Nematic–to–Smectic-*A* Transition in Aerogel

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We study a model for the nematic–to–smectic-*A* (NA) transition in aerogel, and find that even arbitrarily weak quenched disorder (i.e., low aerogel density) destroys translational (smectic) order. Ignoring *elastic* anharmonicities, but keeping anharmonic couplings to disorder, leads to the prediction that there is no "Bragg glass" phase in this system: it is riddled with dislocation loops induced by the quenched disorder. Orientational (nematic) order is destroyed as well, as is the thermodynamically sharp NA transition, in agreement with recent experimental results. [S0031-9007(97)04632-2]

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Recent experiments [1,2] have focused on the study of liquid crystals in the random environment of an aerogel, as a new paradigm system in which to investigate the general problem of quenched disorder in condensed matter systems.

In this Letter we develop a theory of such systems. We find that in a harmonic approximation, even arbitrarily weak disorder destroys both the smectic-*A* phase and the nematic–to–smectic-*A* (NA) transition, and creates unbound dislocation loops. The formalism we use to demonstrate this defect unbinding is new, powerful, and potentially applicable to a wide variety of candidate "Bragg glass" [3] systems.

Once dislocations are present, the phase is best characterized as a nematic in a random tilt field. However, subsequent examination of orientational fluctuations in this nematic leads to the conclusion that tilt disorder destroys the orientational order of the smectic layers as well.

However, a "nematic Bragg glass" and a thermodynamically sharp "nematic glass" transition may occur [4], in agreement with recent dynamic light scattering experiments [1] that show a dramatic slowing down of director fluctuation relaxations in liquid crystals in aerogel below a temperature T_g near the bulk nematic-isotropic (NI) transition.

Near the NA transition the center-of-mass nematogen molecular density $\rho(\mathbf{r})$ develops strong fluctuations dominated by Fourier components near the smectic ordering wave vector $\mathbf{q_0} = \mathbf{n}2\pi/d$ parallel to the nematic director $\mathbf{n}$. Defining the local (complex scalar) order parameter $\psi(\mathbf{r})$ which distinguishes the smectic-*A* from the nematic phase [5] via $\rho(\mathbf{r}) = \mathrm{Re}\,[\rho_0 + e^{i\mathbf{q_0}\cdot\mathbf{r}}\psi(\mathbf{r})]$, where ρ_0 is the mean density of the smectic, we take for our free energy $F = F_{dG} + F_{d\rho} + F_{dn}$, where $F_{dG}[\psi, \mathbf{n}]$ is de Gennes free energy (which includes the Frank free energy) [5], and $F_{d\rho}$ and F_{dn} are the new disorder parts, which couple the smectic density and the nematic director, respectively, to the quenched disorder of the aerogel matrix, and are combined into $F_d = F_{d\rho} + F_{dn}$:

$$F_d = \int d^d r \left[\frac{1}{2}\,\delta t(\mathbf{r})\,(\rho - \rho_0)^2 + U(\mathbf{r})\rho + [\,\mathbf{g}(\mathbf{r}) \cdot \mathbf{n}]^2 \right], \tag{1}$$

where both the quenched random T_c $(\delta t(\mathbf{r}))$ and the quenched random potential $U(\mathbf{r})$ are proportional to the local aerogel density $\rho_A(\mathbf{r})$, and the quenched field $\mathbf{g}(\mathbf{r})$ is random and *short-range* correlated in direction, with strength proportional to the local aerogel density.

Combining the "random field" energy [first two terms of Eq. (1)] with the relation between the smectic order parameter ψ and the density ρ, we obtain

$$F_{d\rho}[\psi] = \int d^d\mathbf{r}\,\frac{1}{2}\,[\delta t(\mathbf{r})\,|\psi|^2 + V(\mathbf{r})\psi + V^*(\mathbf{r})\psi^*], \tag{2}$$

