Decay of a Quasiparticle in a Quantum Dot: The Role of Energy Resolution

P. G. Silvestrov
Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia
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The disintegration of a quasiparticle in a quantum dot due to electron interaction is considered. It was predicted recently that above the energy $\varepsilon^* = \Delta(g/\ln g)^{1/2}$ each one particle peak in the spectrum is split into many components. We show that the observed value of ε^* should depend on the experimental resolution $\delta\varepsilon$. In the broad region of variation of $\delta\varepsilon$ the $\ln g$ term should be replaced by $\ln(\Delta/g\delta\varepsilon)$. We also argue against delocalization transition in the Fock space. Most likely the number of satellite peaks grows continuously with energy. The predicted logarithmic distribution of interpeak spacings may be used for experimental confirmation of the below-golden-rule decay. [S0031-9007(97)04576-6]

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Decay of single-electron excitations in quantum dots has now become the subject of intensive experimental [1,2] and theoretical [2–6] investigations. For a closed quantum dot instead of real decay of a quasiparticle one should consider the disintegration of one δ peak in the single-particle spectral density $\rho(\varepsilon)$ into a relatively dense bunch of peaks. Each component of this bunch represents the one particle contribution to a complicated exact eigenstate. The quasiparticle lifetime in a large Fermi system is usually associated with decay into two-particle–one-hole configuration. The corresponding width may be found using the usual golden rule [2]. The energy ε' at which this width becomes of the same order of magnitude with three-particle level spacing gives us the natural threshold for the decay of a quasiparticle. However, it was shown by the authors of Ref. [4] that due to the effective interaction with five-particle, seven-particle, and so on excitations [they call all states consisting of $n + 1$ particles and n holes the $(2n + 1)$th generation] the actual threshold for particle disintegration is much lower. The more detailed investigation of statistics of states constituting the single-particle excitation below ε' is the subject of this Letter. The statistical approach to finite interacting Fermi systems has a long history [7–11]. However, the main attention was paid to the investigation of the fully developed chaos. Here we are interested in the very beginning of the chaotic behavior, then quasiparticles may be coupled with only a few many-particle states.

The convenient quantity, which describes the splitting of a noninteracting quasiparticle peak into many peaks, is the participation ratio (PR) $P = \sum_i \alpha_i^4$. Here α_i^2 is the relative strength of an individual peak in $\rho(\varepsilon)$ and the sum over a bunch of peaks corresponding to one-particle excitation is $\sum \alpha_i^2 = 1$. Physically the PR is the inverted effective number of exact many-particle eigenstates constituting the quasiparticle excitation. From a technical point of view, the authors of Ref. [4] have summed up starting from the small excitation energies the series of special perturbative contributions leading to quasiparticle disintegration and then estimated at which energy ε^* this series blows up. In terms of PR this procedure gives

$$P = 1 - \frac{\varepsilon^2}{g\Delta^2}\, p(\varepsilon)\,, \qquad p(\varepsilon) = \sum_{n=0} p_n \left(\frac{\varepsilon^2}{g\Delta^2}\ln g\right)^n, \tag{1}$$

where Δ is the averaged single-particle level spacing, $g \gg 1$ is the dimensionless conductance, $\varepsilon \gg \Delta$ is the energy of our quasiparticle, and p_n are some numerical coefficients. Each new term in the sum in Eq. (1) corresponds to taking into account more and more complicated admixtures to quasiparticle. The first term ($n = 0$) describes the mixing with two particles and one hole, second to three particles and two holes, and so on.

