretical predictions, an unambiguous experimental observation of the phase transition has not been reported. One reason is that disorder, which has been shown to inhibit the transition [4], confines the search to small even filling factors. The low g factor and effective mass of electrons in GaAs/AlGaAs heterostructures, which represent the cleanest experimental system currently available, require very large tilt angles (from perpendicular) and therefore prohibitively high magnetic fields. The high effective

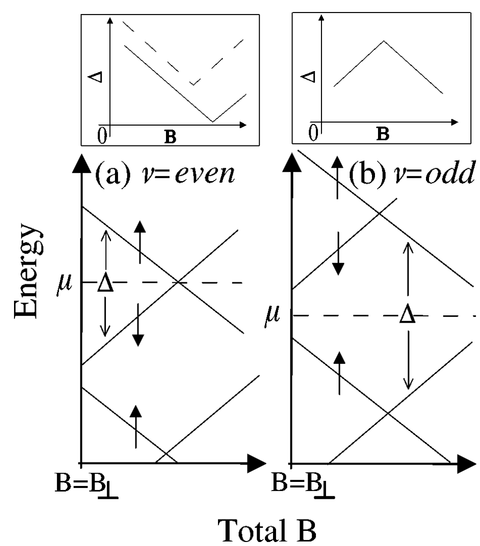

FIG. 1. (a) Plot of the Landau levels against B for fixed $B_\perp$ at $\nu =$ even, in the absence of interactions. The chemical potential μ is represented by the dashed line. The arrows represent the spins of the levels. Inset: the variation of gap Δ with B, for the noninteracting (solid line) and interacting (dashed line) cases. (b) Same as (a) but for odd filling factors.

mass and g factor of holes in GaAs eliminate this problem while retaining a low level of disorder.

In the quantum Hall regime, the energy gaps which occur at integral filling factors lead to zeros in the diagonal resistivity ρ_{xx}. Experimental evidence that the minimum in ρ_{xx} at $\nu = 2$ failed to vanish as a function of E_Z, indicating that the energy gap did not fall to zero, has been presented for electrons in the InGaAs/InP system [6]. Although the authors were unable to measure the magnitude of the gap, it was shown that the higher even filling factor energy gaps did vanish and that samples of low mobility did not show the effect.

Although such evidence is a necessary consequence of the phase transition, it does not demonstrate its occurrence. In particular, the failure of two opposite-spin Landau levels to cross can arise because of mixing between them, for example, by the spin-orbit interaction [7]. This explanation is also consistent with the disappearance of the energy gap in low quality samples and at higher filling factors.

In this Letter, we report on the energy gaps, determined from transport measurements, at fixed even and odd filling factors in a very high mobility two-dimensional hole gas (2DHG) as the total magnetic field is varied. At $\nu = 4$, we shall argue that the dependence of the gap on B is entirely inconsistent with an anticrossing explanation and that the evidence strongly favors the first-order phase transition described above. The dependence of the odd gaps, which are not predicted to exhibit a phase transition, can, however, be explained by a picture in which the Landau levels do, in fact, anticross. We extract coefficients from these data which characterize the exchange interactions and show that they are consistent with the size of the energy gap at which the phase transition occurs for $\nu = 4$.

The two-dimensional hole gas used in this study was grown by molecular beam epitaxy on the (311)A surface of a semi-insulating GaAs substrate. The hole gas is confined to a 200 Å quantum well that is symmetrically modulation doped using Si acceptors. The confinement potential is thought to be close to symmetric, because the low-field Shubnikov–de Haas (S-dH) oscillations exhibit no beating [8]. Devices were fabricated into Hall bars using standard lithographic techniques, and were contacted using annealed AuBe Ohmic contacts. Hall bars of this wafer have a carrier concentration of $1.8 \times 10^{15}\ \mathrm{m}^{-2}$ and a very high mobility in excess of $120\ \mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$ at temperatures below 1.5 K.

