Low-Frequency Noise Probe of Interacting Charge Dynamics in Variable-Range Hopping Boron-Doped Silicon

J. G. Massey and Mark Lee*
Department of Physics, University of Virginia, Charlottesville, Virginia 22903
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Low-frequency voltage noise is used to probe stochastic charge dynamics in nonmetallic boron-doped silicon. A "$1/f$" noise spectrum is observed down to 0.1 Hz. The noise magnitude is suppressed and the frequency dependence strengthens at low temperature. The data are inconsistent with single-particle hopping fluctuations, but are compatible with thermally activated rearrangements of configurations involving many charges. Such configurational fluctuations indicate that many-electron excitations are important to charge dynamics in the interacting regime. [S0031-9007(97)04529-8]

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In doped semiconductors, the disordered impurity distribution leads to localization of charge carrier wave functions below a critical dopant density n_c. Localization degrades the screening of Coulomb interactions, so that correlations should be particularly important at nonmetallic densities. Below n_c, dc charge transport occurs by inelastic hopping among localized sites. Standard treatments of hopping conduction regard single-particle excitations as the current carriers. However, several authors [1] have argued that, including interactions, a single-particle description of dc transport is insufficient since the hopping of one charge changes the local potentials of many surrounding charge sites. In large part, the controversy persists because the nature of excitations in interacting localized insulators have not been well characterized experimentally.

In a 3D variable-range hopping (VRH) conductor, Efros and Shklovskii [2] (E-S) showed that the Hartree interaction leads to a quadratic Coulomb gap in the single-particle density of states. For single-particle transport, such a gap explains a hopping exponent of $\frac{1}{2}$ found in many localized insulators [3]. Alternatively, the dc current has been proposed [1,4] to consist of correlated motions of many electrons, resulting in a many-particle dressing of single-particle excitations. Efros [4] originally proposed, and recent analytical [5] and numerical [6] calculations agree, that if many-electron quasiparticles carry the current, then the interactions among quasiparticles can lead to a quadratic gap in the quasiparticle spectrum and thus to a hopping exponent of $\frac{1}{2}$. However, the dressing should provide additional screening and reduce the Coulomb energy scale for quasiparticles compared to the E-S case [5,6].

Coulomb gaps in nonmetals have been studied by charge injection experiments. Time-domain capacitance measurements on GaAs:Zn by Monroe *et al.* [7] revealed a complicated dynamical response to an injected charge. A nonlinear screening length was attributed to a single-particle Coulomb gap, and a slow decay of the screening length with activation energy scale $\sim$0.5 meV was interpreted as a many-particle relaxation of the gap. More recently, a quadratic spectral shape of the Coulomb gap with width $\sim$0.7 meV was observed directly in boron-doped silicon (Si:B) by tunneling measurements [8]. Charge injection normally probes primarily single-particle characteristics because the injection time is much faster than the time it takes background charges to rearrange in response to the excess charge. Meir [5] pointed out that the Coulomb gap observed in tunneling is significantly larger than the energy scale deduced from transport and suggested that the discrepancy may reflect a difference in screening between bare tunneling charges and dressed transport quasiparticles.

Measurements of dynamics without injecting excess charge can elucidate the roles of single- and many-particle excitations on transport. In this Letter, we study the stochastic charge dynamics in VRH Si:B using measurements of the low-frequency (0.1 Hz $\leq f \leq$ 12 Hz) electronic noise spectra. The noise power S shows a "$1/f$" form in this frequency band: $S = \beta I^2 f^\alpha$, where β describes the overall magnitude, I is the dc current bias, and the spectral exponent α is slightly larger than one. A suppression of β and increase in α below 3 K are inconsistent with models of single-particle fluctuators, but can be interpreted as fluctuations involving slow thermally activated processes with activation energies of 60 to 100 K. This energy scale and the spectral shapes suggest that the noise results not from fluctuations of individual charges between sites, but from stochastic rearrangements of many-charge configurations among the random impurity sites.