At energies close to $\varepsilon^* = \Delta(g/\ln g)^{1/2}$ all terms of the series in Eq. (1) become of the same order of magnitude. This means that at $\varepsilon > \varepsilon^*$ the PR cannot be close to 1. However, the concrete way of the quasiparticle disintegration with the growth of ε depends on the behavior of the coefficients p_n. In general, the three possibilities for the asymptotics of p_n are

$$p_n \sim n!\,, \tag{2a}$$

$$p_n \sim a^n n^\gamma, \tag{2b}$$

$$p_n \sim 1/n!\,, \tag{2c}$$

which corresponds to zero, finite, and infinite radii of convergence of the series in Eq. (1), respectively. One should naturally expect very different features of the resumed result in these three cases. Mapping the problem of quasiparticle lifetime onto that of particle hopping on the Cayley tree led the authors of Refs. [4,5] to the asymptotics (2b). However, as we will show below, the actual asymptotics is close to Eq. (2c).

In general, in order to observe experimentally the splitting described by Eq. (1) one should be able to resolve all many-particle eigenstates, which means that the experimental errors should be exponentially small $\delta\varepsilon \sim \Delta\exp(-2\pi\sqrt{\varepsilon/6\Delta})$ [12]. Therefore, in this Letter we find how the mechanism considered in [4] will manifest itself for more realistic $\delta\varepsilon$. First of all, even in order to see the decay of quasiparticle into three-particle configurations one should have sufficiently good resolution $\delta\varepsilon \sim \Delta^3/\varepsilon^2$ (any few peaks falling into the segment $\sim\delta\varepsilon$ are seen as one of joint strength $\sum_{\delta\varepsilon}\alpha_i^2$). The most interesting is the result for $\Delta^5/\varepsilon^4 \ll \delta\varepsilon \ll \Delta^3/\varepsilon^2$. Physically this means that accuracy is much better than needed to resolve the three-particle levels, but not enough to see the five-particle ones. In this case

$$P = 1 - \frac{\varepsilon^2}{g\Delta^2} b(\varepsilon), \qquad b(\varepsilon) = \sum_{n=0} b_n(\varepsilon/\varepsilon_c)^{2n}, \tag{3}$$

where $\varepsilon_c = \Delta\sqrt{g/\ln(\Delta/g\delta\varepsilon)}$. The transition from pure single-particle to split spectrum now takes place at $\varepsilon \sim \varepsilon_c$. In particular if $\delta\varepsilon \sim \Delta^3/\varepsilon^2$ one has $\varepsilon_c \sim \varepsilon' = \Delta\sqrt{g}$ in accordance with the golden rule prediction [2,4]. At $\delta\varepsilon \sim \Delta^5/\varepsilon^4$ the expansion (1) formally is restored, but the coefficients of this new series p_n^* are much smaller than those of Eq. (1). For better accuracy $\delta\varepsilon \ll \Delta^5/\varepsilon^4$ the coefficients p_n^* become a function of the resolution $p_n^* = p_n^*(\delta\varepsilon)$. Only at extremely small $\delta\varepsilon$ one has $p_n^*(\delta\varepsilon \sim \Delta^{n+1}/\varepsilon^n) \approx p_n$.

As shown in Refs. [3,4] the values of the matrix elements (MEs) of two-particle interaction are Gaussian distributed with the variance:

$$\overline{V^2} = \Delta^2/g^2. \tag{4}$$

Here the numerical factors ~ 1 (see, e.g., Ref. [4]) are included into the definition of $g \gg 1$. The estimate (4) was done for the diffusive quantum dot. However, our approach may be valid for the ballistic dot also. The only necessary condition is that the MEs of interaction should be random with the amplitude $|V| \ll \Delta$.

Consider first the mixing of particle with three-particle states. The density of these states is

$$\nu_3 = \varepsilon^2/4\Delta^3. \tag{5}$$

Here one factor $1/2$ comes from the integration over the three energies at fixed $\varepsilon_{p1} + \varepsilon_{p2} + \varepsilon_h = \varepsilon$ and another is added due to the Fermi statistics of two produced particles. We are interested in energies $\varepsilon \ll \Delta\sqrt{g}$. Therefore $|V|\nu_3 \ll 1$, which means that the majority of one-particle states is almost nonperturbed. In this case the main contribution to the PR comes from the very small part of levels, for which the energy difference between one- and three-particles excitations $\varepsilon^{(1)} - \varepsilon^{(3)}$ occasionally turns out to be of the same order of magnitude with the ME V. The relative fraction of such states is small $\sim |V|\nu_3$, but their PR differs by 100% from $P = 1$. Therefore, the averaged contribution of such events to P is $\delta P_3 \sim |V|\nu_3 \sim \varepsilon^2/g\Delta^2$. The accurate calculation [13] allows one to find also the numerical factor

