Gyro-Bohm Scaling of Ion Thermal Transport from Global Numerical Simulations of Ion-Temperature-Gradient-Driven Turbulence

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The ion gyroradius scaling of ion thermal transport caused by ion-temperature-gradient-driven turbulence is studied with a global fluid simulation code in three-dimensional toroidal geometry. It is found that the effective conductivity scales like the ion gyroradius (gyro-Bohm scaling). [S0031-9007(97)04510-9]

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Understanding the origin of the empirical scaling laws of tokamak confinement as a function of the various plasma parameters is one of the challenging areas of theoretical investigations. Particularly crucial for the extrapolation of present day scaling laws to future devices is the assessment of the dependence of tokamak transport on the scale separation parameter $\rho_* = \rho_s/a$, where ρ_s is the ion sound Larmor radius (the Larmor radius measured at the electron temperature) and a is some macroscopic machine scale length (usually the tokamak minor radius).

Whereas the actual scaling laws may eventually turn out to be the result of different scaling regimes in different regions of the discharge, attempting to understand the transport in individual regions has been the favored approach to this complex problem. In recent years, the leading candidate for ion thermal transport in the "good-confinement" region (the intermediate region between the surface at which the safety factor is equal to 1 and the plasma edge) has been the ion-temperature-gradient-driven (ITG) turbulence [1,2]. Thus it is natural to investigate the ρ_* dependence of the ion thermal transport caused by ITG turbulence.

In the following brief review of the ρ_* scaling problem, "large scale" normalizations are adopted; that is, lengths are measured in units of a and times in units of $a^2/(cT_e/eB)$, where T_e is the electron temperature, B the toroidal magnetic field, e the electron charge, and c the speed of light. In these units, the transport coefficients are measured in Bohm units cT_e/eB. Thus, in general, the ion thermal conductivity χ will scale as $\chi \sim \rho_*^\alpha$, where $\alpha = 1$ for gyroradius scaling (the so-called gyro-Bohm scaling) and $\alpha = 0$ for Bohm scaling, and the dependence on other parameters like the temperature gradient, the safety factor, the magnetic shear, and the aspect ratio is not of concern in the present discussion.

Turbulent transport coefficients are often estimated by means of heuristic dimensional arguments, such as $\chi \sim \lambda^2/\tau$, where λ and τ are some suitable length and time. In the case of transport of pollutants (like trace impurities in a tokamak), it is natural to identify λ with the step size of a random walk occurring in a time τ. In the case of ITG turbulence, the relevant quantity to compute is the ion heat flux F which takes the form of a correlation function of the fluctuating velocity $\mathbf{v}_E = c(\mathbf{E} \times \mathbf{B})/B^2$ and the fluctuating ion temperature $\tilde{T}_i$, $F = \langle \mathbf{v}_E \tilde{T}_i \rangle$, where E is the fluctuating electric field and $\langle \bullet \rangle$ denotes average over the fluctuations time scale and (possibly) over the magnetic surface. It is not clear how to relate λ and τ to F in general, and one may well take the attitude to compute only the flux. Still, much research has been carried out attempting to identify suitable λ and τ from the features of the fluctuating field. This is briefly reviewed here.

Linear theory in a cylinder produces eigenfunctions whose radial extension δr_{cyl} scales like ρ_*. Thus, identifying λ with δr_{cyl}, $\lambda \sim \rho_*$, and τ with the inverse of the mode frequency [which for ITG is the drift frequency $\omega_{*T} = (cT_e/eB)(k_\theta/L_T)$ associated with the temperature scale length L_T], $\tau \sim \rho_*$, one could conclude that $\chi \sim \rho_*$ (gyro-Bohm scaling).

However, recent work on the linear theory in a torus [3–5] has shown that the radial extension of the eigenfunctions changes to $\delta r_{\text{tor}} \sim (\rho_s L_T)^{1/2} \sim \rho_*^{1/2}$ due to toroidal coupling, when the calculations are taken to second order [6] in the ballooning formalism [7]. Since the scaling of the frequency is left unchanged by the toroidal coupling, the identification of λ with δr_{tor} leads to $\chi \sim$ const, independent of ρ_* (Bohm scaling).