The samples used were $\frac{1}{4}$ in. $\times$ $\frac{1}{16}$ in. $\times$ 0.0010 in. crystals of Si:B. The surfaces were cleaned and Al Ohmic contacts were made by evaporation and annealing. Room-temperature resistivities and the resistivity ratios (RRs) $\rho(4.2\text{ K})/\rho(300\text{ K})$ were measured. Densities n were estimated using the calibration of Thurber *et al.* [9] and extrapolating the data of Dai *et al.* [10]. Samples having RRs of 23 and 19, corresponding to n/n_c of 81% and 82%, respectively, were used, where we take $n_c = 4.0 \times 10^{18}$ cm^{-3} from Ref. [10]. Differential noise from 0.1 to 12 Hz was measured in the 5-point bridge configuration described by Scofield [11]. At these frequencies, the noise generated in the bulk samples was smaller than the $1/f$ noise of the amplifiers. The samples' noise was measured by up-converting its $1/f$ noise to sidebands around a modulation frequency ($\sim$1 kHz), where amplification could be done in the much lower white noise background of the amplifiers. The modulation frequency minimized the noise contribution from an ultralow noise (0.1 nV/Hz$^{1/2}$) transformer preamplifier. The system was calibrated by measuring spectra from a low-frequency oscillator and a calibrated noise source. Figure 1 inset shows the noise amplitude vs bias current. The data fit reasonably to a line, showing that the measured noise is from the sample, since amplifier noise does not have a significant bias current dependence.

Of more serious concern was the strongly temperature dependent resistance, i.e., large dR/dT, in these nonmetallic samples. Coupling of the resistance to temperature fluctuations could produce spurious low-frequency noise. The 5-point bridge minimized this concern by sending a balanced current through two identical halves of a sample and measuring only the difference signal, giving a large rejection of all external common-mode fluctuations including temperature and current. A rejection ratio of about 10^{-5} was obtained by balancing the current to within this accuracy. The worst-case common-mode temperature fluctuations were 3 mK (rms) at 7 K, diminishing gradually to 1 mK at 1.5 K. The common-mode rejection does not reduce the effects of differential temperature fluctuations coupling to dR/dT, but temperature variations across the 3 mm distance between voltage contacts should be insignificant.

Figure 1 shows the noise amplitude ($S^{1/2}$) spectrum of the 81% sample for several temperatures. Instrumental contributions have been deconvoluted, and the data have been averaged over several spectra at each temperature. For comparison, the dashed line represents what a pure $1/f$ amplitude slope looks like on this plot. The noise amplitudes at 8 and 3 K differ slightly. Below 3 K, the noise amplitude drops rapidly. After taking and averaging many such spectra, the noise amplitude as a function of temperature for 1 and 10 Hz is shown in Fig. 2. Above 3 K, the amplitude is only weakly dependent on temperature, similar to the data of Shlimak *et al.* [12] on doped Ge in the nearest-neighbor hopping regime. Below $\sim$3 K, a decrease in the noise amplitude is clearly seen. The 10 Hz amplitude falls over an order of magnitude in a temperature interval of 3 K, with a corresponding decrease at 1 Hz. Below 1.5 K, the amplitude falls below the noise floor, which is the minimum sample noise that can be reliably extracted from the amplifiers. A similar noise suppression with a lower onset temperature of $\sim$2 K was observed in the 82% sample.

The behavior of the spectral exponent α is plotted as a function of temperature in Fig. 3. The exponents were obtained by smoothing spectral data such as those in Fig. 1, calculating $-\partial \ln S/\partial \ln f$ to obtain local slopes, then averaging over the local slopes in the frequency range measured. For generic $1/f$ noise, the exponent is anticipated to be 1, so the deviation $\alpha - 1$ is plotted. Even at higher temperatures, α is found to slightly exceed unity. The plot shows a systematic increase of $\sim$5% (compared to unity) in the exponent as T decreases. This increase in α occurs over the same temperature range as the decrease of the noise magnitude in Fig. 2.

