Collisional Instabilities in a Dusty Plasma with Recombination and Ion-Drift Effects

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Instabilities of dust acoustic waves in a plasma with a significant background pressure of neutrals have been investigated. A long wavelength mode is found to be unstable due to recombination of electrons and ions on the surface of dust particles. At short wavelengths, a dissipative instability driven by relative drift between ions and the dust particles is found to be important. Nonlinearly, the short wavelength modes lead to the formation of $K - dV$ solitons whereas the long wavelength end is dominated by modulational instabilities. [S0031-9007(97)03537-0]

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In recent years, the physics of dusty plasmas (i.e., plasmas with a significant population of charged dust particles) has attracted a great deal of attention [1] because of its potential application to problems in astrophysics and planetary physics [2,3], plasma processing of surfaces and materials [4], and its direct relevance to the physics of strongly coupled plasmas [5]. Experimental [6–9] and theoretical [10–13] work on collective oscillations in such plasmas has revealed the existence of a novel low-frequency mode, the so-called dust acoustic wave which is often driven to large amplitudes by the free-energy sources in the plasma. Most interpretations [11,12] of the observed excitation of dust-acoustic waves in these experiments rely on a collisionless inverse ion Landau damping mechanism. However, a closer examination of the experimental conditions reveals that the waves are often excited when there is a significant background pressure of neutrals (e.g., the ion-neutral collisional mean free path may be comparable to or even shorter than the typical wavelengths). Under these conditions the ions no longer behave as a Boltzmann fluid and the use of a collisionless theory is unjustified. It is therefore necessary to reexamine the dynamics of dust-acoustic instability in a plasma with a background pressure of neutrals; this is the objective of the present Letter. Such an investigation is particularly important because many astrophysical, planetary physics, and plasma processing situations do have significant neutral pressure backgrounds. In this Letter we demonstrate that the recombination of the background electrons and ions on the surfaces of dust particles and the momentum loss of ions to neutrals (in the presence of a relative ion-dust drift) act as new processes promoting the excitation of dust-acoustic instability in a collisional plasma.

The basic linearized equations for low-frequency dust-acoustic disturbances in an unmagnetized plasma with a significant pressure of neutrals may be written as

$$\frac{\partial}{\partial t}\tilde{n}_d + \nabla \cdot \tilde{v}_d = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}\tilde{v}_d = c_d^2 \nabla \tilde{\varphi}\,, \tag{2}$$

$$\tilde{n}_e = \tilde{\varphi}\,, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + V_o \cdot \nabla\right)\tilde{n}_i + \nabla \cdot \tilde{v}_i = -\nu_R \tilde{n}_d\,, \tag{4}$$

$$\tilde{v}_i = -\frac{c_i^2}{\nu_{in}} \nabla(\tau\tilde{\varphi} + \tilde{n}_i)\,, \tag{5}$$

$$\frac{n_{io}}{n_{eo}}\tilde{n}_i = \tilde{n}_e + Z_d \frac{n_{do}}{n_{eo}}\tilde{n}_d\,, \tag{6}$$