where $V(\mathbf{r}) \equiv U(\mathbf{r})e^{iq_0 z}$. Note that, despite the long-ranged correlations of $U(\mathbf{r})$, which arise due to the fractal structure of the aerogel [1], $V(\mathbf{r})$ has only *short-ranged* correlations. This is because correlations of V near $\mathbf{q} = \mathbf{0}$ are related to those of U near $\mathbf{q} = q_0\hat{\mathbf{z}}$ and the aerogel itself has no particular spatial structure at the wave vector of the smectic ordering, q_0. Thus, the correlations of $V(\mathbf{r})$ are short ranged, and hence we can accurately capture the long distance physics by taking the real space correlations to be zero ranged, and write $\overline{V(\mathbf{r})V^*(\mathbf{r}')} = \Delta_V \delta^d(\mathbf{r} - \mathbf{r}')$ where $\Delta_V = C_U(q_0\hat{z})$ (overbar denotes quenched disorder average). Expanding in small deviations from perfect nematic order $\mathbf{n}_0 = \hat{\mathbf{z}}$, writing $\hat{\mathbf{n}}(\mathbf{r}) = \hat{\mathbf{z}} + \delta\mathbf{n}(\mathbf{r})$, to linear order in $\delta\mathbf{n}(\mathbf{r})$, F_{dn} [the last term in Eq. (1)] becomes

$$F_{dn} \approx \int d^d r\,\mathbf{h}(\mathbf{r}) \cdot \delta\mathbf{n}\,, \tag{3}$$

where we have defined a quenched random tilt field $\mathbf{h}(\mathbf{r}) \equiv g_z(\mathbf{r})\mathbf{g}(\mathbf{r})$.

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Since we expect $\mathbf{g}(\mathbf{r})$ to have only short-ranged correlations (with range of order the orientational persistence length of the silica fibers), the above correlation function of the tilt disorder should also be short ranged. Furthermore, it must be isotropic. These considerations lead to the following form for the correlation function: $\overline{h_i(\mathbf{r})h_j(\mathbf{r}')} \equiv \Delta_h \delta^d(\mathbf{r} - \mathbf{r}')\ \delta_{ij}$, which is *short ranged.*

While it is tempting to directly analyze the model $F = F_{dG} + F_{d\rho} + F_{dn}$ written in terms of the smectic order parameter ψ, we will not do so here [4]. Our motivation for this is that such a direct approach is *known* to *incorrectly* predict the lower critical dimension in, e.g., the random field Ising model (as well as, e.g., completely missing the existence of the Kosterlitz-Thouless transition). Indeed, it proves to also do so here. Instead, we proceed by *assuming* the existence of smectic order and writing $\psi(\mathbf{r}) = |\psi_0| e^{iq_0 u(\mathbf{r})}$, with a uniform amplitude $|\psi_0| = \text{const}$ and $u(\mathbf{r})$ the local displacement of the smectic layers from the perfect periodic order. Using this low-T ansatz in Eqs. (1)–(3), and integrating over the nematic director fluctuations $\delta\mathbf{n}$, results in the replacement $\delta\mathbf{n} \to \nabla_\perp u$, everywhere in $F[u, \delta\mathbf{n}]$ (the Higg's mechanism). This leads to the elastic free energy of the disordered smectic-A phase, valid in the long wavelength limit, to quadratic order in gradients of u, and provided dislocations are confined,

$$F[u] = \int d^d\mathbf{r}\Big[\frac{B}{2}(\partial_z u)^2 + \frac{K}{2}(\nabla_\perp^2 u)^2 + \mathbf{h}(\mathbf{r})\cdot\nabla_\perp u - |\psi_0|\,(V(\mathbf{r})e^{iq_0 u(\mathbf{r})} + V^*(\mathbf{r})e^{-iq_0 u(\mathbf{r})})\Big]. \tag{4}$$