The dc transport in the same temperature range is also of interest. The samples' I-V curves showed a slight nonlinearity below 5 K, so the resistances are defined as zero-bias values. The hopping exponent is determined by the Zabrodskii plot in Fig. 4. The slope

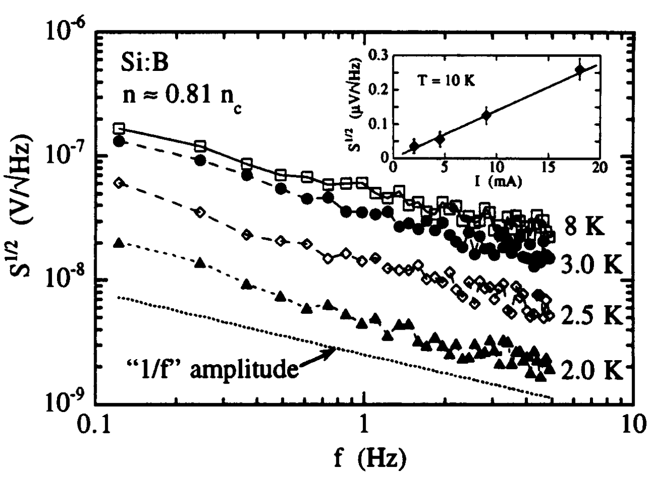

FIG. 1. Noise amplitude ($S^{1/2}$) spectra of an 81% Si:B sample at several different temperatures. The bias current is 4.5 mA. For reference, the plain dotted line shows the amplitude slope of an ideal $1/f$ power spectrum. Inset: Noise amplitude at 2 Hz as a function of the dc bias current.

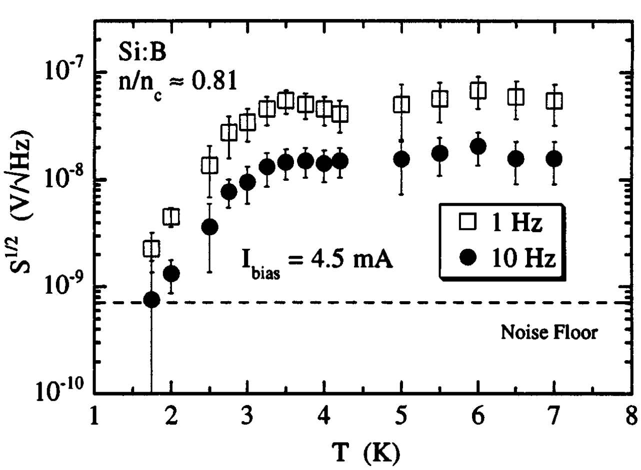

FIG. 2. Noise amplitude ($S^{1/2}$) of an 81% Si:B sample at 1 and 10 Hz as a function of temperature. The bias current is 4.5 mA. The dashed line is the minimum measurable sample noise in the system.

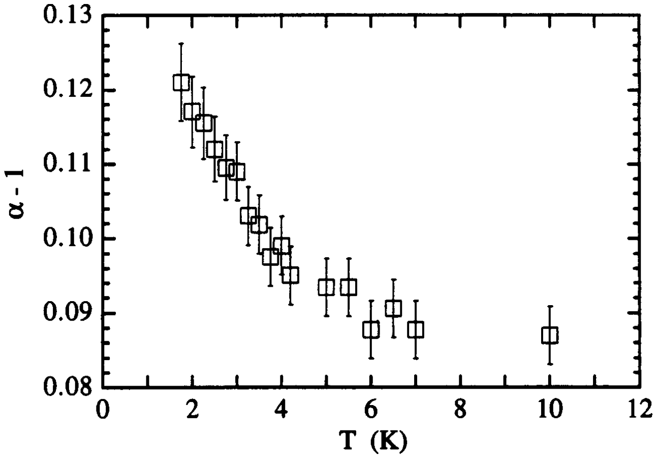

FIG. 3. Deviation from unity of the mean spectral exponent α as a function of temperature. Values for α were obtained by smoothing and averaging the local spectral slopes from data like those in Fig. 1.