Measurements were performed in a ^{3}He cryostat (with a base temperature of 280 mK) designed for the *in situ* rotation of the sample with respect to the magnetic field. The angle of tilt was determined from the periodicity of the Shubnikov–de Haas oscillations, which in an ideal 2DS depends on $B_\perp$ only. Real samples have a thickness characterized by t, which is half the width of the component of the wave function perpendicular to the plane. The system remains essentially 2D for $\sqrt{\hbar/eB} \geq t$. This condition holds for most of the data presented here, although at 12 T, the two terms become comparable. The temperature of the sample was measured using a calibrated Ge thermometer at zero magnetic field and a carbon resistor, of known magnetoresistance, at nonzero B. Standard four-terminal ac measurements were performed using a current of 35 nA to maximize the measured signal, without causing heating effects.

At integral filling factors, the chemical potential μ lies midway between the peaks in the density of states, which are separated by an energy Δ_ν [see Figs. 1(a) and 1(b)]. The energy gap from μ to the upper peak in the density of states may be measured using an activation energy method in which it is assumed that $\rho_{xx} \propto \exp(-\Delta_\nu/2k_BT)$. This relationship strictly holds only when $k_BT \ll \Delta_\nu$, disorder is small, and $\rho_{xx} \ll \rho_{xy}$.

Figure 2 is a plot of $\ln \rho_{xx}$ versus $1/T$, for filling factors 3, 4, and 5 at a particular angle of tilt. The data presented in Fig. 2 were obtained by sweeping the magnetic field back and forth over a particular integral filling factor, while the sample temperature was reduced slowly. Data were also taken by sweeping the magnetic field at stable temperatures; no systematic difference was detected between the two sets of data. The measurement of resistance below 1 Ω is hampered by noise and voltage offsets which can distort the data. The solid lines in Fig. 2 represent linear least-squares fits to the portions of the curves between $-6 < \ln(\rho_{xx}/\mathrm{k}\Omega) < -3$. The data show that the S-dH minima exhibit clear activated behavior, and allow us to deduce $\Delta_\nu/2$ from the gradients of the graphs.

The dependence on B expected for the energy gaps at even and odd filling factors, in a system lacking both

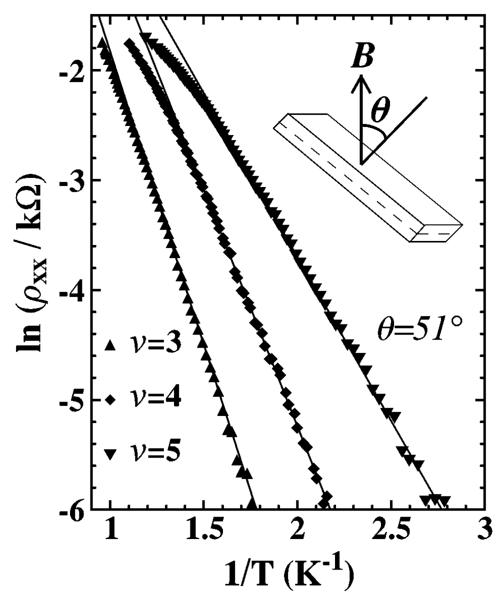

FIG. 2. $\ln \rho_{xx}$ versus $1/T$ measured at filling factors $\nu = 3, 4, 5$ and at an angle of $\theta = 51°$. The low temperature portions of the curves have been least squares fitted to straight lines, which are plotted. Inset is a diagram indicating the sample's orientation to the magnetic field.

interactions and Landau level mixing, is represented by the solid lines in the insets to Figs. 1(a) and 1(b). The measured energy gaps Δ_ν for even and odd filling factors are plotted against *total* magnetic field B in Figs. 3(a) and 3(b), respectively. The dependence of Δ_4 on total B exhibits a clear turning point at a critical field $B_c = 9.13$ T and a critical gap $\Delta_C = 3.5 \pm 0.1$ K. The curve is close to linear on either side of the transition, although the gradients are different. Limited data are available for $\nu = 6$; these are plotted using open symbols in Fig. 3(a) but are less reliable because their gaps are small compared to the cryostat base temperature. In contrast, the odd energy gaps exhibit highly nonlinear characteristics. Δ_5 increases at low B, and exhibits a smooth turning point at 5.7 T. Δ_3 shows qualitatively similar behavior to Δ_5, although the data are truncated by the limited magnetic field available, and the initial gradient is much greater.

