Wigner Solid on the Free Surface of Superfluid ^{3}He

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We report the measurement of electrical transport of the two-dimensional Wigner solid (WS) formed on the free surface of superfluid ^{3}He-B down to 230 μK. The resistance $R(T)$ dramatically decreases as temperature T decreases, obeying Arrhenius' law, $R(T) \propto \exp[-\Delta(T)/k_BT]$, with $\Delta(T)$ which we assign to the superfluid energy gap. That is, the scattering of ^{3}He quasiparticles by the WS determines the transport. The resistance depends systematically on the static electric field which presses the WS toward the liquid surface. We propose that the WS acts as a powerful probe for the study of the superfluid ^{3}He. [S0031-9007(97)04553-5]

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Superfluid ^{3}He has been of much interest during the last two decades [1]. It is a unique example of anisotropic non-s-wave BCS superfluids. Since the Cooper pairs have nonzero angular momenta, the superfluid properties are greatly altered by the presence of boundaries [2]. Near boundaries, the order parameters become highly anisotropic, and textures formed by l and d vectors, which are related to the direction of the orbital and spin angular momenta, are aligned. A number of interesting problems, such as possible new states [3] and Andreev reflection [4], still remain unsolved.

The free surface of liquid ^{3}He provides an ideal boundary of the p-wave superfluids. One may expect that, at the free surface, the quasiparticles scatter *specularly,* and hence the order parameter is much more anisotropic than boundaries between solid walls and the superfluid, where the scattering is diffusive [2]. Despite the importance, few experiments had been done for the superfluid free surface, because of the lack of surface-sensitive experimental means. Development of surface probes is essential for the surface studies. The macroscopic shape of the free surface under rotation was studied by optical means [5]. The surface dynamics may play a crucial role on the U-tube oscillations [6]. Very recently, Andreev reflection of the ^{3}He quasiparticles at the free surface was observed [7].

In this Letter, we report on a new experimental probe for the free surface of superfluid ^{3}He. It is the two-dimensional electron crystal (Wigner solid, WS) trapped on the free surface. We have found that the WS transport is dominated by the scattering of ^{3}He quasiparticles to the periodic surface deformation accompanied by the WS. The WS can be a powerful tool not only for studying the surface properties in superfluid ^{3}He, but also for the sensitive quasiparticle detection.

It is well known that the WS is formed on a ^{4}He surface [8], and it shows interesting properties [9]. On the other hand, on liquid ^{3}He, no experimental searches for the possible WS phase had been made for a long time. It was never thought that changing the underlying liquid from ^{4}He to ^{3}He would produce anything new and important in physics of the WS. Prior to the present work, we performed transport measurements for the surface electrons on normal liquid ^{3}He, in order to search for the WS phase and study the Fermi liquid surface [10]. We observed an abrupt drop of mobility μ, which substantially shows that the WS is formed on liquid ^{3}He as well. The mobility drop is due to the formation of the periodic surface corrugation (dimple lattice) whose wave vectors equal the WS reciprocal lattice vectors, and amplitude (depth) is of the order of 0.1 Å [9]. In the WS phase, μ decreases as temperature T decreases, obeying a power law $\mu \propto T^2$ [11]. This T^2 behavior is explained by taking the Fermi liquid effect into account [12]. The ^{3}He bulk quasiparticles scatter to the dimple lattice. This scattering contributes to the WS mobility as well as the surface excitations (ripplons) do. Since the ripplons are damped on ^{3}He at low temperatures, μ is determined only by the dimple-quasiparticle scattering. The mobility is inversely proportional to the bulk viscosity, so the Fermi liquid viscosity $\eta \propto T^{-2}$ results that $\mu \propto T^2$.