$$P_3 = 1 - 2\pi\overline{|V|}\nu_3 = 1 - \sqrt{\pi/2}\,\varepsilon^2/g\Delta^2. \tag{6}$$

Here due to Eq. (4) $\overline{|V|} = \sqrt{2/\pi}\,\Delta/g$.

Mixing of quasiparticle with higher generations (five particle, seven particle, etc.) may be formally taken into account in the same way:

$$\delta P_{2n+1} = -2\pi|V_{\rm eff}^{(2n+1)}|\nu_{2n+1}, \tag{7}$$

where $\nu_{2n+1} \sim \varepsilon^{2n}/\Delta^{2n+1}$. The only difference from (6) is that now $V_{\rm eff}^{(2n+1)}$ is the effective ME connecting the first and $(2n+1)$th generations via the nth order of the usual perturbation theory. The naive estimate of this effective interaction gives $V_{\rm eff} \sim (\Delta/g)^n(1/\Delta)^{n-1}$, which lead to $\delta P_{2n+1} \sim -(\varepsilon^2/g\Delta^2)^n$. The main advantage of Ref. [4] was in fact the observation that the high order corrections to P have an additional enhancement $\sim(\ln g)^{n-1}$ compared to the naive estimate. In order to demonstrate the origin of this large logarithm, consider the effective ME connecting generations 1 and 5

$$\overline{|V_{\rm eff}^{(5)}|} = \overline{\left|\sum_2 \frac{V_{12}V_{23}}{\varepsilon_1 - \varepsilon_2}\right|} = \frac{2\Delta^2}{\pi g^2}\int_{\Delta/g}^{\Delta}\frac{d\varepsilon}{\Delta\varepsilon} = \frac{2\Delta}{\pi g^2}\ln g\,. \tag{8}$$

Here we have left in the sum over ε_2 only one level closest to ε_1 (contribution of the other levels is $\sim 1/\ln g$ smaller) and then averaged over its position. Therefore, the upper bound of the integral is $|\varepsilon| < \Delta$. More interesting is the origin of the lower bound. The use of the effective interaction requires $|V_{12}|, |V_{23}| \ll |\varepsilon_1 - \varepsilon_2| \approx |\varepsilon_3 - \varepsilon_2|$. Otherwise one should consider the strong mixing of three almost degenerate states $|1\rangle, |2\rangle, |3\rangle$ (accurately taking into account such a three-level interaction leads also to $\sim 1/\ln g$ corrections). That is why the lower bound in the integral in Eq. (8) is $|\varepsilon| > \Delta/g$.

The ME (8) together with $\nu_5 \sim \varepsilon^4/\Delta^5$ allows one to reproduce the first nontrivial term of the expansion (1). Moreover, even if one does not take into account the mixing with higher generations, the correction described by (8) could blow up the PR (6) at $\varepsilon \sim \Delta g^{1/2}(\ln g)^{-1/4}$. However, one more important feature of $\rho(\varepsilon)$ may be demonstrated by Eq. (8). The logarithmic divergence of the integral in (8) shows that the MEs with very different denominators are equally important for the PR. Suppose we are not able to resolve peaks which are closer than some $\delta\varepsilon \ll \Delta/g$. As we mentioned before, one can see two peaks of comparable amplitude at distance $\delta\varepsilon$ only if $\delta\varepsilon \sim V_{\rm eff} \sim \varepsilon_1 - \varepsilon_3$. This means that the upper bound in the integral in Eq. (8) should be chosen via $V_{\rm eff} \sim \Delta^2/g^2\varepsilon_{12} > \delta\varepsilon$ and thus