It will be argued that the distinct differences between the odd and even filling factor data may be explained by the occurrence of a paramagnetic-ferromagnetic phase transition at the sharp turning point in the $\nu = 4$ plot. Before this, the peculiarities of the band structure of the 2DHG are addressed. In general, the g factor of a 2DHG is anisotropic, because the holes are subject to a well-defined axis of symmetry perpendicular to their plane of confinement. At the Γ point of the Brillouin zone, there exists very little mixing of the light and heavy holes and the parallel g factor $g_{||}$ is zero [9]. However,

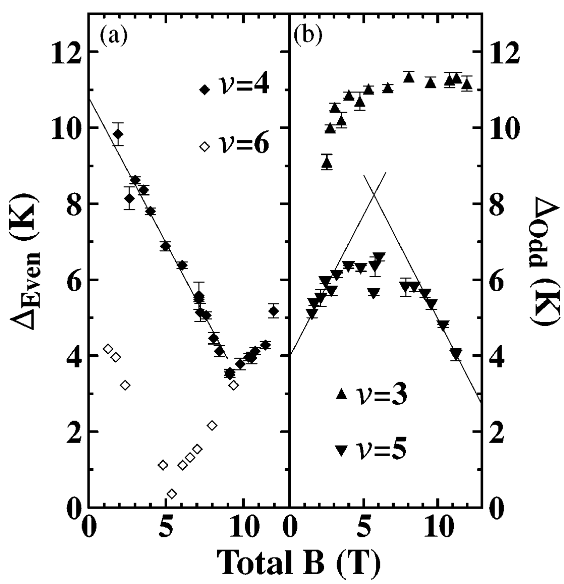

FIG. 3. (a) Filled symbols show the energy gaps measured at $\nu = 4$ as a function of the total magnetic field applied to the sample. Δ_4 drops linearly to a gap of 3.5 ± 0.1 K at its turning point at 9.13 T. The solid line is a fit to the Δ_4 data below 7.2 T with the slope corresponding to a g factor of 1.1. Open symbols show similar data for $\nu = 6$. (b) The energy gaps Δ_3 and Δ_5 at odd filling factors. Δ_5 exhibits distinct curvature near to its turning point at 5.7 T. The solid lines have the same gradients as the line in (a), and demonstrate that away from 6 T the data are consistent with the g factor measured at $\nu = 4$.

the nonzero Fermi wave vector of the hole gas and the application of the magnetic field mix the light and heavy hole states. The Zeeman energy then takes the form $E_Z = \sqrt{(g_{||}^2 B_{||}^2 + g_\perp^2 B_\perp^2)}$, which is only linear in total B in the isotropic case, $g_{||} = g_\perp$.

The lack of curvature at low B in the $\nu = 4$ data indicates that the g factor is isotropic and therefore that E_Z depends on the total magnetic field. The solid line superimposed upon the $\nu = 4$ data in Fig. 3(a) is a linear best fit line for $B < 7$ T. Its gradient suggests that the g factor of the system is 1.1 ± 0.05. The solid straight lines which are superimposed upon the Δ_5 data in Fig. 3(b), at high and low B, have the same gradient and demonstrate that the g factors at $\nu = 4$ and 5 are similar. However, the Δ_3 dependence cannot be described using this g factor. The possibility that the large slope is a consequence of strong anisotropy is rejected because the $\nu = 4$ data are linear.

