Excess Quantum Noise due to Nonorthogonal Polarization Modes

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(Received 7 August 1997)

We show that the quantum-limited linewidth of a laser can be enhanced when the polarization eigenmodes of the laser resonator are nonorthogonal. For the theoretical description of this phenomenon we introduce a simple coupled two-mode model. Experimentally, we observed an enhancement of the quantum noise by a factor of 60 in a He-Xe gas laser. [S0031-9007(97)04703-0]

PACS numbers: 42.50.Lc, 42.55.Lt, 42.60.Da

The coherence of a laser is fundamentally limited by spontaneous emission into the lasing mode. Since the introduction of the fundamental limit to the laser linewidth by Schawlow and Townes [1] several refinements and extensions have been found [2]. One of the most intriguing modifications to the original Schawlow-Townes formula is the so-called excess-noise factor, denoted by K, first proposed by Petermann [3] in 1979 for gain-guided lasers. Siegman [4] showed that excess noise is a general feature of open resonators, directly related to the nonorthogonality of the laser eigenmodes in such resonators. More specifically, it was shown that in unstable resonators the K factor can increase the effect of the spontaneous-emission noise in the laser mode by orders of magnitude. Only recently some of the key issues have received a firm experimental basis. In particular, a large value ($\approx$330) of K in a hard-edged unstable resonator [5] and resonant behavior of K as a function of mirror size [6] have been observed. In a recent paper [7], some of us have speculated that for very large K factors all spontaneous emission would be channeled into the laser mode, thus presenting an alternative approach towards the long-sought thresholdless single-mode laser. Therefore, experimental realization of a laser with a very large K factor is of great importance.

So far, only the *spatial* nonorthogonality of the resonator eigenmodes (transverse and longitudinal) has been considered as a source of excess quantum noise. In this Letter we consider the *polarization* degrees of freedom (in a stable-cavity laser). We show, both theoretically and experimentally, that nonorthogonal polarization eigenmodes can lead to a substantial excess-noise factor, which we denote by K_{pol}. Since large polarization excess noise arises in a simple two-mode description, it may provide further insight into the physical origin of excess noise. Also, by combining spatial and polarization mode nonorthogonality one may hope to obtain even larger K factors. It is to be expected that within the paraxial approximation the overall K factor of a laser factorizes in a spatial K and a polarization K. Thus our work opens the way to even larger K factors, by exploring near-degeneracy of polarization eigenmodes in unstable-cavity lasers.

The polarization state of a laser is in general determined by the anisotropies of the passive cavity and of the gain medium. The linear part of these anisotropies is most conveniently described using the Jones vector formalism [8], in which the polarization dynamics takes the form of a coupled two-mode problem:

$$\frac{d}{dt}\begin{pmatrix} E_+ \\ E_- \end{pmatrix} = iM\begin{pmatrix} E_+ \\ E_- \end{pmatrix} + \begin{pmatrix} \mathcal{L}_+ \\ \mathcal{L}_- \end{pmatrix}, \tag{1}$$

where E_+ and E_- are the (complex) amplitudes of the orthogonal basis vectors which are here chosen to be σ_+ and σ_- for concreteness, and M is a 2×2 Jones matrix describing the laser resonator. A Langevin noise source $\mathcal{L}$ has been added to take into account the quantum noise due to spontaneous emission.

Excess noise now appears in a natural way when Eq. (1) is transformed from the (orthogonal) σ_+, σ_- basis to the eigenmode basis of the resonator matrix M. If the eigenmodes are nonorthogonal, the transformation leads to enhancement of the strength of the transformed Langevin noise into the separate eigenmodes, and to mutual correlations between these two noise sources [9,10]. The enhanced influence of the noise is expressed in terms of the increased diffusion rate of the eigenmode amplitudes, by a factor K. As in the case of the transverse [4] and longitudinal [11] excess-noise factor, the polarization excess-noise factor K_{pol} of an eigenmode $|e_i\rangle$ of M can generally be expressed in terms of the overlap between this eigenmode and the corresponding adjoint mode $|v_i\rangle$, as $K_i = |\langle v_i|e_i\rangle|^{-2}$. Here we have employed the Dirac bra-ket notation to bring out the complete analogy with the usual expressions for transverse and longitudinal excess-noise factors (we have taken $|e_i\rangle$ and $|v_i\rangle$ to be normalized to unity, cf. [12]). For a two-mode system the adjoint modes ($|v_i\rangle$) are easily constructed: the adjoint mode belonging to the first eigenmode corresponds to the polarization state that is orthogonal to the second eigenmode and vice versa. In fact, the excess-noise factors of the two modes are identical, and can be easily rewritten in terms of the eigenmodes only (i.e., without explicit reference to the adjoint modes) as

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$$K = \frac{1}{1 - |\langle e_1|e_2\rangle|^2}. \tag{2}$$

This equation shows explicitly that excess noise only appears when the eigenmodes are nonorthogonal (otherwise $\langle e_1|e_2\rangle = 0$ and $K = 1$), and that K is highest when the two modes are nearly identical ($\langle e_1|e_2\rangle \approx 1$).