$$\varepsilon_{\max} = \frac{\Delta^2}{g^2\delta\varepsilon}, \qquad \overline{|V_{\rm eff}^{(5)}(\delta\varepsilon)|} = \frac{2}{\pi}\frac{\Delta}{g^2}\ln\left(\frac{\Delta}{g\delta\varepsilon}\right), \tag{9}$$

in accordance with (3). In order to illustrate this result we have shown in Fig. 1 the density of couples of peaks as a function of the logarithm of spacing λ between them $dn/d\ln(\lambda)$ (at ε slightly below ε^*). The mixing with third generation leads to the narrow (width ~ 1) peak at $\lambda \sim \Delta^3/\varepsilon^2 \sim \Delta/g$. The contribution from fifth generation at $\lambda \sim \Delta/g$ is in $\varepsilon^2/g\Delta^2$ times weaker, but such events are uniformly distributed over the wide region $\ln(\Delta/g^2) < \ln(\lambda) < \ln(\Delta/g)$.

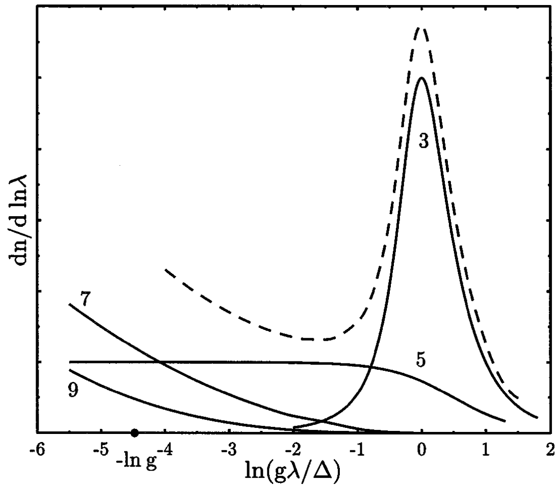

FIG. 1. Distribution of spacings λ for first doubling of the peaks as a function of $\ln\lambda$. The mixing with generations 3, 5, 7, and 9 is shown. The dashed line is the total distribution.

Generalization of (8) for an arbitrary generation gives

$$\overline{|V_{\rm eff}^{(2n+1)}|} = \left(\frac{\sqrt{2}\Delta}{\sqrt{\pi}g}\right)^n \int^{\Delta} \prod_{i<n} \frac{d\varepsilon_i}{\Delta\varepsilon_i} = A\,\frac{2^{\frac{n}{2}}\Delta(\ln g)^{n-1}}{\pi^{\frac{n}{2}}g^n},$$

$$\sum_{i=1}^{k}\ln\left(\frac{g\varepsilon_i}{\Delta}\right) > 0, \qquad \sum_{i=k}^{n-1}\ln\left(\frac{g\varepsilon_i}{\Delta}\right) > 0, \qquad (10)$$

The upper limit for all integrals here is the same as in (8): $\varepsilon_i < \Delta$. The small values of ε_i are restricted due to the requirement that none of the intermediate states in $V_{\rm eff}$ could be mixed strongly with the initial or final state. One may find the lower and upper bounds for $|V_{\rm eff}|$ by considering the simplified version of the logarithmic inequalities in (10): $\ln(g\varepsilon_i/\Delta) > 0$ for any i for lower bound and $\sum_1^{n-1}\ln(g\varepsilon_i/\Delta) > 0$ for upper bound. For large n such a calculation gives

$$1 < A < e^n. \qquad (11)$$

Thus at least the integral (10) could not contain any $n!$ [14]. Equations (7) and (10) together lead to Eq. (1).