where we have written the equations in a frame moving with the equilibrium speed of dust particles, $\tilde{\varphi} = e\tilde{\phi}/T_e$, $\tilde{n}_j = (\tilde{n}_j/n_{jo})$ and $\tilde{v}_j$ refer to density and/or velocity perturbations in species *j* (*e* electron, *i* ion, and *d* dust), n_{jo} is the mean density of species *j*, $\tau = T_e/T_i$, T_j is the mean temperature, Z_d is the charge on the dust particle (assumed constant; see discussion at the end), $c_i^2 = KT_i/m_i$, $c_d^2 = Z_d KT_e/m_d$, V_o is the relative drift between ions and dust grains, $\nu_{in} = n_n \sigma_{in} c_i$ is the ion-neutral collision frequency for momentum loss, n_n is the neutral density, σ_{in} is the collision cross section, and $\tilde{\nu}_R = \nu_R \tilde{n}_d$ is the perturbed recombination frequency of the background electrons and ions on the dust particle surfaces. Equations (1) and (2) are, respectively, the equations of continuity and motion for the cold charged dust fluid. Equation (3) is the Boltzmann electron fluid approximation. This is valid when $\frac{k^2 c_e^2}{\omega \nu_{en}} \gg 1$ (where $c_e^2 = \frac{kT_e}{m_e}$, ν_{en} is the electron-neutral collision rate) and $\tilde{T}_e = 0$. Equations (4) and (5) describe the ion fluid which is non-Boltzmann because of collisional effects and the mean relative ion-dust drift velocity $\vec{V}_o$. Equation (4) contains a sink term because of electron-ion recombination on the surface of the dust particles; it is assumed that the equilibrium sink term $-\nu_R n_{io} \equiv -\beta n_{do} \pi r_d^2 c_s n_{io}$ (r_d is the dust radius and β is a numerical factor of order unity taking account of enhancement of the collision cross section between negatively charged dust particles and positive ions due to electrostatic focusing effects [2]) is being balanced by an appropriate source (so that the equilibrium ion density is time independent) and that only its linearized normalized perturbation $-\tilde{\nu}_R = -\nu_R \tilde{n}_d$ survives in Eq. (4). Furthermore, it is also assumed that the ion temperature fluctuation $\tilde{T}_i = 0$, because $\omega < (m_i/m_n)\nu_{in}$.

In deriving Eq. (5), we have assumed $kV_o < \nu_{in}$. Finally, Eq. (6) is the quasineutrality condition.

Equations (1)–(6) may now be used to obtain the dispersion relation for dust acoustic waves. Assuming perturbations of the form $\exp(-i\omega t + i\underline{k} \cdot \underline{x})$ and following standard methods, we get the dispersion relation

$$\left(1 - \frac{k^2 c_{d*}^2}{\omega^2}\right)\left[1 - i\,\frac{\nu_{in}(\omega - kV_o)}{k^2 c_i^2}\right] + \left[\tau_* - \frac{\nu_{in}\nu_R}{\omega^2}\,\frac{c_d^2}{c_i^2}\,\frac{n_{io}}{n_{eo}}\right] = 0\,, \quad (7)$$

where $\tau^* = \tau n_{io}/n_{eo}$, $c_{d*}^2 = c_d^2 Z_d n_{do}/n_{eo}$. Writing the dispersion relation as $D(k,\omega) = 0$ and assuming that $\mathrm{Re}\,D \gg \mathrm{Im}\,D$ and $\omega = \omega_r + i\gamma$ with $\omega_r > \gamma$, we may write the solution

$$\omega_r^2 = (k^2 c_{d*}^2 + \alpha \nu_{in}\nu_R)/(1 + \tau^*)\,, \quad (8a)$$

$$\gamma \cong \frac{\nu_{in}}{2(1 + \tau^*)^2}\left(\frac{\alpha \nu_{in}\nu_R}{k^2 c_i^2} - \tau^*\,\frac{c_{d*}^2}{c_i^2}\right)\left(1 - \frac{kV_o}{\omega_r}\right), \quad (8b)$$

where $\alpha = (c_d^2 n_{io}/c_i^2 n_{eo})$. It is to be noted that the frequency becomes independent of the wave vector k at the long wavelength end. For $V_o = 0$, instability occurs only for long wavelengths with $\alpha\nu_R > \tau^* \frac{k^2 c_{d*}^2}{\nu_{in}}$; this is because the recombination induced growth must overpower the diffusive damping of the ion-density perturbations by ion-neutral collisional effects. For this range of wavelengths and neutral pressure, the inclusion of ion drift only weakens the recombination instability. For shorter wavelengths (with the above inequality reversed) instability occurs if the ion-dust relative drift V_o is finite and exceeds the phase velocity ω_r/k. This is because the dust acoustic wave is a negative energy wave in the ion frame in this case and therefore dissipative effects on the ion fluid lead to an instability of the waves.