of the data on such a graph yields minus the hopping exponent. A single line does not fit the data over the entire temperature range, but there are ranges that straight lines describe reasonably well. The data for $2 < T < 5.5$ K fit a slope of $-\frac{1}{4}$, which is characteristic of noninteracting hopping. For $T < 2$ K, the slope of $-\frac{1}{4}$ clearly no longer fits, giving way to a larger slope closer to the interacting hopping exponent of $\frac{1}{2}$. Although the temperature range of the data is too limited to perform a thorough analysis of the crossover from noninteracting to interacting hopping, Fig. 4 demonstrates that interactions are becoming important near this temperature range.

Considering single-particle hopping fluctuations, Shklovskii and Kogan [13] (S-K) and Kozub [14] found that a hopping conductor generates $1/f$ noise in certain limits. However, there are substantial differences between the data presented and these calculations that cannot be resolved. The S-K model of charge fluctuations predicts an exponent $\alpha \leq 1$ that is significantly smaller than 1 for n near n_c, climbing to unity only when $n \ll n_c$. Our measured α are all ≥ 1 for n not very far below n_c. In Kozub's calculation of mobility fluctuations, the noise magnitude is shown to increase as $T \rightarrow 0$, with a faster rate of increase in the E-S regime. This is opposite to what we observe. Finally, in both models the rarest fluctuations from the most isolated dopant sites lead to a low-frequency cutoff in the $1/f$ divergence. For our samples, this cutoff should exceed 100 Hz, but no cutoff is observed down to 0.1 Hz in our data or in most of the literature [12,15]. Hence the data of Figs. 1–3 are inconsistent with existing models of single-particle fluctuators.

The noise data is at least qualitatively compatible with the thermally activated fluctuator model of Dutta, Dimon, and Horn (D-D-H) [16]. In this model, if the distribution of activation energies $D(E)$ is constant, then the noise power has the generic $1/f$ form with $\alpha = 1$ and a linear temperature dependence [17], i.e., $\beta \propto T$. If $dD/dE > 0$, the D-D-H model gives a faster decrease in noise magnitude and a concomitant increase in α as T decreases. The frequency and temperature properties of the noise are related by

$$\alpha(T) = -\frac{\partial \ln S(f,T)}{\partial \ln f} = 1 - \frac{1}{\ln(2\pi f \tau_0)}\left[\frac{\partial \ln S(f,T)}{\partial \ln T} - 1\right], \quad (1)$$

where τ_0 is an inelastic lifetime, typically 10^{-11} to 10^{-12} s. In any temperature range where S decreases faster than linearly as $T \rightarrow 0$, i.e., $\partial \ln S/\partial \ln T > 1$, consistency with the D-D-H model means that α should increase [18]. This general trend is clearly evident in the data of Figs. 2 and 3. Attempts to compute $\alpha(T)$ through Eq. (1) using data like that in Fig. 2 yielded qualitative agreement in overall trend and order of magnitude, but the computed curve showed a noticeably stronger low temperature dependence than was measured. Finally, the temperature and frequency ranges over which the noise shows features set an energy scale $E^* = -k_B T \ln(2\pi f \tau_0)$ for the activation energies probed by fluctuations in the measured bandwidth. Using f between 1–10 Hz and T of 2–3 K, $E^* \approx 60$ to 100 K. It is interesting to note that this energy scale governing fluctuations about the equilibrium charge distribution is ten times larger than the Coulomb energies determined from response to single-particle charge perturbations (Refs. [7] and [8]).