The very existence of the WS on ^{3}He gives us two novel aspects: (1) the static and dynamical properties which are peculiar to the WS-^{3}He system, and (2) the utility of the WS for the study of the bulk and surface properties of superfluid ^{3}He. In this Letter we concentrate on the latter. We point out its importance and advantage: The oscillation of the WS and the accompanied dimples induces a dipolarlike fluid motion beneath the surface, so the WS becomes a *mechanical device* similar to vibrating wire or ultrasonic transducer. The WS can be in principle operated in a wide range of frequency, say, from kilohertz to gigahertz. This broad frequency, i.e., energy, range will be advantageous for superfluid ^{3}He studies. In particular, one can employ the coupled collective modes of the WS phonons to the surface dimples, which were observed on superfluid ^{4}He [8] and probably exist also on ^{3}He. The frequencies of such modes are about 100 MHz, which are comparable to the superfluid energy gap. Much information about the energy gap and order-parameter collective modes may be obtained.

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As a first step of investigation, we have performed the measurement of ac transport properties, i.e., resistivity, on the B phase down to 230 μK.

We employ the capacitive coupling method [13] to measure the resistivity. The experimental cell is shown in Fig. 1(a). The copper cell contains a ring-shape sintered silver powder of 30 m^2 surface area, in its bottom half. In the middle of the cell, we set an electrode pair shown in Fig. 1(b) (the Corbino disk), which is made by etching a copper-epoxy print plate. The gap between the inner and outer disks is 0.1 mm. Thin copper tubes (commercially available, originally for making circuit boards with copper-lined holes) are buried to the disk, and the electrical contact is made by soldering copper wires from the bottom of the epoxy plate to the tubes. We introduce 4 cm^3 of liquid ^{3}He so as to set the free surface at 1.0 mm above the electrode. A copper-epoxy disk, whose radius equals the outer Corbino radius, is set on the ceiling of the cell. The liquid level is monitored by measuring the capacitance between the ceiling disk and the Corbino one. A copper guard ring, made of a thin Kapton print plate, is glued on the side wall of the cell.

The electrons are generated by thermionic emission of a tungsten filament, which is mounted 2 mm above the liquid. To decrease the electron energy by vapor gas scattering, the emission is done at 600 mK, at which the ^{3}He vapor pressure is 0.6 Torr. The electrons are kept on the liquid surface by applying a positive dc voltage V_{dc} to the Corbino disk, and the electron density n_s is determined by the shielding condition of the electric field above the liquid. The data reported here are taken at $n_s = 1.5 \times 10^8\ \mathrm{cm}^{-2}$. The WS melting occurs at 265 mK.

The cell is mounted on a copper nuclear demagnetization refrigerator, in which the effective amount of copper under 6.5 T is 37 moles [14]. The temperature is measured by a platinum NMR susceptibility thermometer mounted on the nuclear stage, which is calibrated by a ^{3}He melting curve thermometer with the temperature scale established by Greywall [15].

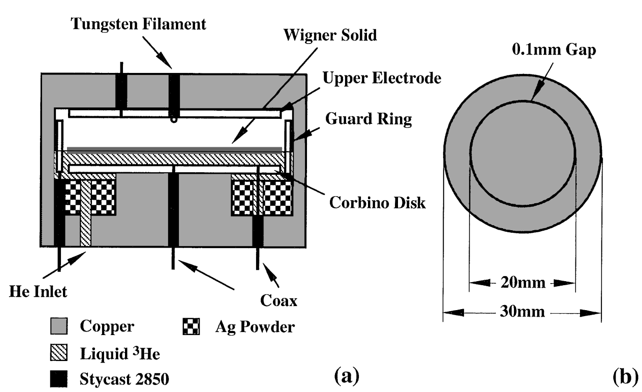

FIG. 1. (a) Schematic cross-sectional view of the experimental cell. (b) The Corbino electrode.