To compute self-averaging quantities, we employ the replica "trick." After replicating and integrating over the disorder [4], we obtain $\overline{Z^n} = \int [du_\alpha] e^{-F[u_\alpha]}$, with

$$F[u_\alpha] = \int d^d r\Big[\sum_{\alpha=1}^{n}\Big(\frac{K}{2}(\nabla_\perp^2 u_\alpha)^2 + \frac{B}{2}(\partial_z u_\alpha)^2\Big) + \sum_{\alpha,\beta=1}^{n}\Big(\frac{\Delta_h}{4}|\nabla_\perp(u_\alpha - u_\beta)|^2 - \gamma\cos[q_0(u_\alpha - u_\beta)]\Big)\Big], \tag{5}$$

where $\gamma \equiv |\psi_0|^2\Delta_v/2$. We have studied this model using the standard momentum shell renormalization group (RG) transformation, generalized to allow for anisotropic scaling: $r_\perp = r'_\perp e^\ell$, $z = z' e^{\omega\ell}$ [4]; the results are the RG flow equations in 3D,

$$\frac{d\gamma(\ell)}{d\ell} = (2 + \omega - \eta)\gamma - A_1\gamma^2, \tag{6}$$

$$\frac{dK(\ell)}{d\ell} = (\omega - 2)K, \tag{7}$$

$$\frac{dB(\ell)}{d\ell} = (2 - \omega)B, \tag{8}$$

$$\frac{d\Delta_h(\ell)}{d\ell} = \omega\Delta_h + A_2\gamma^2, \tag{9}$$

where for simplicity we set the UV cutoff $\Lambda = 1$, and $\eta = q_0^2/4\pi\sqrt{KB}$, $A_1 = 2\eta/3\pi K$, $A_2 = q_0^2\eta/4K$. The statistical symmetry under global rotation requires that the disorder generated replica off-diagonal terms be invariant under $u_\alpha(\mathbf{r}) \to u_\alpha(\mathbf{r}) + \boldsymbol{\theta}\cdot\mathbf{r}_\perp$. In Eq. (5) the nonlinearities only depend on the difference between different replica fields and therefore do not depend on the "center of mass" field $\sum_{\alpha=1}^n u_\alpha$, which is therefore a noninteracting field. This implies that K and B are *not* renormalized by disorder [6,7]; i.e., their flow equations are *exact*, ignoring (for now) the effects of both anharmonic elastic terms and topological defects loops in u. Note that η, which is simply the Caille exponent for the algebraic decay of smectic correlations in the pure smectic, is unrenormalized. The recursion relation for the proper dimensionless coupling constant $\tilde\gamma \equiv 2\gamma/3\pi K$ can be easily obtained by combining Eqs. (6) and (7),

$$\frac{d\tilde\gamma(\ell)}{d\ell} = (4 - \eta)\tilde\gamma - \eta\tilde\gamma^2, \tag{10}$$

and, as required, is independent of the arbitrary anisotropy rescaling exponent ω. From Eq. (10), we then find that for $\eta < 4$ (large elastic moduli, $KB > q_0^4/256\pi^2$), well below the NA transition, the smectic fixed line is unstable to disorder. However, this instability to disorder is stabilized by the nonlinear terms in $\tilde\gamma$, which terminate the flow at a new finite disorder fixed line, $\tilde\gamma^* = (4 - \eta)/\eta$. This new fixed line then controls a glassy smectic-A phase, analogous to the super-rough phase of crystal surface on a random substrate [6,7] and the vortex glass phase of flux-line vortices in type-II superconductors [6,8].

The flow Eq. (10) also implies that the random-field disorder is irrelevant for $\eta > 4$. Since the bulk modulus B vanishes while K remains finite through T_{NA}, η diverges as $T \to T_{NA}^-$, and hence near the bulk NA transition T_{NA}, we are *guaranteed* to have a range of T over which the random-field disorder is irrelevant. However, as we will see below, because tilt disorder (Δ_h) is a strongly relevant perturbation, the 3D quasi-long-range smectic order for $\eta > 4$ will be converted into short-range correlations, even when the random-field disorder given by γ is irrelevant. It is essential to stress that the RG flow described above (i.e., relevance for $\eta < 4$ and irrelevance for $\eta > 4$ of the random-field disorder) survives even despite the strong relevance and runaway of the random tilt coupling Δ_h.