We may now present a more complete physical description of these collisional instabilities. Multiplying Eq. (2) by $\tilde{v}_d^*$, integrating over all space, adding the complex conjugate, and doing some partial integrations, we get the energy conservation equation

$$\frac{\partial}{\partial t}\int [|\tilde{v}_d|^2 + c_{d*}^2|\tilde{n}_d|^2]\,d^3r = c_d^2\,\frac{n_{io}}{n_{eo}}\int \tilde{n}_i(\nabla \cdot \tilde{v}_d^*)\,d^3r + \text{c.c.} \quad (9)$$

The two terms in the integral on the left side are, respectively, the normalized kinetic energy and thermal energy terms associated with the dust acoustic wave. The right hand side thus describes the basic source or sink of this wave energy. We may now use the linearized Fourier mode description to rewrite the right side as (if $\gamma \ll \omega_R$)

$$\int d^3k|\tilde{\varphi}_k|^2\left(\nu_{in}\nu_R - \omega_r^2\,\frac{c_i^2}{c_d^2}\,\tau\right)\left(\frac{k^2 c_d^2}{\omega_r^2}\right)^2\left[\frac{\nu_{in}\omega_r(\omega_r - \underline{k} \cdot \underline{V}_o)}{\nu_{in}^2(\omega_r - k \cdot V_o)^2 + k^4 c_i^4}\right]. \quad (10)$$

As stated earlier, this expression leads to wave excitation either when $V_o = 0$ and ν_R terms dominate (i.e., recombination driven instability at long wavelengths) or when $\underline{k} \cdot \underline{V}_o > \omega_r$ and the conventional wave dissipation terms lead to growth because the waves have negative energy in the moving ion frame. Fundamentally, both instabilities can be traced to a modified phase relationship between $\tilde{n}_i$ and $\tilde{\varphi}$ in a collisional plasma, viz,

$$\tilde{n}_i = -\tilde{\varphi}\left(\tau - \frac{k^2 c_d^2}{\omega^2}\,\frac{\nu_{in}\nu_R}{k^2 c_i^2}\right)\Big/\left[1 - i\,\frac{\nu_{in}(\omega - \underline{k} \cdot \underline{V}_o)}{k^2 c_i^2}\right], \quad (11)$$

which permits a positive feedback between the dust acoustic wave and the ion fluid. The short wavelength instability with $V_o \neq 0$ extracts free energy from the relative drift between the ion and dust fluids. The long wavelength recombination instability arises because the introduction of dust has a tendency to extinguish the basic plasma discharge through surface recombination effects; thus the homogeneous steady state is prone to localized quenching at dust compressions by an interplay between dust-acoustic disturbances and recombination phenomena, when they have similar time scales.

For a general numerical investigation of the dispersion relation Eq. (7), we express it as a cubic, viz,

$$-\frac{ib}{k^2}\,\hat{\omega}^2(\hat{\omega} - \hat{k}\hat{V}_o) + \hat{\omega}^2(1 + \tau^*) + ib(\hat{\omega} - \hat{k}\hat{V}_o) - (a + \hat{k}^2) = 0\,, \quad (12)$$

where $\hat{\omega} = \omega L/c_{d*}$, $\hat{k} = kL$, $\hat{V}_o = V_o/c_{d*}$, $a = n_n n_{io} \times \pi r_d^2 \sigma_{in} L^2 \tau^{1/2}/Z_d$, $b = n_n \sigma_{in} L Z_d\ (n_{do} m_i \tau/n_{eo} m_d)^{1/2}$, and L is a normalizing length (such as the system dimension). Figures 1 and 2 illustrate the variations of $\hat{\omega}_r$ and $\hat{\gamma} = \frac{\gamma L}{c_{d*}}$ with $\hat{k}$ for two interesting parameter regions. As a specific example, we consider the experiment of the type of Prabhuram and Goree [7], where the typical parameters are $n_n \simeq 2 \times 10^{16}$ cm^{-3}, $n_i \simeq 5 \times 10^{10}$ cm^{-3}, $r_d \simeq 10^2$ nm, $m_i/m_d \simeq 5 \times 10^{-8}$, $\sigma_{in} \simeq 10^{-14}$ cm^2, $Z_d^2 n_{do}/n_{eo} \simeq 500$, $\tau \sim 4$, $Z_d \sim 200$, $L \sim 20$ cm giving the dimensionless parameters $a \simeq 10^4$ and $b \simeq 50$. Figure 1 shows that the recombination driven growth rate for this case maximizes around $kL \simeq 30$ giving $\lambda \simeq$ few cm, which compares favorably with the typical scale sizes observed in this experiment. The best examples of dust-acoustic instability driven by ion drift are found in the low pressure limit where recombination effects are