The method of the resistivity measurement is similar to our previous works [10,11,13]. An ac voltage V_{in} of 100 kHz and 2.0 $\mathrm{mV_{P\text{-}P}}$ is applied to the inner electrode. The current flowing inside the WS is detected from the outer electrode as a voltage V_{out} induced on a capacitor which is connected between the outer electrode and the ground. The in phase and quadrature components of V_{out} are monitored by a lock-in amplifier. The conductivity σ_{xx} is derived by fitting V_{out} to a formula which is given elsewhere [13]. In this Letter no magnetic field is applied. Therefore, the superfluid B phase is always realized. Moreover, the electrical current flows only in the radial direction of the Corbino disk, and the resistance R is simply given by $\sigma_{xx}{}^{-1}$. (In two dimensions, resistance is equivalent to resistivity.) The experimental data reported here were taken during slowly sweeping temperature by adiabatically demagnetizing or magnetizing the nuclear stage.

In Fig. 2, we show the resistance R as a function of temperature, for various static electric fields $E_\perp$, which press the WS toward the surface, and hence determine the depth of the dimples. R is independent of temperature down to 930 μK. This temperature-independent resistance is seen up to about 20 mK, above which the T^{-2} behavior appears. We interpret this T-independent behavior as the so-called mean free path effect of the ^{3}He quasiparticles [12]. When the quasiparticle mean free path exceeds the WS lattice constant, which is about 10^{-4} cm, the exchange of momenta at the dimples is no longer represented by the bulk viscosity, but by the individual scattering of the quasiparticles which has the Fermi momentum p_F. Then the force exerted to the corrugated surface is independent of temperature. We note that, although the details will be discussed elsewhere, the calculated resistance agrees well with experiment.

At $T = 930\ \mu$K, R decreases abruptly. It is remarkable that, from 930 to 230 μK, R decreases more than

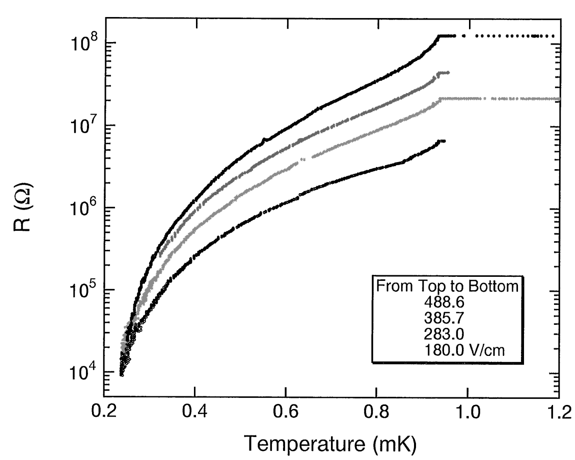

FIG. 2. Temperature dependence of resistance R for various pressing electric fields $E_\perp$, 180.0, 283.0, 385.7, and 488.6 V/cm.

3 orders of magnitude. The temperature 930 μK is exactly the superfluid transition temperature T_c of ^{3}He at saturated vapor pressure (SVP) [15]; that is to say, the abrupt change of resistance is caused by the superfluid transition of ^{3}He.

We find that R increases as $E_\perp$ increases. We show in Fig. 3 the dependence of R on $E_\perp$, for various fixed temperatures. The resistance obeys a power law, $R(T) \propto E_\perp^\alpha$. The exponent α is temperature dependent: In the normal phase, α is slightly larger than 2. In the superfluid phase, α decreases slowly from 2, as T decreases from T_c.

The tremendous decrease of the WS resistance at T_c shown in Fig. 2 is due to the change of the scattering mechanisms, and/or the decrease of the number of scatterers. In Fig. 4, we plot R normalized by R at T_c, $R(T)/R(T_c)$ with logarithmic scale, as a function of inverse normalized temperature T_c/T with linear scale. The substantial linearity observed in all the data for $T_c/T > 1.5$ ($T < 620$ μK) shows that $R(T)$ obeys *Arrhenius' law*,

$$R(T)/R(T_c) \propto \exp(-\Delta/k_B T),$$

where k_B is the Boltzmann constant. The parameter Δ increases from $1.76 k_B T_c$ to $2.0 k_B T_c$ as $E_\perp$ increases from 180 to 489 V/cm. These values of Δ are close to the weak-coupling BCS energy gap at $T = 0$, $\Delta(0) = 1.764 k_B T_c$, and also to the estimated gap at SVP, $1.774 k_B T_c$ [1].