As can be seen from the recursion relations, Eq. (9), even if the bare $\Delta_h = 0$, tilt disorder is generated by the random-field disorder γ upon renormalization. In contrast to the 2D random-field XY model, where the generated Δ_h disorder is only marginally relevant and only weakly affects the quasi-long-range order found for $\Delta_h = 0$ (replacing log phase correlations by $\log^2$) [6,7], for the 3D smectic-A phase Δ_h tilt disorder is strongly relevant. The effect of the tilt disorder is controlled by the dimensionless coupling $g \equiv \Delta_h/B\lambda^3$, where $\lambda \equiv (K/B)^{1/2}$. From the recursion relations Eqs. (6)–(9) we find $dg/d\ell = 2g + (9\pi^3\eta^2/4)\tilde\gamma^2$. For $\eta > 4$, $\tilde\gamma(\ell) \to 0$ (as we have seen), and so $dg/d\ell = 2g$, which is

trivially solved to give $g(\ell) = g(0)e^{2\ell}$, where $g(0)$ is a constant. Thus, the tilt disorder *is* strongly relevant. We expect this on physical grounds since random tilt disorder explicitly breaks the rotational invariance of the smectic-*A* phase.

For $\eta < 4$, $\tilde{\gamma} \to \tilde{\gamma}^* > 0$. Now, in the 2D random-field *XY* model, the existence of a nonzero $\tilde{\gamma}^*$ in the low-T phase implied completely different behavior for the tilt coupling $g(\ell)$ than in the high-T phase. In *our* problem, however, solving Eq. (6) gives $g(\ell) = [g(0) + 9\pi^3\eta^2\tilde{\gamma}_*^2/8]e^{2\ell} - 9\pi^3\eta^2\tilde{\gamma}_*^2/8$, which asymptotically runs away to $+\infty$ as $\sim e^{2\ell}$ in *both* phases, *in exactly the same way.* Nonuniversal constants (like g_0) change but the scaling ($e^{2\ell}$) does not. This implies that *equal time* correlation functions scale in exactly the same way in both the "glassy" ($\eta < 4$) and "nonglassy" ($\eta > 4$) phases. We will therefore calculate them in the nonglassy ($\eta > 4$) phase, where we can set $\gamma = 0$; our results, however, will apply to *both* phases. The difference between these two phases is that the *dynamics* are slower in the glassy phase.

For $\gamma = 0$ we can calculate anything. The quantity of interest $C(\mathbf{r}_\perp, z) = \overline{\langle [u(\mathbf{r}_\perp, z) - u(\mathbf{0}, 0)]^2 \rangle}$ is

$$C(\mathbf{r}_\perp, z) = 2\int \frac{d^2q_\perp dq_z}{(2\pi)^3}(1 - e^{i\mathbf{q}\cdot\mathbf{r}})\frac{\overline{\langle u(\mathbf{q})u(\mathbf{q}')\rangle}}{\delta^d(\mathbf{q} + \mathbf{q}')}, \tag{11}$$

where the quenched and thermal averaged $\overline{\langle u(\mathbf{q})u(\mathbf{q}')\rangle} = \langle u_\alpha(\mathbf{q})u_\alpha(\mathbf{q}')\rangle = \delta^d(\mathbf{q} + \mathbf{q}')G_{\alpha\alpha}(\mathbf{q})$ is expressed in terms the replicated correlation function, where no sum on α is implied, and the replica propagator $G_{\alpha\beta} = \delta_{\alpha\beta}/\Gamma_q + \Delta_h q_\perp^2/\Gamma_q^2$, with $\Gamma_q \equiv Kq_\perp^4 + Bq_z^2$, can be read off from Eq. (5). This together with Eq. (11) gives, in the $\mathbf{r} \to \infty$ limit,

$$\begin{aligned} C &= 2\Delta_h \int \frac{d^2q_\perp dq_z}{(2\pi)^3}\frac{q_\perp^2[1 - e^{i\mathbf{q}\cdot\mathbf{r}}]}{(Kq_\perp^4 + Bq_z^2)^2}, \\ &= \frac{\Delta_h}{32\pi B\lambda^3}\left\{4\lambda|z|e^{-r_\perp^2/4\lambda|z|} + r_\perp^2\left[Ei\left(\frac{-4\lambda L_z}{L_\perp^2}\right) + Ei\left(\frac{-r_\perp^2}{4\lambda|z|}\right) + 2\ln\left(\frac{L_\perp}{r_\perp}\right)\right]\right\}, \end{aligned} \tag{12}$$