Since single-particle fluctuations are inconsistent with the observed noise behavior, the problem of interest is what mechanism leads to the noise properties. One strong possibility lies in thermally activated rearrangements of

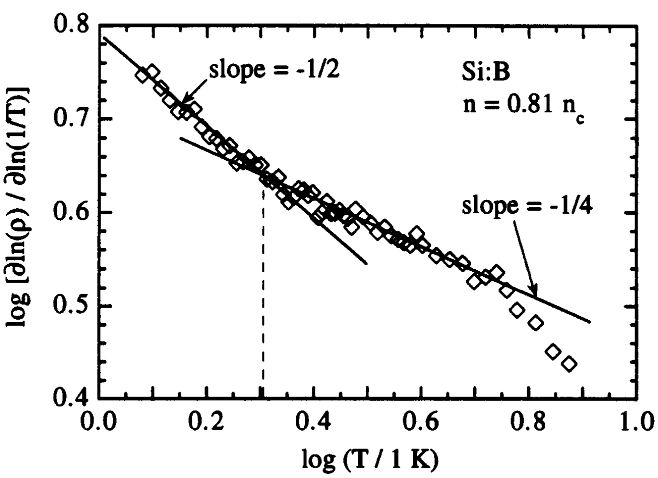

FIG. 4. Zabrodskii-style plot of the dc resistivity as a function of temperature for an 81% Si:B sample, showing a crossover from the noninteracting Mott exponent of $1/4$ to the interacting E-S exponent of $1/2$.

many electrons among random sites. Kogan [19] recently calculated that $1/f$ noise in interacting VRH systems can arise from fluctuations among low energy, many-electron configurations. His simulations show that the charge arrangements have a large number of nearly degenerate total energy minima all lying within a Coulomb gap energy, of order 1 K, of each other. These energy valleys are separated by a broad distribution of energy barriers which can greatly exceed the gap energy, leading to thermally activated configurational fluctuations between energy minima when the temperature is of the order of the Coulomb gap. The distribution breadth and the activated nature of the fluctuations make a D-D-H analysis plausible as an approximate description, and the 60 to 100 K activation energy scale derived is reasonable for the barrier heights being probed. Kogan further showed that configurational fluctuations give α very close to unity over a wide range of dopant levels and down to the lowest attainable frequencies, in agreement with measured results and in contrast to the single-particle models of Refs. [13] and [14]. There are similarities between this many-particle description and a spin glass, whose stochastic dynamics were investigated by analyzing non-Gaussian noise in mesoscopic samples [20]. However, non-Gaussian fluctuations cannot be resolved in our macroscopic samples so that no useful comparisons to spin glasses can be made at present.

Although the stochastic dynamics of the $1/f$ noise cannot be connected to the dc conductivity via a simple fluctuation-dissipation theorem, the noise data do emphasize the importance of many-electron effects on charge dynamics. In a closely related vein, Pérez-Garrido *et al.* [6] recently introduced a treatment of dc hopping transport that considers continuous paths through the space of many-charge configurations. Their results show that many-electron hops lead to a dc hopping exponent of $\frac{1}{2}$. However, the characteristic temperature T_0 for many-electron hopping is computed to be an order of magnitude smaller than the single-particle E-S result. This configurational picture can help explain the discrepancy between the tunneling and resistivity Coulomb gap energy in Si:B, and the factor of 4 to 10 reduction between the measured values of T_0 and the calculated E-S values in transport experiments on Ge:Ga [21] and on Si MOSFETs [22].

In summary, we have measured the low-frequency noise in nonmetallic Si:B in the VRH regime. The noise amplitude decreases and the spectral exponent increases for temperatures below $\sim$3 K in samples with density $n/n_c \approx 81\%$. These features are inconsistent with existing models of noise due to single-particle fluctuators. The noise can be analyzed in terms of activated fluctuators with a distribution of activation energies $\gg k_B T$. A plausible mechanism for such fluctuators is in terms of stochastic charge rearrangements involving many electrons. This points out the importance of many-electron processes in the fluctuations and likely also in the dc hopping transport, where a recent theory based on charge configurations can account for observed anomalies in transport data.

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*Electronic address: marklee@virginia.edu

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