In the nonlinear regime, Cowley *et al.* [8] have shown that radially elongated structures tend to be unstable to secondary instabilities which reduce their aspect ratio, making the resulting vortices somewhat roundish. Such a tendency to isotropization is well known from many numerical simulations of various models. Thus one can conclude that the radial extension of the vortices should scale like the poloidal extension, but this is

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not enough to determine the gyroradius scaling. In a recent work [9] it has been suggested that the poloidal scale length (and hence the radial scale length) can be estimated as the inverse of the poloidal wave number of marginally stable eigenmodes. The rationale behind this hypothesis is the tendency of turbulent fluctuations to cascade towards large scales; it is then natural to assume that this process would stop where the fluctuations are damped. Naive linear theory carried out by replacing the parallel derivative operator $\nabla_\parallel$ with a constant, $\nabla_\parallel \to 1/(qR)$, where q is the safety factor and R the major radius, shows that modes of sufficiently long wavelength are stabilized by ion Landau damping when $\omega_{*T} \approx c_s/(qR)$. This implies $\delta r \sim \rho_s(qR/L_T)$, which yields again a gyro-Bohm scaling. The natural objection to the latter estimates is that naive linear theory is incorrect. Indeed it is known that unstable linear eigenmodes that defy Landau damping by assuming parallel derivatives smaller than $1/(qR)$ are possible [10]. Although it is plausible that these long wavelength eigenmodes may not play an important role in the turbulent dynamics because of their small growth rate, their existence still casts a doubt on the above construction.

It is the scope of this Letter to assess the problem of the gyroradius scaling by investigating ITG transport by means of direct numerical simulations of a relevant model. Our main conclusion is that ITG thermal transport, at least well above the instability threshold, has gyro-Bohm scaling.

Direct numerical simulation has been the favored tool to investigate tokamak turbulence in recent years. One can refer to Ref. [11] for a partial review. Two types of approaches have been pursued, gyrokinetic and gyrofluid. Furthermore, the various codes can be characterized as either global or local. Local codes are unsuitable to study the scaling with ρ_* because they rely on an ordering of the fluctuation scale length that assumes gyro-Bohm scaling. This is often done for the practical purpose of simplifying the nonlinear terms. Global codes can advance the model equations either in the full torus or in a toroidal annulus (the volume contained between two specified flux surfaces). They can in principle address the ρ_*-scaling problem, provided that the simulations are run to steady state for at least one (ion) energy confinement time. However, the global ITG simulations available in the literature (which are all gyrokinetic [12–14]) have been carried out in "decaying mode," i.e., initializing the code with some temperature profile and allowing it to relax under the effect of the ensuing fluctuations, without energy injection. The consequence is that transport is studied on some intermediate time scale when turbulence is in a relaxed state, often with temperature profiles close to marginal stability. The implications of this approach are discussed below.

In this Letter the ITG scaling problem is analyzed with a global fluid code, with a focus on forced turbulence steady states well above marginality. The minimal ITG model can be written as

$$dw/dt + 2\epsilon\omega_d(\Phi + T_i) + A\nabla_\parallel v = D_w\nabla^2 w - \gamma_{\rm pfd}\rho_*^2\langle\Phi\rangle, \quad (1)$$

$$dv/dt + A\nabla_\parallel(\Phi + T_i) = D_v\nabla^2 v, \quad (2)$$

$$dT_i/dt + \Gamma\langle T_i\rangle A\nabla_\parallel v = -A\langle T_i\rangle^{1/2}|\nabla_\parallel|T_i + D_T\nabla^2 T_i, \quad (3)$$