where $Ei(x)$ is the exponential integral function, we have considered a finite system whose shape is a rectangular parallelepiped of linear dimensions $L_\perp \times L_\perp \times L_z$, L_z being the length of the system along the ordering (z) direction, and dropped the subdominant thermal (finite T) contribution to $C(\mathbf{r}_\perp, z)$. In the usual $\lambda L_z \ll L_\perp^2$ limit, the asymptotic behaviors of $C(\mathbf{r}_\perp, z)$ are

$$C \approx \begin{cases} \frac{\Delta_h}{32\pi B^2\lambda^3}[4\lambda|z| + r_\perp^2 \ln|\frac{L_z}{z}|], & \lambda|z| \gg r_\perp^2, \\ & \lambda L_z \ll L_\perp^2, \\ \frac{\Delta_h}{16\pi B^2\lambda^3} r_\perp^2 \ln(\frac{2\sqrt{\lambda L_z}}{r_\perp}), & \lambda|z| \ll r_\perp^2, \\ & \lambda L_z \ll L_\perp^2, \end{cases} \tag{13}$$

An unusual feature of this result is that even the *relative* displacement of two points with *finite* separations $(r_\perp, z)$ diverge as the system sizes $(L_\perp, L_z)$ go to infinity. This is because the mean squared real space *orientational* fluctuations $\overline{\langle|\delta\mathbf{n}(\mathbf{r})|^2\rangle} = \overline{\langle|\nabla_\perp u(\mathbf{r})|^2\rangle}$ also diverge as $L_{\perp,z} \to \infty$,

$$\begin{aligned} \overline{\langle|\delta\mathbf{n}(\mathbf{r})|^2\rangle} &= 2\Delta_h \int \frac{d^2q_\perp\, dq_z}{(2\pi)^3}\frac{q_\perp^4}{(Kq_\perp^4 + Bq_z^2)^2}, \\ &= \frac{\Delta_h}{4\pi B^2\lambda^3}\ln(\min[\sqrt{\lambda L_z}, L_\perp]). \end{aligned} \tag{14}$$

Defining the *translational* correlation lengths $\xi_\perp$ and ξ_z, as the distances $r_\perp$ and z at which $C(\mathbf{r}_\perp, z)$ is of order a^2, where a is a lattice constant, gives $\xi_z = a^2 8\pi KB/\Delta_h$, and

$$\xi_\perp = 4a\left(\frac{\pi B^2\lambda^3}{\Delta_h \ln(2\sqrt{\lambda L_z}/\xi_\perp)}\right)^{1/2}. \tag{15}$$

Furthermore, because the liquid crystal in aerogel lacks long-ranged *orientational* order, as well, to obtain the x-ray scattering from aerogel, one must powder average; the broad *ring* of x-ray scattering that results has width $\kappa_{\text{powder}} \cong (\xi_z)^{-1} = \Delta_h/(8\pi BKa^2)$.

The *orientational* correlation lengths $\xi_{\perp,z}^o$ can similarly be defined as the values of $L_{\perp,z}$ beyond which the mean squared orientational fluctuations $\overline{\langle|\delta\mathbf{n}|^2\rangle}$ of Eq. (14) get to $O(1)$. This gives $\xi_\perp^o = ae^{4\pi B^2\lambda^3/\Delta_h} = ae^{\xi_z\lambda/2a^2}$, $\xi_z^o = (a^2/\lambda)e^{2\pi B^2\lambda^3/\Delta_h} = (a^2/\lambda)e^{\xi_z\lambda/4a^2}$. Thus, orientational order persists out to *much* larger distances than translational order, in the limit of weak disorder where *all* the correlation lengths get large.

All of the above results apply subject to our two initial assumptions that (1) dislocations were *not* generated by the disorder, and (2) anharmonic terms in the elastic free energy could be neglected.

