Cloning, Dragging, and Parametric Amplification of Solitons in a Coherently Driven, Nonabsorbing System

Gautam Vemuri,[1,*] G. S. Agarwal,[2] and K. V. Vasavada[1]
[1]*Indiana University-Purdue University Indianapolis (IUPUI), 402 N. Blackford Street, Indianapolis, Indiana 46202-3273*
[2]*Physical Research Laboratory, Ahmedabad and Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India*
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Through an investigation of the dynamics of solitons in three-level atoms, we demonstrate the possibility of optical pulse control and shaping in coherently driven media. It is also shown that solitons generated in three-level atoms, in contrast to two-level atoms, can propagate at the speed of light. [S0031-9007(97)04563-8]

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The development of new techniques for pulse shaping and control is central to generating tailored pulses for communications, study of ultrafast processes, and preparation of atoms and molecules in desired quantum states. In this Letter we investigate solitons in three-level atoms and, in addition to revealing the striking differences from two-level atom results, we also demonstrate that a coherently driven resonant medium can be utilized for pulse shaping and control. Driven atom dynamics continue to receive tremendous attention, and our work represents a qualitatively different way of exploiting these dynamics. We focus on solitons due to their special fundamental properties [1] and their many applications [2–5]. The principal results are that in a nonabsorbing, resonant Λ system [inset of Fig. 1(a)] (i) a weak field of *arbitrary profile* at the Stokes transition is parametrically amplified into the replica of a soliton at the pump transition (*cloning*), (ii) the degree of overlap between the input pump and Stokes pulses permits a control over the temporal location of the Stokes soliton (*dragging*), as well as its amplitude and phase, and (iii) the cloned soliton, which has a different frequency from the pump soliton, travels at the speed of light, *c*, and hence is a steady state pulse [i.e., dependent only on pulse-local coordinate $\tau(=t - z/c)$, and not on $\zeta(=z)$]. This is an unusual property of the cloned pulses generated in a three-level system since solitons generated in two-level atoms always propagate with a speed less than *c*, and so depend on both τ and ζ.

We begin by referring to the inset of Fig. 1(a) where a soliton, with Rabi frequency $\Omega_p(z,t)$, is applied at the pump ($|1\rangle \leftrightarrow |3\rangle$) transition. In a two-level system, this pulse would propagate unchanged, as known from self-induced transparency (SIT) [1]. Now we apply a weak field with arbitrary profile, of Rabi frequency $\Omega_s(t,z)$, at the Stokes ($|1\rangle \leftrightarrow |2\rangle$) transition. Note that we are studying a nonabsorbing medium, and so the physical problem and the associated results are different from other work on absorbing media [6–8], as we discuss later. The results are also distinct from simultons [9]. The governing equations are the coupled Schroedinger-Maxwell equations in the slowly varying envelope approximation, which are given by [6–8]

$$\partial ic_1/\partial t = -(1/2)\Omega_p c_3 - (1/2)\Omega_s c_2\,, \tag{1a}$$

$$\partial c_2/\partial t = (1/2)\Omega_s^*(ic_1) + i\Delta_2 c_2\,, \tag{1b}$$

$$\partial c_3/\partial t = (1/2)\Omega_p^*(ic_1) + i\Delta_1 c_3\,, \tag{1c}$$

$$(\partial/\partial z + \partial/\partial ct)\Omega_p = 2ic_3^* c_1 \mu_p\,, \tag{1d}$$

$$(\partial/\partial z + \partial/\partial ct)\Omega_s = 2ic_2^* c_1 \mu_s\,, \tag{1e}$$

where c_i $(i = 1,2,3)$ are the probability amplitudes of the atomic levels, $\mu_p(\mu_s)$ is the propagation constant for the pump (Stokes) pulse with dimensions of frequency/length, and Δ_1 (Δ_2) is the detuning of the pump (Stokes) pulse from its transition. In Eq. (1), all quantities are made dimensionless by using the width of the input pump soliton, σ, as the normalization factor. The pump soliton, in pulse-local coordinates, is given by

$$\Omega_p(\tau,\zeta) = \Omega_p^0 \operatorname{sech}\left(\frac{\tau - \tau_0 - \zeta\mu_p\sigma^2}{\sigma}\right), \tag{2}$$

where $\Omega_p^0 = 2/\sigma$ for a 2π pulse. Once the form of the Stokes field is specified, Eq. (1) can be integrated to investigate the evolution of the two fields. The initial conditions are that the atomic population is in $|3\rangle$ at $t = 0$, for all *z*, and the form of the input fields is specified at $z = 0$, for all *t*. Note that Eq. (1) is analytically intractable except under certain approximations, and so one must resort to numerical solutions.

Soliton cloning.—We begin with an input Stokes pulse that has the same profile as the initial pump soliton but has an amplitude, Ω_s^0, which is 5% of Ω_p^0. For specificity, we take $\mu_p = 1/c\sigma^2$ and $\mu_s = \mu_p/1.5$. Note that while the pump pulse is a soliton with an area equal to 2π, the input Stokes pulse is not. Figure 1(a) [Fig. 1(b)] depicts time profiles of the pump (Stokes) pulse at different propagation distances. It is evident from these figures that the pump is continuously depleted as it propagates through the medium, and the Stokes field is progressively amplified and transformed into a pulse whose shape is the clone of the pump soliton. The Stokes pulse becomes