A pair of Landau levels that are mixed by some interaction do not cross. In simple perturbation theory, the levels anticross according to an equation of the form $\delta = \sqrt{\lambda^2(B - B_0)^2 + \delta_0^2}$, where δ represents the energy gap between the interacting levels. The essential properties of this equation are that, for $\lambda(B - B_0) \gg \delta_0$, the variation is linear and its extrapolation passes through $\delta = 0$ at $B = B_0$. In the vicinity of this anticrossing, however, the dependence is expected to be curved.

The Δ_4 dependence presented in Fig. 3(a) is not accounted for by the anticrossing picture. The extrapolation of the low-field data reaches the $\Delta_4 = 0$ axis at a much higher field than the turning point. Furthermore, the dependence is close to linear for all B. Thus we argue that at the sharp turning point of $\nu = 4$, a paramagnetic-ferromagnetic phase transition occurs in which charge transfers between approaching Landau levels.

Figure 1(b) shows that at $\nu = 5$, the approaching levels are either both full (for energies less than μ) or both empty (energy more than μ). Thus, a phase transition is not possible and the levels will anticross through mixing. The curvature evident for the $\nu = 5$ data in Fig. 3(b) confirms this anticrossing interpretation. The maximum value of Δ is close to $\hbar\omega_C = eB_\perp/m^*$ where m^* is the hole effective mass which takes values between $0.2m_0$ and $0.3m_0$ in this system. The energy difference between the point of intersection of the two solid lines in Fig. 3(b) and the actual data at that field demonstrates that the anticrossing gap δ_0 is approximately 1.6 ± 0.1 K, and is more than a factor of 2 smaller than Δ_C. The source of this Landau level mixing is likely to be the spin-orbit interaction, which is of particular importance in the valence band.

In the limit of strong magnetic field ($\hbar\omega_C \gg E_X$), the characteristic energy E_X of the exchange interaction at integer filling factors is $e^2/4\pi\epsilon\epsilon_0 l_B$, where ϵ is the dielectric constant and $l_B = \sqrt{(\hbar/eB_\perp)}$ is the magnetic length. In this regime there is no mixing of the Landau levels by the Coulomb interaction, which is therefore unscreened. However, for $\nu > 1$, the 2DHG enters a different regime because of the large effective mass, which is not well defined but takes values between $0.2m_0$ and $0.3m_0$. Using these values, we find E_X exceeds $\hbar\omega_C$ by a factor of 5 to 8. Consequently, strong Landau level mixing by the Coulomb interaction is expected, which should screen the interparticle Coulomb potential [10].

The size of the screened exchange interaction, and its consistency with the value of Δ_C measured at $\nu = 4$, may be extracted from the data at odd filling factors. In the strong magnetic field limit, the Hartree-Fock energy ϵ_N^σ of a particle may be written as

$$\epsilon_N^\sigma = \hbar\omega_c(N + 1/2) + \sigma g\mu_B B - \Sigma_M A_{NM} n_M^\sigma, \quad (1)$$

where N is the Landau level index and σ is the spin [1]. In the third term, n_M^σ represents the occupancy of Landau level (M, σ). The coefficients A_{NM} represent the exchange corrections due to interactions between particles of the same spin in Landau levels N and M. In the strong field limit, they take values calculated elsewhere [1], the most familiar of which is $A_{00} = \sqrt{\pi/2}\,E_X$. To account for the strong Landau level mixing in the 2DHG, we generalize Eq. (1) by treating the A_{NM} as phenomenological parameters to be deduced from the data.

The magnetic field at which the phase transition occurs may be determined by equating the total energy of the system in the paramagnetic and ferromagnetic phases. The ferromagnetic phase is obtained by transferring all the particles with spin down from Landau level $N = 1$ to the spin-up Landau level $N = 2$. The energy gap at $\nu = 4$ at this field can be shown to be $(A_{11} + A_{22})/2$.

