One-Channel Time Reversal of Elastic Waves in a Chaotic 2D-Silicon Cavity

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(Received 17 March 1997; revised manuscript received 25 March 1997)

Acoustic time-reversal experiments usually need large arrays of transducers. It is shown that in a reflecting cavity with negligible absorption one is able to perform a time reversal of elastic waves using a single element. The field is measured at one point over a long period of time and the time-reversed signal is reinjected at the same position. Numerical simulations illustrate the process. Experiments carried out in silicon wafers show that it is possible to obtain an excellent temporal and spatial focusing quality. [S0031-9007(97)03576-X]

PACS numbers: 43.20.+g, 05.45.+b, 62.30.+d

In the last few years, time reversal of elastic waves has grown from its initial experimental setup to a method opening a large field of applications as well as a powerful tool for fundamental investigations [1]. An acoustic time-reversal (TR) device uses the time-reversal invariance of the wave equation in lossless media. It tries to create the time-reversed counterpart of a given propagating wave, i.e., it attempts to generate a wave that has the same shape as the given one, but which propagates in the reverse direction, just as if time were counting backwards. In a time-reversal experiment the wave generated by an acoustic source is first measured by an array of reversible piezoelectric transducers located around the source. The acoustic field is sampled, recorded, time reversed, and reemitted by the array. The time-reversed wave thus created backpropagates and eventually focuses on the location of the initial source, now remaining passive. In theory, the transducers must cover a closed surface around the medium to obtain information about all wave fronts propagating in all directions. This fact leads to the concept of the closed time-reversal cavity [2]. In practice, TR cavities are difficult to realize and the TR operation is usually performed on a limited angular area (TR mirror—TRM), thus limiting reversal and focusing quality. The TRM consists typically of some hundred elements, or channels. Derode *et al.* [3] have found that the focusing quality is improved if the wave traverses a multiple scattering medium before arriving on the transducer array. Through media with high order multiple scattering, the large length of paths involved in the experiment widens the effective focusing aperture. After the time-reversal operation, the whole multiple scattering medium behaves as a coherent focusing source, with a large angular aperture for enhanced resolution. As a consequence, in multiple scattering media, one is able to reduce the number of channels participating in the TR process. However, in this case, reversal quality depends crucially on the duration of the time-reversal window, i.e., the length of the recording to be reversed.

In this paper, we are interested in another aspect of multiply scattered waves: waves confined in closed reflecting cavities such as elastic waves propagating in a silicon wafer. With such boundary conditions, no information can escape from the system and a reverberant acoustic field is created. If, moreover, the cavity shows ergodic properties and negligible absorption, one may hope to collect all information at only one point. We present in this Letter experiments and numerical simulations proving that a time reversal can be obtained *using only one TR channel* operating in a closed cavity. The improvement of reversal quality with increasing time-reversal window is quantified. However, it is shown that, even for windows of infinite duration, reversal quality is limited because of multiple reflections passing over the TR transducer.

First, we present a finite difference simulation of a scalar wave field inside a 2D cavity in the shape of a disk with one segment cut off. The corresponding billiard of the same form shows chaotic properties [4], so this form has been chosen to ensure ergodicity. In addition, it mimics the silicon cavity used in experiments described below. The simulation has been performed using a method of second order central differences in space and time on a grid of 320×320 points with Dirichlet boundary conditions. The resolution is $\Delta x = 0.13$ mm with a time step $\Delta t = 0.011\ \mu$s for a sound speed of 8000 m/s = 8 mm/μs. The finite difference scheme is (numerically) weakly anisotropic and dispersive, but this fact does not seem to harm the process.

Inside the cavity, we introduce two particular points A and B, both able to act as a source or as a measuring instrument. A starts at $t = 0$ by emitting a short pulse of Gaussian shape of width $\tau = 0.04\ \mu$s [Fig. 1(a) at $t = 0.5\ \mu$s]. After several reflections 1(b), 1(c), the wave field acquires a foggy distribution in which wave fronts can no longer be distinguished 1(d), 1(e). At point B, the resulting wave field is recorded during a long period of time $T = 700\ \mu$s without perturbing propagation. At the end, we stop wave motion and clear the cavity. Then B reemits a part $\Delta T = 685\ \mu$s of the measured signal, which has been time reversed. Note that the reemission is omnidirectional, the information about the direction of each measured wave front is lost. Using a new origin of the