Monarkha and Kono [12] theoretically studied the WS transport on normal and superfluid ^{3}He. When the ordinary quasiparticle scattering is relevant, and the scattering is elastic, the collision frequency is given by

$$\nu = \frac{\hbar k_F^4}{8\pi^2 m_e n_s} 2f\left(\frac{\Delta}{k_B T}\right) \sum_{\mathbf{g}} |\mathbf{g}|^2 |\xi_{\mathbf{g}}^{(0)}|^2, \qquad (1)$$

where f is the Fermi distribution function, $\Delta(T)$ the superfluid energy gap, k_F the Fermi wave number, m_e the electron mass, $\mathbf{g}$ the WS reciprocal lattice vector, and $\xi_{\mathbf{g}}^{(0)}$ the Fourier component of the surface displacement due to the electrons, i.e., the dimple depth at the lattice site. The resistance is given by $R = m_e \nu/(n_s e^2)$.

Equation (1) immediately leads to some important conclusions. In the normal phase, where $\Delta = 0$ and hence $2f(\Delta/k_B T) = 1$, the WS resistance is independent of temperature. This agrees with the experimental observation. Furthermore, as $\xi_{\mathbf{g}}^{(0)}$ is proportional to $E_\perp$ [12,13], we find that $R \propto E_\perp^2$. This power law is close to the observed $E_\perp$ dependence of the resistance near the superfluid transition, shown in Fig. 3.

In the superfluid phase, $R(T)/R(T_c)$ is obtained from Eq. (1) as

$$R(T)/R(T_c) = 2\{\exp[\Delta(T)/k_B T] + 1\}^{-1}. \qquad (2)$$

In Fig. 4, we show the result of Eq. (2) using the weak-coupling BCS energy gap function given by [1]

$$\Delta(T) = \Delta(0) \tanh\left[\frac{3.067 k_B T_c}{\Delta(0)} \left(\frac{T_c}{T} - 1\right)^{1/2}\right], \qquad (3)$$

where $\Delta(0) = 1.764 k_B T_c$. The theoretical curve is in good quantitative agreement with the experimental data of smallest $E_\perp$ (the saturation case, in which the electric field is completely terminated at the surface electrons).

We conclude that, in the superfluid phase, the WS transport is dominated by the scattering of the Bogoliubov quasiparticles, whose distribution is controlled by the superfluid energy gap $\Delta(T)$. Two observations should be stressed: (1) The $R(T)/R(T_c)$ data for the saturation case is perfectly described by Eq. (2). This fact suggests that the contribution of Andreev reflection (AR), which decreases the momentum transfer and hence the WS resistance, is negligible. On the other hand, a recent experiment showed that AR does exist at the free surface [7]. The quasiparticle momentum transfers which prevail the WS resistance increase as the incident angle θ

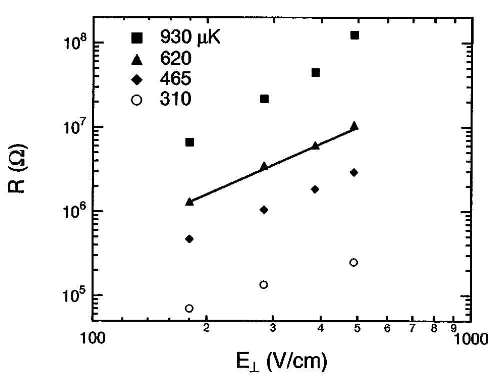

FIG. 3. Resistance as a function of pressing electric field $E_\perp$, for various fixed temperatures. The data are taken from $R(T)$ shown in Fig. 2. The solid line represents $R \propto E_\perp^2$.