A simple way to create nearly identical polarization eigenmodes is to have M take the form [13]

$$M = \begin{pmatrix} \Omega & iA \\ iA & -\Omega \end{pmatrix}, \tag{3}$$

where 2Ω is the frequency splitting of the σ_+ and σ_- modes and iA represents a (dissipative) coupling between the two. The frequencies and damping rates of the eigenmodes are given, respectively, by the real and imaginary parts of the eigenvalues $\lambda_\pm$ of the matrix M,

$$\lambda_\pm = \pm\sqrt{\Omega^2 - A^2} \equiv \pm\omega/2. \tag{4}$$

These are shown in Fig. 1. Two regimes can be distinguished: (i) the lock band $|\Omega| < |A|$, where the eigenmodes are linearly polarized, and have the same frequency but different damping, (ii) outside the lock band $|\Omega| > |A|$, where the eigenmodes are elliptically polarized and have different frequencies but equal losses. When, for instance, $|\Omega| \downarrow |A|$, outside the lock band, the eigenmodes become more and more identical; they evolve from σ_+, σ_- (when $|\Omega| \gg |A|$) towards the same linear polarization (when $|\Omega| = |A|$). As the overlap of the eigenmodes is given by $|\langle e_1|e_2\rangle| = |A/\Omega|$, outside the lock band the polarization excess-noise factor as given by Eq. (2) becomes

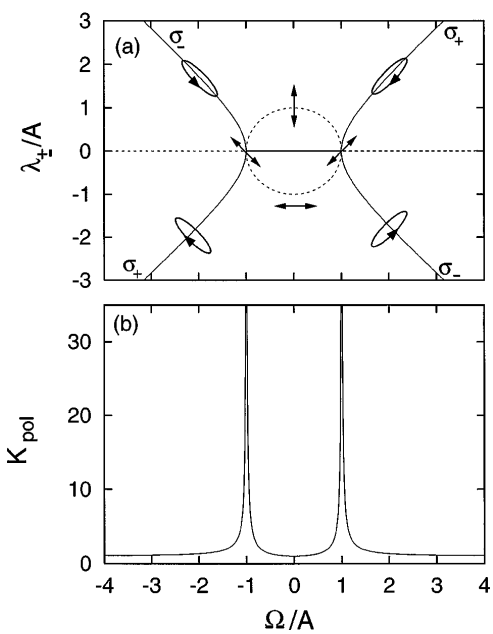

FIG. 1. (a) Frequencies (solid lines) and net gains (dashed lines) of the eigenmodes, as a function of Ω/A. The polarization states of the eigenmodes are also indicated. (b) The polarization excess-noise factor $K_{\rm pol}$, as derived from the nonorthogonality of the polarization eigenmodes.

$$K_{\rm pol} = \frac{\Omega^2}{\Omega^2 - A^2}. \tag{5}$$

Inside the lock band $K_{\rm pol}$ is obtained from Eq. (5) by exchanging the role of Ω and A. The resulting polarization excess-noise factor is shown in Fig. 1(b). The key point is that as $\Omega \to A$ the polarization eigenmodes become nearly identical, and the excess-noise factor diverges.

The linear model considered above describes essentially the situation at threshold; to describe a laser above threshold, the linear analysis needs to be supplemented with the nonlinearities introduced by the gain medium. Although the essential features of the analysis discussed above are preserved, it turns out that saturation somewhat modifies the excess-noise characteristics of the laser, depending on the specific case considered. For concreteness, we limit the discussion below to the case that is relevant for our experiments; namely, that of a gas laser where the σ_+ and the σ_- modes are weakly coupled by the gain saturation, leading to linear polarization of the laser field [14]. We also assume that the laser is operated outside the lock band ($|\Omega| > |A|$). The dynamics reduces then to the evolution of the angle of linear polarization θ, which is governed by the Adler equation [15]:

$$\dot{\theta} = \Omega - A\sin(2\theta). \tag{6}$$

The polarization angle θ will rotate at a frequency $\omega/2$ [Eq. (4)]. For $|\Omega| \gg |A|$ the rotation will be harmonic, but this rotation will become increasingly anharmonic as the lock band is approached. Including the Langevin noise terms of Eq. (1) leads to a Langevin noise source in Eq. (6), with a strength $\propto I^{-1/2}$. This is the source of the well-known inverse-power dependence of the diffusion rate of the phase, that is at the heart of the Schawlow-Townes formula [1,2]. Straightforward analysis shows that the excess noise that arises due to the nonorthogonality of the eigenmodes in the linear model [Eq. (2)], is mapped onto an enhancement of the sensitivity of θ to noise, due to the anharmonicity implied by Eq. (6) [16]. We find that the diffusion rate of the angle θ is enhanced by a factor