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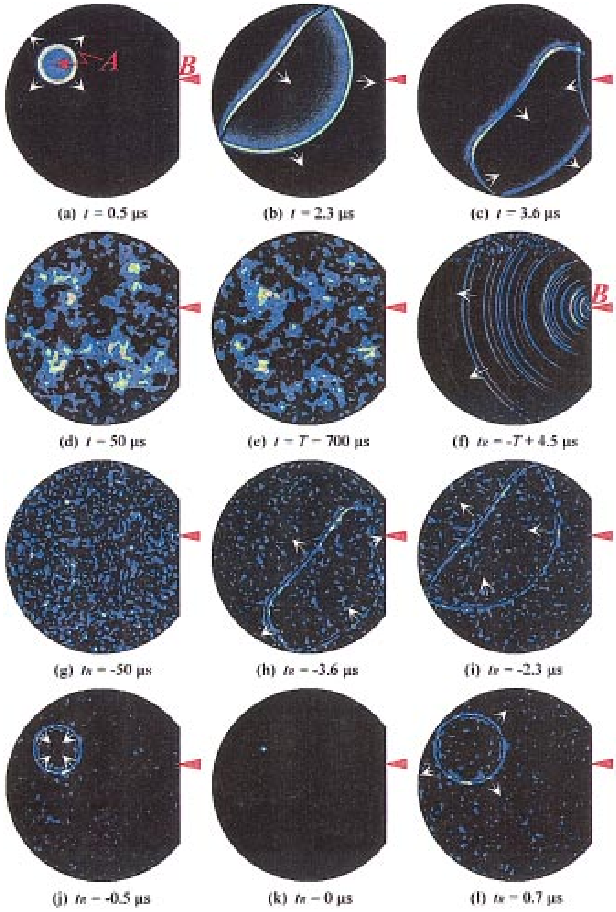

FIG. 1(color). Finite difference simulation of a one-channel time reversal. (a)–(e) Injection of a short pulse in A and recording in B. (f)–(l) Emission of the time-reversed signal in B and refocusing.

time scale, the reemission 1(f) begins at $t_R = -T$. The wave field passes again by a foggy distribution 1(g) until, shortly before refocusing, a wave front emerges from noise 1(h), 1(i). It has the same shape as the initially emitted one, but propagates in the reverse direction. So it generates a collapsing circular wave 1(j) that focuses in A at $t_R = 0$, 1(k). After focusing, the wave diverges again 1(l).

The success of this time-reversal experiment is particularly interesting in two aspects. First, *it proves the feasibility of time reversal in wave systems with chaotic ray dynamics.* An equivalent experiment would not be possible in the case of classical trajectories: Despite the time-reversal invariance of the governing equations, a time reversal would fail due to strong sensitivity to initial conditions. Paradoxically, in the case of a one-channel time reversal, chaotic dynamics is not only harmless but even useful, as it guarantees ergodicity. Second, using a source of vanishing aperture, we obtain an almost perfect focusing quality. The procedure approaches the performance of a closed time-reversal cavity which has an aperture of 360°. A one-point time reversal in a chaotic cavity hence produces better results than a TRM in an open system. Using reflections at the edge, focusing quality is not aperture limited, and in addition, the collapsing wave front approaches the focal spot from all directions.

However, though one obtains excellent focusing, a one-channel time reversal is not perfect, as a weak noise level throughout the system can be observed. To describe the origin of this noise, it is convenient to consider the temporal behavior of the elastic wave at a given point. This will be done through some basic experiments carried out on silicon cavities.

We have carried out experiments using elastic waves propagating in a monocrystalline silicon wafer. The material was chosen for its weak adsorption. In fact, the decay periods of 10 to 20 ms are much greater than the largest time-reversal windows used (1.5 ms), so that the mean intensity level inside can be considered as being constant.

The wafers used are disks of thickness 525 μm and diameter between 100 and 125 mm, with a segment of variable size cut off. There are several modes of propagation possible (*SH* and two others corresponding to the Lamb modes $S0$ and $A0$), propagation is dispersive and anisotropic. But the observed phenomena can be described successfully without taking these facts into account. Wave speeds vary between 2000 and 8000 m/s and rays in the cavity encounter typically between 80 and 400 reflections during a time-reversal experiment.