where $w = (\Phi - \langle\Phi\rangle)/T_e - \rho_*^2\nabla^2\Phi$ is the generalized vorticity (effectively the ion guiding center density), Φ is the electric potential, v the parallel ion velocity, T_i the ion temperature, $d/dt = \partial_t + \mathbf{v}_E \cdot \nabla$ the advection operator, $\omega_d = (1/r)\cos\theta\,\partial_\theta + \sin\theta\,\partial_r$ the curvature operator, $\nabla_\parallel = (1/q)(q\partial_\phi + \partial_\theta)$ the parallel derivative operator, $\langle\bullet\rangle$ denotes flux surface average, $A = \epsilon/\rho_*$, and Γ is a constant. Units of T_e for the temperature, T_e/e for the potential, and $c_s = (T_e/M_i)^{1/2}$ for the velocity are employed. The main control parameters are ρ_* and the aspect ratio ϵ. Furthermore, D_w, D_v, and D_T are small artificial perpendicular dissipation coefficients set to damp the smallest scales and $\gamma_{\rm pfd}$ models the poloidal flow damping. The model is written for a low-β plasma with circular magnetic surfaces identified by the radial coordinate r; θ and ϕ are the poloidal and toroidal angles, respectively.

This model can be viewed as a simplified version of the "3 + 1" gyrofluid model [15], where the pressure tensor is taken isotropic and a number of finite Larmor radius (FLR) terms are dropped. This is justified by the interest in the dynamics of long wavelengths: if long wavelengths play a dominant role in determining the transport scaling, then FLR terms are irrelevant. In general, a shift of the peak of the spectrum from $\rho_s k_\theta \sim 1$ to larger scales is expected in the saturated nonlinear state.

The dominant damping mechanism is ion Landau damping, which is modeled by the $|\nabla_\parallel|$ term in Eq. (3), following the prescription employed in gyrofluid models [16]. Since the goal is to study the scaling behavior and not to reproduce accurate predictions, the constant in front of the Landau damping operator is set to unity. Similarly Γ is also set to unity. Unlike the 3 + 1 gyrofluid model, enforcing the isotropy of the pressure tensor prevents the model from effectively damping the self-generated poloidal flow. Therefore the poloidal flow damping must be introduced artificially by setting $\gamma_{\rm pfd}$ with the correct scaling. Since the actual damping is proportional to the ion transit frequency $v_i/(qR)$ [15], one must take $\gamma_{\rm pfd} = \gamma_0(\epsilon/q\rho_*)$, where γ_0 is a constant of order one ($\gamma_0 = 0.25$ throughout this study). It turns out that, with this choice of γ_0, the poloidal $\mathbf{E} \times \mathbf{B}$ flow is comparable to the diamagnetic flow. It was previously verified [15,17] that, when the flow damping is set to zero, the plasma accelerates and the transport drops substantially.

A further simplification is introduced by setting $\langle T_i\rangle = T_e$, where $\langle T_i\rangle$ appears as a coefficient in front of the operators of Eq. (3) and taking the electron temperature constant $T_e = 1$. Thus Eqs. (1)–(3) are reinterpreted as the equations of the evolution of the ion temperature gradient normalized to T_e/a.

In the following, the simulation domain is an annulus with inner radius $r_a = 0.5$. Flux boundary conditions, with prescribed heat flux, are taken at $r = r_a$, while $T_i = 0$ at $r = 1$. The fluctuating components are set to zero at the boundaries. In order to inject the desired amount of energy, the product $F_{\text{in}} = D_T \nabla T_i$ must be fixed at $r = r_a$. In order to avoid excessively high gradients at the inner boundary, it is convenient to set D_T sufficiently large at $r = r_a$ and gradually decrease it as a function of radius until it reaches its nominally small value at some $r = r_b$. The range $r_a < r < r_b$ defines a buffer region where the turbulence is gradually switched on. In this Letter $r_b = 0.6$. In order to simplify the analysis the shear parameter $\hat{s} = rd \ln q/dr$ is taken constant, $\hat{s} = 1$, and the safety factor profile is therefore linear in the region of interest, $q = q_a r$ with $q_a = 4$, so that $2 < q < 4$.

Equations (1)–(3) are advanced with a hybrid code, spectral in the angles and finite difference in radius. Time advancing is carried out with a modified leap-frog algorithm that has been found convenient for turbulence simulations due to its weak dissipativity.