$$K_{\rm pol} = \frac{\Omega^2 + A^2/2}{\Omega^2 - A^2}. \tag{7}$$

The problem of diffusion of the phase in the Adler equation due to a Langevin noise term has in fact been analyzed in great detail in a series of papers by Cresser *et al.* [17], in the context of quantum noise in laser gyros. In the limit of weak noise the same result as in Eq. (7) was found. The difference between Eq. (7) and the excess-noise factor from the linear model, as given by Eq. (5), is a factor $1 + A^2/2\Omega^2$, which is in the order of unity (for the case $|\Omega| > |A|$ which we consider here), so the excess-noise factor is expected to be only slightly modified by saturation.

We have experimentally verified the polarization excess-noise factor predicted above. The setup was very

similar to that used previously to determine quantum-limited linewidths [15,18]. A He-Xe gas laser was operated on line center of the 3.51-μm Xe transition, the capillary being filled with a He pressure of 1.2 kPa and a Xe pressure of a few Pa. The discharge was maintained by rf excitation. The capillary was terminated by two quartz plates with a single-pass transmission of 92% each. The 8.6-cm long stable-cavity laser consisted of a concave dielectric mirror (32% reflection, radius of curvature 30 cm) and a flat, highly reflective gold mirror placed around the capillary. Coils around the laser cavity produced an axial magnetic field of $\lesssim$1 mT, inducing a circular birefringence through the Faraday effect in the gain medium. This controls the detuning of the two modes (Ω). The dissipative coupling (iA) of the σ_+ and σ_- modes was produced by an intracavity 2-mm-thick CaF_2 plate, which introduces linear dichroism. This plate, with its normal at roughly 35° from the resonator axis, created a loss difference between the s- and p-polarized light ($\approx$15% per round trip). The output of the laser was sent through a polarizer, detected on a cryogenic InSb photo diode with a bandwidth of 12 MHz, and monitored on an rf-spectrum analyzer.

As mentioned above, the output of the laser is linearly polarized, due to weak coupling of the σ_+ and σ_- modes by the gain saturation. When operated outside the lock band ($|\Omega| > |A|$), the angle of polarization will rotate as described by Eq. (6) at a fundamental frequency $\omega/2$. In the experiment this is detected after the polarizer as an intensity modulation at a fundamental frequency ω, leading to spectra as in Fig. 2. For large magnetic fields ($\Omega \gg A$) the rotation is harmonic and the spectrum consists of a single Lorentzian [Fig. 2(a)], while close to the lock point ($\Omega \gtrsim A$) the rotation becomes anharmonic [Fig. 2(b)], and higher harmonics appear in the spectrum. Very close to the lock point all the harmonics melt together [Fig. 2(c)] and no information about the first harmonic in the spectrum can be obtained.

In Fig. 3 the fundamental rotation frequency ω as determined from the rf spectra is plotted as a function of the current generating the axial magnetic field. The solid line is a fit to the data, from which all the relevant parameters under operating conditions can be determined, namely, the value of the dichroism present (A, typically a few MHz), the proportionality between the applied current and the Faraday rotation, and an offset in the magnetic field caused by stray magnetic fields.

The widths of the peaks in the spectra are proportional to the quantum-noise strength [6,7,15,18]. We have measured the fundamental laser linewidth (determined from the width of the first harmonic in the spectra) as a function of the axial magnetic field [19]. For $\Omega \gg A$ (orthogonal polarization eigenmodes, $K_{\mathrm{pol}} = 1$) we recover the usual Schawlow-Townes linewidth [2,15,18]. By dividing all the measured linewidths by the asymptotic linewidth ($\Omega \gg A$) we find the polarization excess-noise factor, K_{pol}. This is plotted in Fig. 4 as function of the Faraday rotation Ω. The experimental data clearly show a dramatic increase in the linewidth as the lock point is approached (an enhancement of more than a factor of 60), in good agreement with both the linear theory based on mode nonorthogonality [solid line, Eq. (5)], and the enhancement expected on the basis of the Adler equation [dashed line, Eq. (7)]. We cannot distinguish between the quality of the two predictions. Note that the theoretical