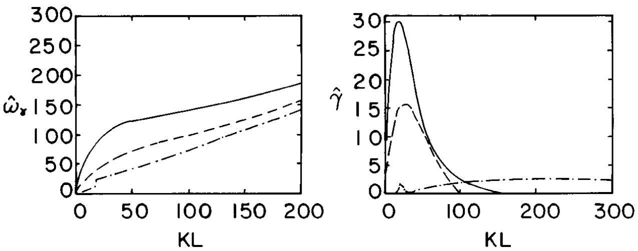

FIG. 1. Real frequency $\hat{\omega}$ and growth rate $\hat{\gamma}$ versus $\hat{k}$. Solid lines (—): $a = 30\,000$, $b = 10$, $\tau^* = 1$, $\hat{V} = 0$; dashed lines (– –): $a = 10\,000$, $b = 50$, $\tau^* = 1$, $\hat{V} = 0$; dash-dotted lines (– · –): $a = 1000$, $b = 50$, $\tau^* = 1$, $\hat{V} = 1$.

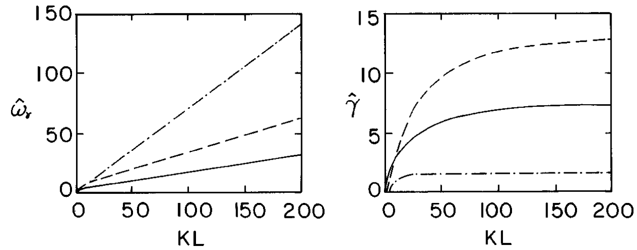

FIG. 2. Real frequency $\hat{\omega}$ and growth rate $\hat{\gamma}$ versus $\hat{k}$. Solid line (—): $a = 0$, $b = 1$, $\tau^* = 40$, $\hat{V} = 100$; dashed lines (– –): $a = 0$, $b = 10$, $\tau^* = 10$, $\hat{V} = 10$; dash-dotted lines (– · –): $a = 10$, $b = 1$, $\tau^* = 10$, $\hat{V} = 10$.

negligible (Fig. 2). These results are more relevant to the experiments of Prabhakara and Tanna [9] and many plasma processing situations where the dust levitates in a sheath region into which ions are drifting typically with velocities satisfying the Bohm stability condition $V_o \geq c_i$. In this case strong instabilities with typical sizes ~few cm are excited, even when V_o is only a few times c_d.

Let us now investigate the nonlinear evolution of the dust-acoustic perturbations. We first consider the dissipative short wavelength instability driven by the relative ion-dust drift velocity. The linearized perturbation for $\nu_R = 0$ is described by the equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2}\right)\tilde{n}_d = -\delta\left(\frac{\partial}{\partial t} + V_o \frac{\partial}{\partial x}\right)\tilde{n}_d , \qquad (13)$$

where $c^2 = c_{d*}^2/(1 + \tau^*)$ and $\delta = \nu_{in}\tau^*(c_{d*}/c_i)^2/(1 + \tau^*)^2$. This equation reproduces all the salient features of the short wavelength instability. Nonlinear dispersive wave packets associated with these perturbations can be described if one retains quadratic nonlinearities ("convective" nonlinearities in the dust hydrodynamic equations and the "Boltzmann" nonlinearities in the electron and/or ion perturbations) and deviations from quasineutrality [an additional $\nabla^2\tilde{\varphi}$ term in Eq. (6)]. Considering nonlinear wave packets traveling to the right with a velocity close to the group velocity c, introducing the variables $\lambda_D^{-1}(x - ct) = X$, $\omega_{pd}t = T$, $U = -\tilde{n}_d/6$, we finally get the nonlinear evolution equation