The ρ_* scaling study is carried out by first running a long simulation at $\rho_* = 1/50$ until a steady state was reached. A suitable resolution for this run is $81 \times 128 \times 32$ (radial $\times$ poloidal $\times$ toroidal) points. Other parameters are $\epsilon = 1/2$, $F_{\text{in}} = 0.01$, $D_w = D_v = D_T = 0.001$ [with $D_T(r_a) = 0.01$]. The instantaneous confinement time $\tau_E = E_{\text{th}}/[D_T(r = 1)\nabla T_i(r = 1)]$ (where E_{th} is the total ion thermal energy) turns out to be $\tau_E \approx 18$, which is shorter than the total simulation time ($t_{\text{sim}} = 20$), thus confirming that a steady state is indeed achieved.

In a second run ρ_* is set to $\rho_* = 1/100$ and the energy injection is halved, $F_{\text{in}} = 0.005$. The dissipation coefficients are also halved, all the other parameters being held fixed. The initial conditions are given by the configuration obtained at the end of the first run. The code has been run at a resolution of $121 \times 192 \times 48$ for another 15 units of time. The main result is that the confinement time doubles to $\tau_E \approx 36$ (Fig. 1), while the temperature profile remains almost unchanged (Fig. 2). In terms of the effective conductivity $\chi = F_{\text{in}}/\nabla T$, one can deduce $\chi \sim \rho_*$ from this numerical experiment.

The contour plots of a poloidal cross section of the electric potential are shown in Fig. 3 for the two cases. Note that the vortices are almost isotropic, in contrast with the elongated structures predicted by linear theory and in agreement with Cowley *et al.* [8]. It is apparent that the vortex size (both poloidal and radial) decreases with ρ_*, suggesting a scaling $\lambda_c \sim \rho_*$ for the radial correlation length λ_c. A more precise way to evaluate λ_c is to compute the radial correlation function $C(\delta r) \equiv \langle \phi(r + \delta r, \theta)\phi(r, \theta)\rangle$, where the average is taken over the poloidal angle and over the time when the fluctuation level is roughly stationary. It is found that the correlation function essentially vanishes for $\delta r \approx 10\rho_*$. This result provides an estimate for the radial correlation length, $\lambda_c \approx 10\rho_*$ (for our set of parameters). The nonlinear correlation time τ_c was also found consistent with the scaling $\tau_c \sim \rho_*$. Thus the nonlinear time and length scales are compatible with the observed scaling for the effective conductivity $\chi \sim \lambda_c^2/\tau_c \sim \rho_*$. Finally, we verified that the fluctuation level is also about halved in the $\rho_* = 1/100$ case.

A simulation was also run by reinitializing the fluctuations to a very low level, while maintaining the same profiles for the macroscopic fields. After a transient phase, the fastest growing eigenfunctions are selected and grow exponentially. These are radially extended modes composed by several poloidal harmonics, as predicted by the linear theory of toroidal drift waves. At a later time, the

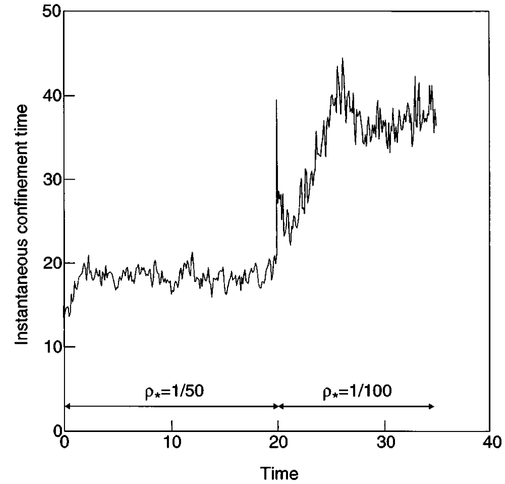

FIG. 1. Instantaneous confinement time for both simulations. At $t = 20$, ρ_* is switched from $\rho_* = 1/50$ to $\rho_* = 1/100$.