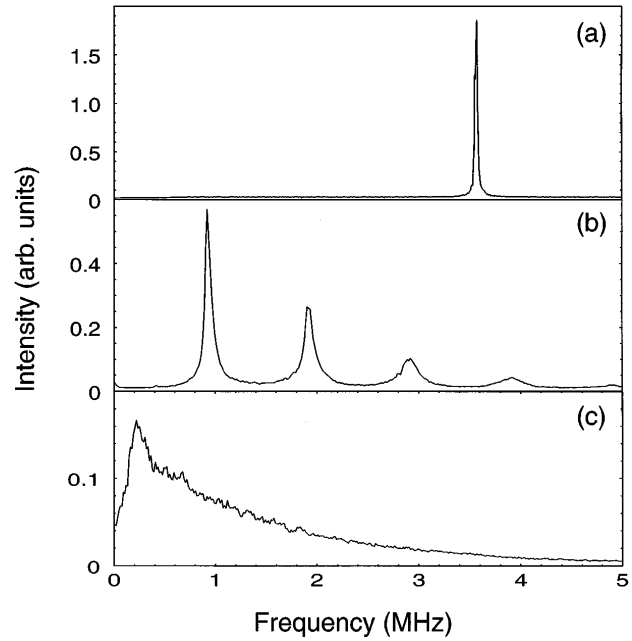

FIG. 2. Measured rf polarization-rotation spectra as the magnetic field is progressively reduced. (a) For high magnetic fields the spectrum contains a Lorentzian peak at the fundamental frequency ω, with no harmonics visible. (b) As the magnetic field is lowered and the lock point is approached, the rotation becomes increasingly anharmonic, and the widths increase, due to the enhanced quantum noise. (c) Very close to the lock point the harmonics are not separated anymore.

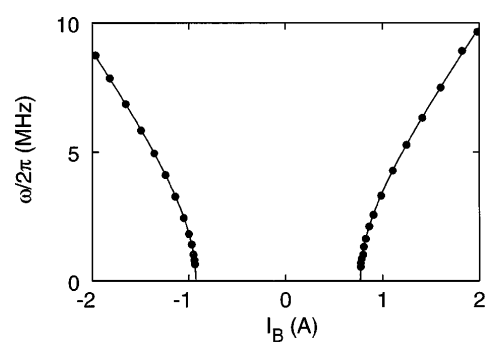

FIG. 3. Fundamental rotation frequency ω of the polarization as a function of the current (I_B) generating the axial magnetic field. Points: measured frequencies, as determined from spectra such as in Fig. 2. The solid line is a fit to the data from which the linear dichroism (A) and the Faraday rotation (Ω) can be determined (see text).

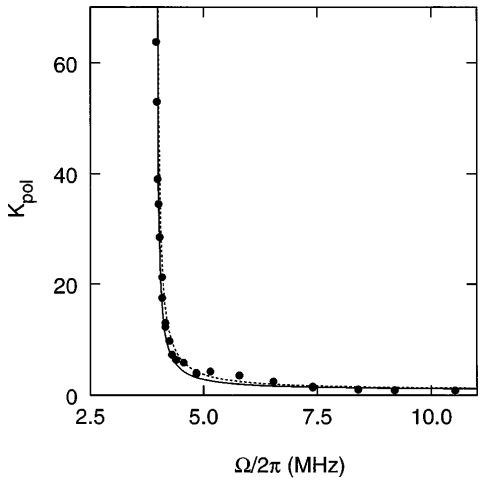

FIG. 4. Measurement of the polarization excess-noise factor, which shows strong enhancement of the quantum noise close to the lock point. Points: measured K_{pol} as a function of the Faraday rotation Ω. The solid curve is the theoretical expectation from mode nonorthogonality theory [Eq. (5)], the dashed curve displays the expected enhancement based on the Adler equation; see Eq. (7).

curves were drawn without using any fit parameter, all the information needed for calculating K_{pol} was obtained from the fit in Fig. 3.

It is surprising that the mode-nonorthogonality theory based only on the linear model [Eq. (1)], gives such a good quantitative description of the excess quantum noise, despite the fact that in our experiment the nonlinearity implied by the gain saturation plays an important role in the polarization behavior.

In conclusion, we demonstrated the existence of a polarization excess-noise factor, by exploring the near-degeneracy of polarization eigenstates defined by a circular birefringence and a linear dichroism. A quantum-linewidth enhancement of up to a factor of 60 was observed. It was found that outside the lock band the saturation only slightly modifies the excess-noise behavior. It is interesting to note that, as the lock point is approached from outside the lock band, the excess-noise factor increases dramatically, while the losses of the eigenmodes remain constant [see Fig. 1(a)]. This shows explicitly that there is no direct, simple relation between eigenmode losses and excess noise.

We acknowledge support of the Stichting voor Fundamenteel Onderzoek der Materie (FOM) which is supported by NWO. We also acknowledge support from the European Union under the TMR Contract No. ERB4061PL95-1021 (Microlasers and Cavity QED) and the ESPRIT Contract No. 20029 (ACQUIRE).

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