Elastic waves are injected by aluminum cones fixed to transverse transducers (Fig. 2). Their tip touches the wafer and, vibrating in a given polarization, excites the different propagation modes. In addition, they can act as measuring instruments for the same polarization. After excitation, their influence on the billiard is negligible. All transducers are lined to the electronics of a time-reversal mirror, able to send, receive, and time reverse arbitrary signals. We use transducers with a central frequency of 1 MHz and a bandwidth of 100%. The electronics of the TRM samples each channel at a 10 MHz rate and digitizes them on 8 bits. Each one receives and emits up to 15 000 sampling points, allowing a total of 1.5 ms.

The experiment is a two-step process as described above: In the first step, we inject with one of the transducers, called A, a short signal of duration 0.5 μs (central frequency 1 MHz) into the wafer [Fig. 3(a)].

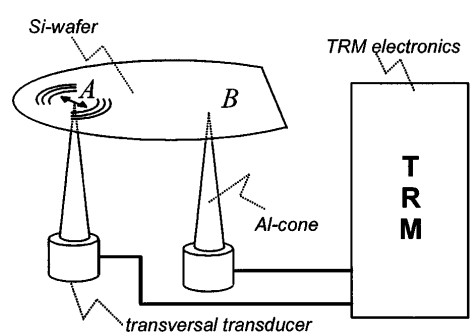

FIG. 2. Experimental setup.

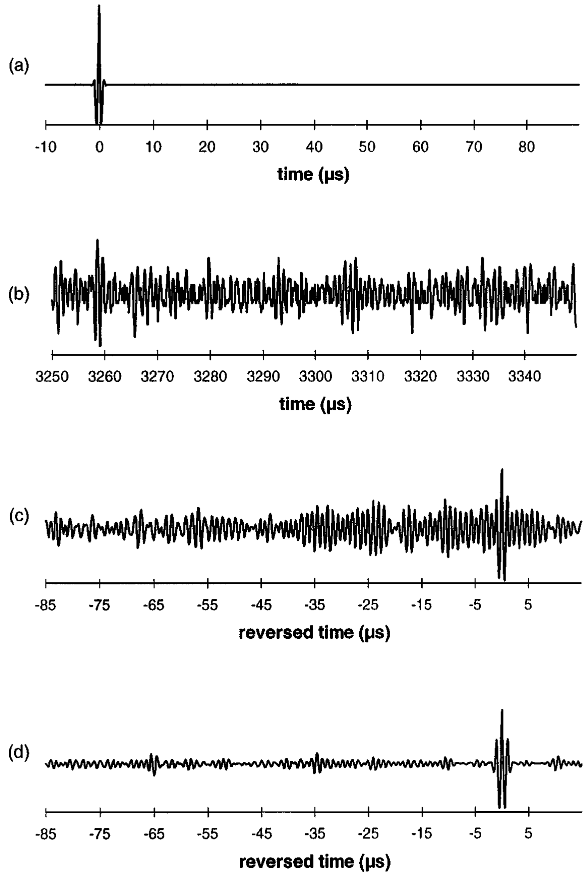

FIG. 3. Some characteristic signals. (a) Excitation pulse introduced at point A into the wafer. (b) Typical shape of a signal received at point B. (c) and (d) Typical refocusing peaks obtained at point A, applying a time-reversal window of $\Delta T = 40\ \mu$s and $\Delta T = 1500\ \mu$s, respectively.