$$U_T - 6UU_X + U_{XXX} = \frac{\delta}{2}(V_o - c)U + \frac{\delta}{2}\int U_T\, dX , \qquad (14)$$

where we have introduced the standard normalizations for x, t, etc. and the subscripts denote partial derivatives. The left side is the standard $K - dV$ equation for dust-acoustic perturbations, derived earlier by Rao *et al.* [10] and others [14]. The right side of Eq. (14), which we intend to treat perturbatively, describes the growth and saturation of the solitons due to the ion drift. We use the perturbation theory of solitons due to Karpman and Maslov [15] to write the solution

$$U \equiv 2\kappa^2(T)\,\mathrm{sech}^2 Z , \qquad (15)$$

where $Z = \kappa(T)[X - \xi(T)]$, $\xi = 4\int \kappa^2\, dT$ + corrections of order δ and

$$\kappa^2 = \tfrac{1}{4}(V_o - c) \Bigg/ \left\{1 + \left[\frac{V_o - c}{4\kappa_o^2} - 1\right] \times \exp\left[-\frac{2\delta}{3}(V_o - c)T\right]\right\}. \qquad (16)$$

Equation (16) shows that the amplitude of the soliton increases from an initial value $2\kappa_o^2$ to a saturated value $(V_o - c)/2$. Physically, the soliton saturates because as its amplitude increases, it accelerates and eventually reaches a velocity V_o; at this velocity, it can no longer extract energy from the ion drift. The above analysis shows that the short wavelength collisional instability will nonlinearly lead to a collection of $K - dV$ solitons which interact relatively weakly with each other.

We next discuss the nonlinear physics for the long wavelength instability driven by recombination processes. In this limit, the modification of the real part of frequency due to collisions assumes importance and we may write the linearized wave equation (ignoring the growth terms) as

$$\frac{\partial^2}{\partial t^2}\tilde{n}_d + \omega_c^2\tilde{n}_d = c^2\frac{\partial^2\tilde{n}_d}{\partial x^2} , \qquad (17)$$

where $\omega_c^2 = \alpha\nu_{in}\nu_R/(1 + \tau^*)$. In the long wavelength limit ($kc, \tilde{v}_d \cdot \nabla \ll \omega_c$, etc.), the convective nonlinearities are negligible. The dominant nonlinear effect arises because of "slow" modulations of n_d (which modulate ν_R and hence ω_c). We may thus write $n_d = n_{do}(1 + \tilde{n}_d) + \delta n_{ds}$, where δn_{ds} is the low frequency $[\Omega \ll k(T_d/m_d)^{1/2}]$ response of the dust density to beat-frequency perturbations generated by the dust-acoustic wave. It may be noted that this problem has many similarities to the description of nonlinear Langmuir waves [16,17]. At very low frequencies ($\Omega \ll kc_{td}, k^2c_i^2/\nu_{in}$), ion and electron response will be Boltzmann-like: $\delta\tilde{n}_{es} = \delta\tilde{\varphi}_s$; $\delta\tilde{n}_{is} = -\tau\delta\tilde{\varphi}_s$. The low-frequency dust dynamics is given by $\langle\tilde{v}_d \cdot \nabla\tilde{v}_d^*\rangle = c_d^2\nabla\delta\tilde{\varphi}_s - c_{td}^2\nabla\delta\tilde{n}_{ds}$. Using the quasineutrality condition, we finally get $\delta n_{ds}/n_{do} \simeq -|\tilde{v}_d|^2/[c_{td}^2 + c_{d*}^2/(1 + \tau^*)]$.