We will show now that if we continue to assume (2) (whose validity we will investigate in a future publication [4]), assumption (1) is wrong: in the *harmonic* elastic approximation, dislocations *are* created even by arbitrarily weak disorder. However, they are felt only on length scales longer than $\xi_{\perp,z}^o$, and hence much longer than the translational correlation lengths. Thus, our above calculations of these lengths remain valid.

We can include dislocations in the "tilt only" model, i.e., Eq. (4) with $V(\mathbf{r}) = 0$. As discussed earlier, this theory correctly reproduces all of the static correlation functions in *both* the glassy and the nonglassy regimes.

The dislocations are characterized by an integer-valued 3D vector field $\mathbf{m}(\mathbf{r})$ defined on the sites $\mathbf{r}$ of a lattice connected to the displacement field u via $\nabla \times \mathbf{v} = \mathbf{m}$, with $\mathbf{v} \equiv \nabla u$, and a dislocation line continuity constraint $\nabla \cdot \mathbf{m}(\mathbf{r}) = 0$. Standard manipulations [4,9] lead to a Coulomb gas theory of these dislocation loop defects,

$$H_d = \frac{1}{2}\int_{\mathbf{q}}\left[\frac{Kq_\perp^2}{\Gamma_q}P_{ij}^\perp m_i(\mathbf{q})m_j(-\mathbf{q}) + \mathbf{m}(\mathbf{q})\cdot\mathbf{a}(-\mathbf{q})\right], \tag{16}$$

where the inverse of the pure smectic propagator $\Gamma_q \equiv q_z^2 + \lambda^2 q_\perp^4$, $P_{ij}^\perp(\mathbf{q}) = \delta_{ij}^\perp - q_i^\perp q_j^\perp / q_\perp^2$, and $\mathbf{a}(\mathbf{q})$ is a Fourier transform of the quenched field related to the original random tilt field $\mathbf{h}(\mathbf{q})$ via $-i\mathbf{a} = \mathbf{q} \times \mathbf{h}/q^2 - (\hat{\mathbf{z}} \times \mathbf{q})\mathbf{q} \cdot \mathbf{h}\, q_z(1 - \lambda^2 q_\perp^2)/(\Gamma_q q^2)$.

The partition function for this model is then $Z[\{\mathbf{h}\}] = \sum'_{\{\mathbf{m}(\mathbf{r})\}} e^{-S[\{\mathbf{m}\}]}$, where $S \equiv H_d/T + E_c/T \sum_\mathbf{r} |\mathbf{m}(\mathbf{r})|^2$, the sum is over all integer-valued configurations of $\mathbf{m}$ satisfying the dislocation line continuity constraint $\nabla \cdot \mathbf{m} = 0$, H_d is given by Eq. (16), and we have added a core energy term $E_c \sum_\mathbf{r} |\mathbf{m}(\mathbf{r})|^2$. To proceed, we enforce the constraint $\nabla \cdot \mathbf{m} = 0$ by introducing a new auxiliary field $\phi(\mathbf{r})$, and introduce a dummy gauge field $\mathbf{A}$ to mediate the long-ranged interaction between defects loops in the Hamiltonian Eq. (16) obtaining

$$Z = \prod_\mathbf{r} \int d\phi(\mathbf{r})\, d\mathbf{A}(\mathbf{r}) \times \sum_{\{\mathbf{m}(\mathbf{r})\}} e^{-S[\{\mathbf{m}\},\phi,\mathbf{A}]} \delta(\nabla \cdot \mathbf{A}) \delta(A_z),$$

$$S = \beta \sum_\mathbf{r} [\mathbf{m}(\mathbf{r}) \cdot (-i\nabla\phi(\mathbf{r}) + i\mathbf{A}(\mathbf{r}) + \mathbf{a}(\mathbf{r})) + E_c |\mathbf{m}|^2] + \frac{1}{2} \sum_\mathbf{q} \frac{\Gamma_q}{K q_\perp^2} |\mathbf{A}|^2. \quad (17)$$