The predicted dependence of the energy gap on magnetic field is included in the inset to Fig. 1(a) using a dashed line. The effect of disorder is to increase the downward curvature in the vicinity of the transition. Little evidence of this is observed near to $\Delta_4 = \Delta_C$ in Fig. 3(a), probably because Δ_C is much larger than the expected disorder width of the Landau levels which is estimated to be less than 1 K [11].

Manipulation of Eq. (1) demonstrates that the energy gap between a pair of Landau levels at odd filling factors, in the limit of zero disorder and neglecting anticrossing with other higher energy levels, is given by $\Delta_{2M+1} = g\mu_B B + A_{MM}$. A_{MM} may be estimated experimentally by extrapolating Δ vs B to $B = 0$. From the $\nu = 5$ data, $A_{22} = 4.0 \pm 0.2$ K. Unfortunately the same extrapolation to A_{11} is impossible for the $\nu = 3$ curve, because of the particularly steep gradient at low fields.

By scaling the measured value of A_{22} with l_B to account for the dependence of E_X on $B_\perp$, and using the theoretical result that A_{22} and A_{11} are close in magnitude [12], the energy gap at the transition point at $\nu = 4$ is predicted to be 4.5 ± 0.3 K. Given the approximations made at all stages of the theory, the agreement with the directly measured value $\Delta_C = 3.5 \pm 0.1$ K is good. An indication that the magnitude of the exchange energy is reasonable may be found by including screening in the Thomas-Fermi approximation. In this simple approximation, the coefficients A_{NM} are reduced from their unscreened values by an order of magnitude, so that they are comparable with, but larger than, the measured gap. The inclusion of the effect of the finite width of the system, which softens the Coulomb interaction, should lower the calculated energy gap further.

The difference in gradients measured before and after the transition at $\nu = 4$ and the observation that the initial slope at $\nu = 3$ is much larger than the g factor consistent with the data at $\nu = 4$ and 5 are not explained by this mean-field picture. A more complicated theory, possibly incorporating spin textures, may be required.

In conclusion, we have measured energy gaps at even and odd filling factors in a very strongly interacting fermion liquid. We observe evidence of a first-order phase transition from a paramagnetic to a ferromagnetic state. We can explain most features of the transition using a phenomenological model, the parameters of which are estimated from the odd energy gaps. These observations highlight the utility of low-dimensional hole systems for the study of interactions and should stimulate further work to quantify the predictions in the limit of large interaction energies.

We thank C. H. W. Barnes for useful discussions. This work was supported by the EPSRC. One of us (D. A. R.) acknowledges support from the Toshiba Cambridge Research Centre.

[1] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1974).
[2] R.J. Nicholas, R.J. Haug, K.v. Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).
[3] G.F. Giuliani and J.J. Quinn, Phys. Rev. B **31**, 6228 (1985); G.F. Giuliani and J.J. Quinn, Solid State Commun. **54**, 1013 (1985); G.F. Giuliani and J.J. Quinn, Surf. Sci. **170**, 316 (1986).
[4] S. Yarlagadda, Phys. Rev. B **44**, 13 101 (1991).
[5] C. Kallin and B.I. Halperin, Phys. Rev. B **30**, 5655 (1984). See also I.V. Lerner and Yu.E. Lozovik, Zh. Eksp. Teor. Fiz. **80**, 1488 (1981) [Sov. Phys. JETP **53**, 763 (1981)].
[6] S. Koch *et al.,* Phys. Rev. B **47**, 4048 (1993).
[7] V.I. Fal'ko, Phys. Rev. B **46**, 4320 (1992).
[8] J.P. Eisenstein *et al.,* Phys. Rev. Lett. **53**, 2579 (1984).
[9] H.W. van Kesteren *et al.,* Phys. Rev. B **41**, 5283 (1990).
[10] I.L. Aleiner and L.I. Glazman, Phys. Rev. B **52**, 11 296 (1995).
[11] Shubnikov–de-Haas oscillations may be discerned below magnetic fields of 0.2 T at 50 mK.
[12] In the limit of no mixing, A_{11} and A_{22} can be shown to differ by about 16%.