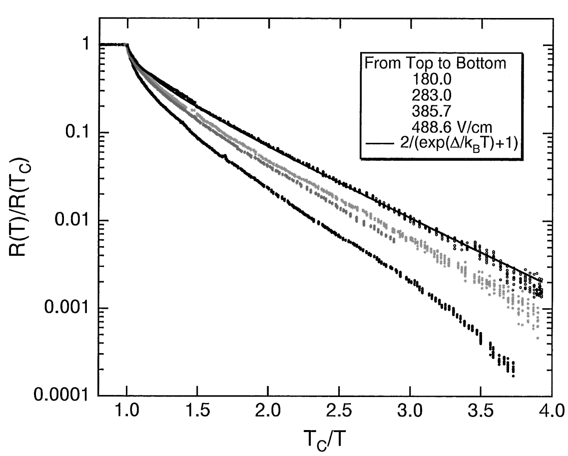

FIG. 4. Resistance normalized by the resistance at $T_c = 930$ μK, $R(T)/R(T_c)$, as a function of normalized inverse temperature T_c/T. The original $R(T)$ data are shown in Fig. 2. The solid line represents Eq. (2).

decreases to zero, because only the surface normal component $2p_F \cos\theta$ is transferred to the WS. It is shown theoretically [4] that the AR rate rapidly decreases from unity to zero, as θ changes from 90° to 0°. Therefore, the AR rate may be small in the scattering processes which contribute to the WS resistance. Quantitative calculation is needed for further discussion.

(2) $R(T)/R(T_c)$ deviates systematically from Eq. (2) as $E_\perp$ increases. $E_\perp$ only increases the depth of the surface dimples. As the dimple depth increases, the flow velocity induced beneath the surface increases. This flow field might cause an enhancement of AR rate, which was recently observed [16], and hence might decrease the WS resistance. In the experiment of Enrico *et al.* [16], the AR rate increased up to 5% only when the flow velocity reached about 10^{-3} m/s, which is an order of magnitude larger than the WS-induced flow velocity just beneath the surface ($\sim 10^{-5}$–10^{-4} m/s). It seems unlikely that the dimple-induced flow enhances the AR contribution.

There are strong similarities between the WS resistance below 20 mK and the mobility of electron bubbles in liquid ^{3}He [17]. The inverse mobility of the bubbles is independent of temperature below 100 mK, and rapidly decreases below the superfluid transition. The mobility is essentially determined by quasiparticle-bubble collisions. The T-independent mobility in normal ^{3}He is explained by the diffusive nature of bubble motion. This is not the case for the WS on ^{3}He, where the electrons are strongly correlated by Coulomb repulsion. In the superfluid phase, a substantial discrepancy between the experimental data and the calculated mobility, which is exactly the same as Eq. (2), was found. It was resolved by taking into account the effect of superfluid correlations on intermediate scattering states. It may also be worthwhile to consider as a possible explanation for the $E_\perp$ dependence of the WS resistance.

The data presented here were taken at the effective current density of 10^{-10} A/cm, which corresponds to the electron velocity 10 cm/s. In this velocity regime, the resistance shows an Ohmic behavior. However, at higher currents, we have observed a strongly nonlinear resistivity in both superfluid and normal phases. Such an anomalous transport will be reported elsewhere.

We have shown that the WS transport is prevailed by the quasiparticles coming from *bulk.* The WS can be developed as a sensitive quasiparticle detector, as well as vibrating wire technique [18], which has been applied to particle detection [19] and cosmic-string simulation [20]. Our next step is to study the surface properties employing the WS. One may expect that the WS-dimple system interacts with textures and defects, especially vortices which terminate at the free surface [21]. Measurements in the A phase, in which the l vectors are perpendicular to the surface, are also intriguing.

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