We may now modify characteristic frequency in Eq. (17) as $\omega_c^2 = \omega_{co}^2(1 + \delta n_{ds}/n_{do})$, express the resulting nonlinear equation in terms of $\tilde{v}_d$, and eliminate the

ω_{co}^2 terms by the ansatz $\tilde{v}_d = \tilde{V}_d \exp(-i\omega_{co}t) +$ c.c., to finally get the nonlinear equation

$$iU_T + U_{XX} + |U|^2 U = 0, \tag{18}$$

where $T = \omega_{co}t$, $X = \sqrt{2}(\omega_{co}x/c)$, $U = \tilde{V}_d\sqrt{2}/[c_{td}^2 + c_{d*}^2/(1+\tau^*)]^{1/2}$ and we have assumed that the modulation wavelengths are much longer than that of the basic waves. Equation (18) is the well known nonlinear Schrödinger equation [16,17]. It shows that in one dimension, large amplitude harmonic waves are susceptible to modulational instabilities, which finally saturate to give envelope soliton solutions. A standard envelope soliton solution has the form

$$U = A\,\mathrm{sech}\left[\frac{A}{\sqrt{2}}(X - 2k_o T)\right] \times \cos\left[k_o X - \left(1 + k_o^2 + \frac{A_o^2}{2}\right)T\right], \tag{19}$$

where $A = \bar{A}\sqrt{2}/[c_{td}^2 + c_{d*}^2/(1+\tau^*)]^{1/2}$ is the dimensionless amplitude of the perturbed dust velocity ($\bar{A}$) and $k_o = \bar{k}_o c/\sqrt{2}\,\omega_{co}$ is the dimensionless wave number of the propagating modulated wave ($\bar{k}_o$ being the usual wave number). When instability terms in the original Eq. (17) are retained, we get a nonlinear Schrödinger equation with sources, which is also known to generate envelope solitons [17]. We thus expect the long wavelength recombination instability in one dimension to generate a collection of envelope solitons. However, the behavior for multidimensional perturbations is expected to be more complex. It is well known that in such cases the nonlinear Schrödinger equation gives no stationary states and instead leads to a collapse of wave packets [17]. Such a collapse leads to the formation of localized regions of high concentrations of wave fields, from which dust density is expelled; furthermore, intense wave-particle interactions in such localized regions can lead to strong heating of the dust particles. The formation of "voids" and "filamentary structures" in some dusty plasma experiments [7] may be related to such phenomena.

In conclusion, we have investigated dust-acoustic instabilities in a plasma with a background of neutrals. We find a short wavelength branch driven by ion drift and collisional ion-neutral momentum transfer. Nonlinearly this instability leads to the formation of $K - dV$ solitons. We also find a long wavelength branch, particularly relevant for high neutral pressure situations, which is driven by recombination effects on the surface of dust particles. This instability nonlinearly favors the formation of envelope solitons in one dimension and collapsing wave packets in more than one dimension. We finally comment on some effects not included in the above treatment. It is widely recognized now that self-consistent fluctuation of the charge on dust particles must also be incorporated in a complete analysis [18–20]. This effect leads to a novel wave damping which has to be overcome by any excitation mechanisms. Since the instability growth rates by collisional mechanisms are fairly strong ($\gamma \leq \omega_r$), it is likely that these damping rates will be exceeded in many realistic experimental situations. Another wave damping mechanism which may be important in high pressure experimental situations [21,22] arises through dust neutral collisions. The damping enters through a collisional drag term $-\nu_{dn}\tilde{v}_d$ on the right side of Eq. (2) and would proceed at a rate of order $\nu_{dn}/2 \simeq (n_n/n_d)(M_n/2M_d)\nu_{nd}$, where ν_{nd} is the neutral-dust collision rate $\sim \pi r_d^2 v_n n_d$. Comparing this damping rate with the typical growth rate driven by recombination effects we find that the growth dominates for $k\lambda_f < 1 \leq \sqrt{T_e/T_n}\,(n_d M_d/n_n M_n)$, a condition which is readily satisfied. Another effect not included in our present calculation is that of strong correlations in the dust fluid, which may be important in some experiments (where $\Gamma_d = Z_d^2 e^2/dkT_d > 1$, d being the inter-dust-particle spacing). The basic analytic methods of treating such strongly coupled systems either using lattice models [23] or simple viscoelastic transport phenomena [24] are still in their infancy. Detailed investigations of such effects are therefore left for future work.

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