For finite accuracy one should take into account only the MEs exceeding the experimental error $V_{\rm eff} > \delta\varepsilon$, which is equivalent to the additional restriction on the domain of integration

$$\sum_{1}^{n-1}\ln(g\varepsilon_i/\Delta) < \ln(\Delta/g\delta\varepsilon). \qquad (12)$$

If in addition $\ln(\Delta/g\delta\varepsilon) \ll \ln(g)$, the integration in Eq. (10) may be performed explicitly

$$\overline{|V_{\rm eff}^{(2n+1)}(\delta\varepsilon)|} = \frac{1}{n-1}\,\frac{2^{\frac{n}{2}}\Delta}{\pi^{\frac{n}{2}}g^n}\,[\ln(\Delta/g\delta\varepsilon)]^{n-1}. \qquad (13)$$

In terms of log distribution of level spacings $dn/d\ln(\lambda)$ shown in Fig. 1 the contribution of generation $2n+1$ leads to correction $\sim[\ln(\Delta/g\delta\varepsilon)]^{n-1}$.

Consider now the physical consequences for the spectrum of the different variants of asymptotic behavior of the coefficients p_n (b_n) shown in Eq. (2): (a) In fact, there is no real danger in divergence of the asymptotic series. One should simply break the summation at the smallest term (with the number $n_c \sim \sqrt{\varepsilon^*/\varepsilon}$ or $n_c \sim \sqrt{\varepsilon_c/\varepsilon}$). The same smallest term gives the order of magnitude estimate of the rest (nonperturbative) part of the sum. $p(\varepsilon), b(\varepsilon)$ become completely nonperturbative at $\varepsilon > \varepsilon^*, \varepsilon_c$. (b) The series in ε^2 has finite radius of convergence $R = \varepsilon^{*2}/a$ (ε_c^2/a) and the γ is responsible for the kind of singularity of the resummed result at $\varepsilon^2 = R$ (both $a, \gamma \sim 1$). Close to this point all terms of the series become equally important. It is natural to consider such a behavior as an indication of the localization-delocalization transition in the Fock space [4,5]. (c) The series is absolutely convergent. We consider this as the indication of absence of delocalization transition.

The estimates of ME [(8),(10),(13)] were done for a given tree-type Feynman diagram connecting given initial and final states. Now we have to estimate the number of such diagrams, first of all, the density of final states:

$$\nu_{2n+1} = \frac{\varepsilon^{2n}}{\Delta^{2n+1}}\,\frac{1}{(2n)!\,(n+1)!\,n!}. \qquad (14)$$

Here $(2n)!$ appears after the integration over energies of final particles (holes), $(n+1)!$ and $n!$ account, respectively, for the $n+1$ identical particles and n holes. The number of diagrams for the fixed final state is easy to estimate for the Schrödinger perturbation theory. The examples of diagrams for the screened Coulomb interaction $V(x-y) \sim \delta(x-y)$ are shown in Fig. 2 [15]. Each individual ME of $V(x-y)$ corresponds to decay of one particle into two particles and one hole, or one hole into two holes and one particle. In order to find the number of diagrams it is convenient to start from the final state. At first step there are $(n+1)n^2/2$ ways to join two $(n+1)$ particles and one n hole into one particle and $(n+1)n(n-1)/2$ ways to join one particle and two holes into one hole. Then the same procedure may be repeated with n particles and $n-1$ holes. The number of diagrams connecting the same initial and final states found

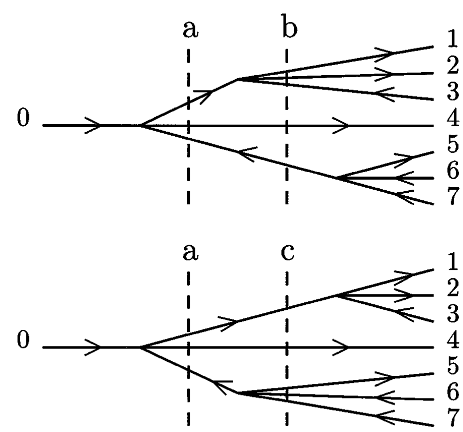

FIG. 2. The examples of diagrams. Energy denominators are associated with transverse sections (dashed lines).