Performing the summation over the dislocation loop degrees of freedom, replacing the resulting Villain potential by a cosine, generalizing to a "soft spin" model described by a "disorder" parameter $\psi = |\psi| e^{i\phi}$, we finally obtain [4] a *complex* "action" S

$$S = \sum_\mathbf{r} [(\nabla + i\mathbf{A} + \mathbf{a})\psi^* (\nabla - i\mathbf{A} - \mathbf{a})\psi + t|\psi|^2 + u|\psi|^4] + \sum_\mathbf{q} \frac{\Gamma_q}{2K q_\perp^2} |\mathbf{A}(\mathbf{q})|^2, \quad (18)$$

A complete discussion of the behavior of the above model [4] is outside the scope of this Letter; here we are only interested in the question of whether the dislocations loops will or will not unbind at any, even infinitesimal, amount of disorder. Using replicas and computing the disorder-averaged free energy, we find that the lowest order contribution to the renormalized *dual* temperature t_R comes from the average of the "diamagnetic" term $\delta S = \sum_\mathbf{r} (\langle |\mathbf{A}|^2 \rangle - \overline{|\mathbf{a}|^2})\, |\psi|^2$, which gives

$$t_R = t_0 + (d-2) \int \frac{d^d q}{(2\pi)^d} \left[\frac{K q_\perp^2}{\Gamma_q} - \frac{\Delta_h q_z^2 q_\perp^2}{q^2 \Gamma_q^2} \right], \quad (19)$$

where we have used the connection between $\mathbf{a}(\mathbf{r})$ and the quenched tilt disorder $\mathbf{h}(\mathbf{r})$, averaged over $\mathbf{h}(\mathbf{r})$ using its distribution function, and generalized to d dimensions.

The second, disorder, term in this integral dominates the first as $\mathbf{q} \to 0$. Indeed, this integral diverges in the infrared for $d \leq 3$

$$\int d^d q \, \frac{q_z^2 q_\perp^2}{q^2 (q_z^2 + \lambda^2 q_\perp^4)^2} \propto \int \frac{d^{d-1} q_\perp}{q_\perp^2}, \quad (20)$$

where we used the fact that the dominant regime of the integral is $q_z \sim \lambda q_\perp^2$. This divergence implies that t_R is driven to $-\infty$ (note the minus sign) by the disorder in $d = 3$. Indeed, we find in 3D $t_R = t_0 - \frac{\Delta_h}{\lambda} \ln(L/a) \times O(1)$, where L is an IR cutoff (e.g., the lateral extent of the smectic layers) and a is the UV cutoff (e.g., ~ 10 Å, the size of the liquid crystal molecules).

This implies that the *dual* (dis-)order parameter is always in its ordered phase, which, in turn, implies that the dislocation loops of the original smectic model are always *unbound*, thereby destroying the smectic order, even at $T = 0$, for any infinitesimal amount of disorder.

These results of the *harmonic* theory imply that for 3D disordered smectics the dislocations are unbound, even at $T = 0$, for arbitrarily weak disorder. This means that there is no thermodynamically *sharp continuous* NA transition, and the low temperature phase must have a *finite* smectic translational correlation length (even at $T = 0$), once the liquid crystal is put in aerogel. A *first* order transition between nematics with short and long (but finite) smectic correlation lengths, or a smeared analytic crossover, are, of course, always possible and have been observed in experiments of Refs. [2] and [1], respectively. We have also shown [4] that once the dislocations are unbound, the static *director* fluctuations are precisely those of a *nematic* in a random tilt field. This implies that the system can be thought of as a nematic in such a random field, at all temperatures and all disorder strengths. This, in turn, can be shown [4] to imply that long-ranged orientational (nematic) order is destroyed as well, again in agreement with experiments [1].

Finally, preliminary investigation of *anharmonic* elastic effects suggests that they *may* prevent dislocation unbinding and stabilize orientational order [4]. Furthermore, our results *do not* imply that there is no thermodynamically sharp phase transition in this system analogous to the *nematic to isotropic* phase transition in the pure system. Indeed, preliminary investigation suggests that a kind of nematic Bragg glass phase may exist in these systems. This possibility will be further discussed in a future publication [4].

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