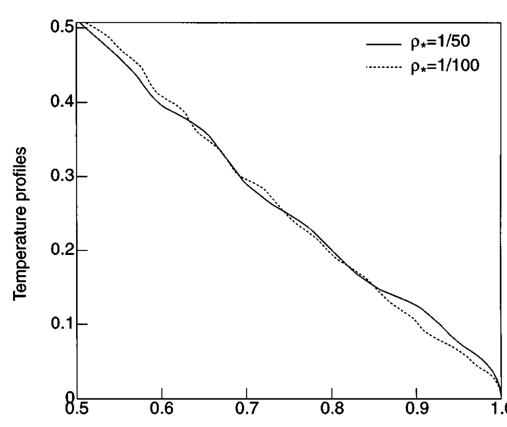

FIG. 2. Temperature profiles at the end of the simulations with $\rho_* = 1/50$ ($t = 20$) (solid) and $\rho_* = 1/100$ ($t = 35$) (dashed).

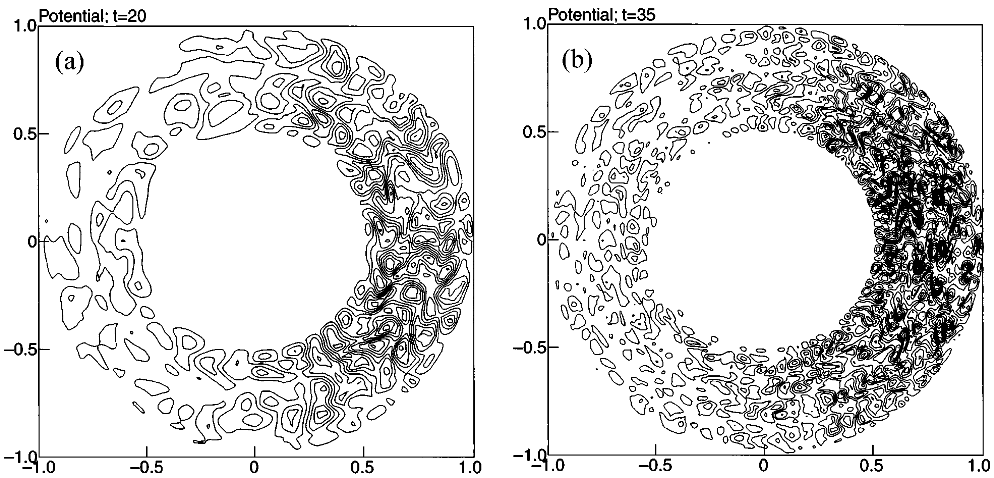

FIG. 3. Poloidal cross section showing the isolines of Φ. (a) $\rho_* = 1/50$; (b) $\rho_* = 1/100$.

nonlinear coupling causes these structures to break down and the characteristic radial length is reduced.

Thus, all our observations support the main conclusion of this Letter, that ion transport in the ITG model scales like gyro-Bohm, at least in the present regime, which is well above the stability threshold.

This conclusion differs from what was obtained in recent papers based on global gyrokinetic simulations [12–14], where transport is close to Bohm. However, as explained before, those simulations were carried out in decay mode. The consequence is that transport is studied in a slowly evolving relaxed state close to marginal stability. In these conditions, one expects the dynamics to be dominated by a small number of degrees of freedom associated with few radially extended linear eigenfunctions, which are the first modes to be destabilized when the stability boundary is crossed.

An alternative explanation of the scaling behavior near threshold is put forth by Garbet and Waltz [18]. By employing a reduced model with prescribed radial shape of each (m, n) components, these authors found that the effect of the $\mathbf{E} \times \mathbf{B}$ flow is to introduce corrections to the basic gyro-Bohm scaling in the form $\rho_* \rightarrow \rho_*(1 - \alpha_*\rho_*)$, where α_* is a constant, nominally of order 1, which measures the strength of the flow. The effect is important at moderate ρ_* and when the flow is large, especially near threshold where the leading term is small. Thus Garbet and Waltz attribute the deviations from gyro-Bohm to a change of the stability properties near threshold rather than to a change of the scaling of the correlation length.

In summary, we have performed numerical experiments based on a simplified 3D fluid model which retains the fundamental terms needed for a correct description of toroidal ITG turbulence. Simulations run for over an ion energy confinement time strongly indicate that the effective ion thermal conductivity obeys a gyro-Bohm scaling law. This is in disagreement with recent gyrokinetic results obtained, however, in a different regime, close to marginal stability. Further work with our code is required to clarify this point.

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