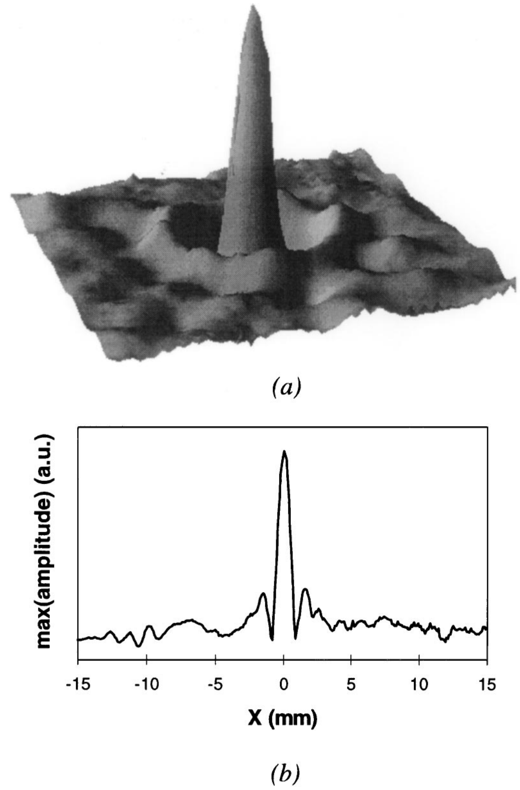

FIG. 4. $A0$ component of the focal spot. (a) Wave field at $t_R = 0$ on a square of 15×15 mm. (b) cut of the focal spot.

Another transducer, called B, observes a long signal whose appearance is simple noise [Fig. 3(b)], but which in fact still contains information. It starts recording after a given time t_0 and ends at time T. (The beginning can be immediately after the emission or after a long period time, so t_0 is typically between 10 μs and 6 ms = 6000 μs.) In the second step, a part of this signal, the time-reversal window ΔT (typically 10 μs–1.5 ms), is time reversed and reemitted by B. We now observe, in A, a signal of the same noisy shape as before in B. But exactly at time T after the reemission, one can see a sharp peak, standing out more or less from the surrounding (temporal) side lobes, depending on the size of the time-reversal window ΔT. Figures 3(c) and 3(d) show two of these refocusing peaks: the first one for small ΔT, the second one for a large value of ΔT. A large window is favorable to obtain a sharp peak.

We have used an optical interferometer to analyze the spatial distribution of the refocusing peaks. In fact, $A0$ is the only propagation mode which is detectable by the interferometer in this frequency range. Figure 4(a) shows the wave field at $t_R = 0$ for a time-reversal window of $\Delta T = 1.5$ ms on a square of 15×15 mm around the focal spot, sampled in steps of 0.25 mm. We observe a focal spot of circular symmetry. Figure 4(b) shows a cut of the focal spot using a step of 0.05 mm. Its -6 dB width of 1.1 mm corresponds to a perfect focusing (i.e., $\lambda/2$) of an $A0$ wave whose phase velocity has been evaluated experimentally to 2600 m/s at the observed signal's central frequency of 1.2 MHz.

As ΔT grows to infinity, the refocusing signal in A tends, after normalization, to an asymptotic signal $S_\infty(t_R)$. The amplitude of the refocusing signal observed in A increases linearly with ΔT: $s_{\Delta T}(t_R) = \Delta T S_\infty(t_R)$ as $\Delta T \to \infty$. This is an interesting aspect of focusing experiments: The amplitude of the focal spot can be increased not only by amplification of the emitted signal, but also by emission of a *longer* signal. Hence this technique permits high-amplitude focusing using a single transducer that is limited in power.

However, S_∞ does not, as may be hoped for focusing experiments, consist of a single peak at $t_R = 0$. In fact, one still observes low-level residual side lobes around the peak [see Fig. 3(d); as ΔT is large, the shape of the observed signal here is close to S_∞].

The origin of the residual side lobes is described below, but it is useful to understand first how the refocusing signal converges to $\Delta T\ S_\infty$ for finite ΔT. A qualitative response can be found by the following reasoning: Let $f(t)$ be the signal emitted by the initial source A. Imagine that B receives a signal which is a superposition of multiply reflected signals of the same shape. During reemission, B sends each reversed signal not only in the direction where it came from, but in all directions. A signal taking the "good" direction traces exactly its way back and arrives at $t_R = 0$ in A. Thus we obtain at this point a constructive interference, giving a peak of shape $f(-t_R)$. Its amplitude increases linearly with the size of the time-reversal window. The signals taking the "bad" direction may also arrive in A, but their superposition may be considered as a purely random process. Therefore the amplitude of the noise created in this way increases linearly with $\sqrt{\Delta T}$. The peak-to-noise ratio is finally found to increase with the square root of the time-reversal window, and hence the intensity ratio increases in a linear way. The proportionality factor can be predicted by a more rigorous reasoning and compared to experimental data. In Fig. 5, we compare the theoretical and experimental intensity ratio for time-reversal windows of sizes between 40 and 1500 μs. The agreement is excellent. The experimental noise intensity level has been estimated by evaluating $\int [s_{\Delta T}(t_R) - \Delta T S_\infty(t_R)]^2 dt_R$.