in this way is

$$2^{-n} n! (n + 1)! (2n - 1)!! . \quad (15)$$

The doubling of single-particle peaks is based on very rare events of almost coincidence of the small ME and small energy difference. This means that the probability to find two equally large MEs is small and one should simply multiply the correction (7) by the number of statistically independent diagrams. However, not all of the diagrams (15) are statistically independent. First of all, we have not taken into account the Fermi statistics of particles in the intermediate states. This means that some of the diagrams should cancel each other. Second, we have estimated the number of diagrams of Schrödinger perturbation theory. If one goes to the Feynman technique, many of the diagrams having the same MEs and different energy denominators will be joined into one. For example, for the two diagrams in Fig. 2 one has

$$1/\varepsilon_a \varepsilon_b + 1/\varepsilon_a \varepsilon_c = 1/\varepsilon_b \varepsilon_c , \quad (16)$$

because $\varepsilon_b + \varepsilon_c = \varepsilon_a$ (for almost degenerate initial and final states). Here $\varepsilon_{a,b,c}$ are the energy denominators for corresponding cross section in the figure. Therefore, Eq. (15) gives only the upper bound of the number of independent diagrams. Combining together (14) and (15) and the estimate of $\overline{|V_{\text{eff}}|}$ one finds

$$p_n < (\text{const})^n \frac{n! (n + 1)! (2n - 1)!!}{(2n)! (n + 1)! n!} \sim \frac{(\text{const})^n}{n!} . \quad (17)$$

We see that combinatorics of the diagrams (15) could not compensate the decrease of phase space and the asymptotics of p_n (as well as b_n) is described by Eq. (2c). Slightly above $\varepsilon = \varepsilon^*, \varepsilon_c$ due to the mixing with finite number $(\sim\sqrt{\ln g})$ high order (with $n \sim \ln g$) generations the PR becomes sufficiently smaller than 1. This finite number of connected generations constitutes the main difference of our result from what happens on the Cayley tree [4,5], where even the first splitting of the quasiparticle peak into two proceeds through the interaction with all generations. For higher energies our perturbative approach formally is not valid. We are able to consider rigorously only the first splitting of quasiparticle peak into two. In order to go further one should be able to diagonalize exactly the three-levels almost degenerate events, then the four-levels and so on. Mathematically, this means that one has to sum up the series of $\sim 1/\ln(g)$ corrections to P. Nevertheless, it is natural to suppose that further disintegration of the quasiparticle also proceeds through the interaction with a finite number of generations. If so, the number of peaks constituting one excitation most likely will grow smoothly with energy (crossover instead of phase transition). The delocalization in the Fock space will not take place in this scenario (although, it may be difficult to find the experimental evidence of presence or absence of such delocalization).

Even more informative than P is the distribution of spacings inside the quasiparticle bunch. The distribution of spacings for first decay into two peaks (two distinct bunches) has complicated hierarchical structure. The natural variable to describe this distribution is $\ln \lambda$ (Fig. 1). In particular this means that the disintegration threshold ε_c should depend on the experimental accuracy. It is natural to expect that this log distribution of spacings will survive after further disintegration into three or more peaks. Moreover, both new delocalization thresholds ε^* and ε_c differ only by the square root of the logarithm from the golden-rule prediction, which makes them quite difficult to be observed in the direct experiment. However, the wide logarithmic distribution of spacings within the single particle bunch of peaks (like that in Fig. 1) may be easily distinguished from, e.g., Poisson or Wigner-Dyson distribution. Thus we may conclude that the investigation of spacings distribution in the single particle spectral density should open the easiest way to observe the below-golden-rule decay of quasiparticles in a quantum dot predicted in Ref. [4].

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[15] Taking into account the noncontact interaction $V(x - y)$ may add the factor const^n to Eq. (15), but not $n!$.