In order to find a more quantitative expression of the residual temporal sidelobes, consider the signal observed in A, the source location, after the signal emission at this point. The signal can be described by $h_{AA}(t)^* f(t)$, where the impulse response $h_{AA}(t)$ from A to itself is convolved with the signal function $f(t)$. It is composed of a first peak during emission around $t = 0$ and some multiple reflections afterwards, which pass over the source point even after the excitation has ended [as in Fig. 1(b)]. $h_{AA}(t)$ describes backscattering properties of A due to the boundaries of the cavity. If we had a perfect time-reversal operator (i.e., we simply count time backwards), we would obtain in A the signal $h_{AA}(-t_R)^* f(-t_R)$, i.e., some multiple reflections with a final peak around $t_R = 0$. Thus multiple reflections passing over the source point are one reason for temporal sidelobes in S_∞. However, due to reciprocity, the signal obtained in A by a time reversal in B must be the same as a single in B obtained by a time reversal in A. Thus the impulse response $h_{BB}(t)$ from B to B must have the same influence on S_∞ as h_{AA}.

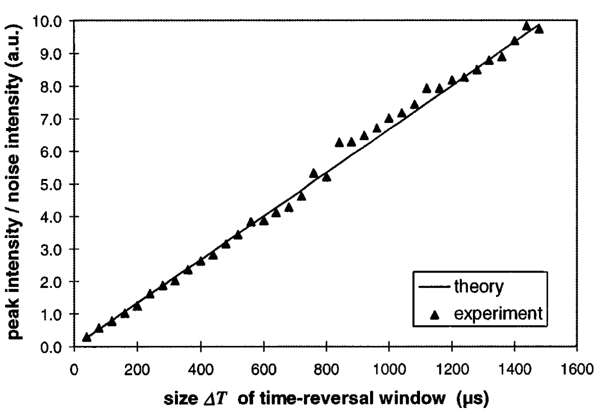

FIG. 5. Comparison of experimental and theoretical focusing quality for varying time-reversal window sizes.

It is possible to quantify this influence by an analysis of the system's eigenmodes: B measures and injects each eigenmode proportionally to it amplitude at this point. In cavities with weak level degeneracy (condition presumably enhanced by level repulsion of chaotic systems), one can show that this effect leads to a closed-form expression of S_∞:

$$\Delta T S_\infty(t_R) = (h_{AA}(-t_R)^* h_{BB}(t_R))^* f(-t_R) \quad \text{as} \quad \Delta T \to \infty .$$

This theorem is confirmed by numerical simulations. A demonstration and detailed discussion cannot be presented in this Letter and will be published elsewhere. Note the interest of this expression: It describes entirely the possibilities and limitations of a one-channel time reversal. First, the theorem proves that the received signal actually resembles a perfectly reversed signal $h_{AA}(-t_R)^* f(-t_R)$, and second, the loss of reversal quality is completely described by the convolution with h_{BB}.

In this Letter, we have presented experiments and numerical simulations of a time reversal in a closed reflecting cavity. It is proven that, using a single time-reversal channel, a good reversal quality can be obtained. Unlike in experiments with open systems using a time-reversal mirror, the resulting reversed wave field is not aperture limited. The signal-to-noise intensity ratio depends linearly on the size of the time-reversal window. The focusing amplitude can be increased by amplification and use of larger TR windows. However, residual temporal sidelobes persist even for time-reversal windows of infinite size. They are due to multiple reflections passing over the locations of the TR transducers, and their expression can be given in a closed form.

Presently, we are working towards making the most of this expression. For example, the choice of the reversing point B has a considerable influence on reversal quality. Furthermore, we attempt to quantify the dependence on the chaotic behavior of the system. It can be shown that, in rectangular cavities, a one-channel time reversal does not work properly.

The authors would like to thank J.-P. Nicolovsky for the production of the aluminum cones, J.-L. Thomas and D. Royer for their experimental help, A. Derode, B. van Tiggelen, F. Mortessagne, O. Legrand, C. Vanneste, and P. Sebbah for useful discussions, and P. Cahill for his help in editing the manuscript.

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