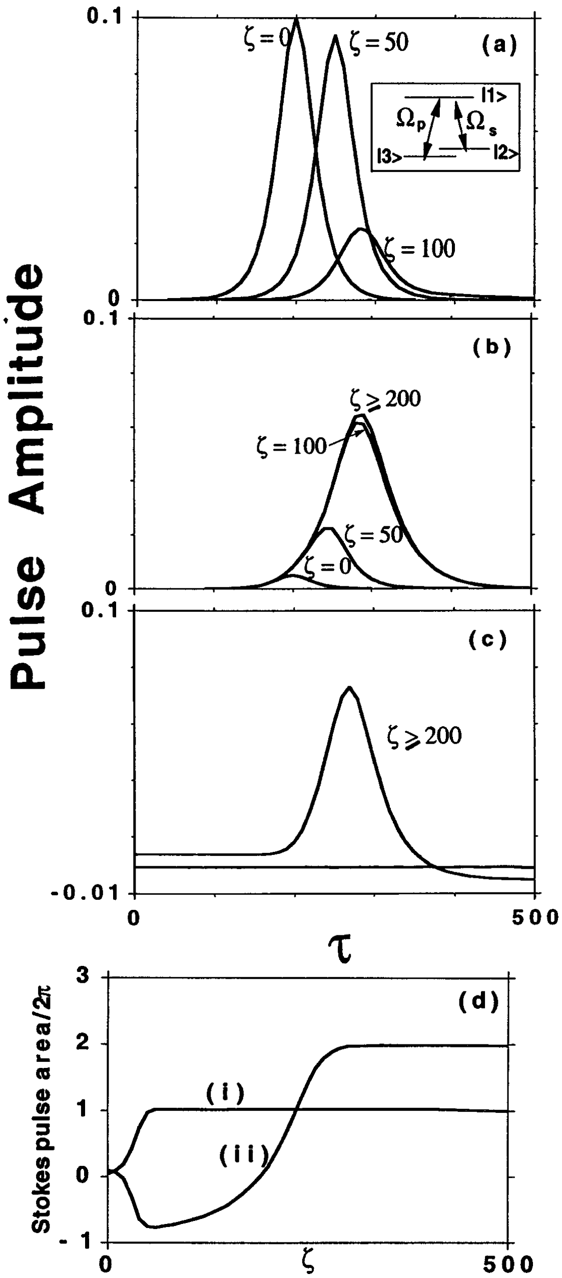

FIG. 1. Temporal profiles of the (a) pump and (b) Stokes pulses at different propagation distances within the medium. Input pulse parameters are $\sigma = 20$, $\Omega_p^0 = 0.1$, $\Omega_s^0 = 0.05\Omega_p^0$, and $\mu_s = \mu_p/1.5$. $\tau_0 = 200$, for both pulses. Inset: Schematic representation of a Λ system with ground state $|3\rangle$ and two excited states $|1\rangle$ and $|2\rangle$. Ω_p (Ω_s) is the Rabi frequency of the pump (Stokes) pulse. (c) Stokes pulse at output of the medium when the input Stokes field is cw, with amplitude of $0.05\Omega_p^0$. Solid line is the extinguished pump pulse. (d) Evolution of Stokes pulse area with propagation distance, when input pump pulse has area of (i) 1.5π and (ii) 3.5π.

a soliton-shaped pulse of area 2π at approximately $\zeta = 200$, and can be fit to a sech profile of the form given by Eq. (2), with $\sigma = 31$ and $\tau_0 = 290$. Once the cloned soliton is formed, it travels *unaltered* through the medium. The pump soliton, which in the absence of the Stokes pulse would have propagated unattenuated at the pump transition, is now completely extinguished at $\zeta = 200$. Clearly, the energy of the input pump soliton goes into the formation of the Stokes soliton at a different frequency. A remarkable feature of the cloning technique is that it works for any profile of the input Stokes fields, as shown in Fig. 1(c), where a continuous wave (cw) field is transformed into a pulse that is almost identical to the pulse in Fig. 1(b). Figure 1(c) highlights an important difference between pulse propagation in two-level and three-level atoms. A cw field corresponds to a pulse of ∞ area, and such a field would not lead to a soliton in two-level atoms. Results for the spatial evolution of pulse areas [Fig. 1(d) are analogous to those for two-level atoms. In SIT, a pulse of area between π and 2π evolves into a soliton of area 2π, and a pulse of area between 3π and 4π into two 2π pulses. Here if the input *pump* soliton has an area between π and 2π, the final *Stokes* clone has an area of 2π, whereas if the initial pump pulse has an area between 3π and 4π, the Stokes field evolves into two 2π solitons.

Soliton dragging.—The ability to control the temporal location (hence speed) of a pulse is critical to many applications [2], and the scheme proposed here can achieve this also. First consider a pump soliton, of speed v, at the pump transition, and no Stokes fields. Since we use the transformation $\tau = t - z/c$, and depict our results in terms of τ, Eq. (2) indicates that the temporal position of the soliton will appear shifted to higher values of τ with increasing ζ. Specifically, a soliton with speed v, at a distance of ζ, would have its peak temporally shifted by $\tau = \mu_p \zeta \sigma^2$. Therefore, an input soliton that is peaked at $\tau = 200$ would be peaked at $\tau = 1200$ by the time it propagates to $\zeta = 1000$. Now we discuss the consequences of introducing a Stokes pulse in addition to the pump soliton. Once the Stokes clone is formed (at $\zeta = 200$), and the pump extinguished, it has a speed of c. We have confirmed this by observing that the Stokes pulse has a peak at $\tau = 290$ for all $\zeta > 200$. Thus, at distance of, say, $\zeta = 1000$, a soliton at a two-level transition would have been peaked at $\tau = 1200$, whereas the Stokes pulse in a three-level atom is dragged to $\tau = 290$. *Thus, a crucial difference between solitons produced in two-level and three-level atoms is that in the former case the pulse is a function of both ζ and τ, whereas the pulse in the latter case is independent of ζ and becomes a function of τ only.*

The extent of dragging can be influenced by the *initial temporal overlap* of the input pulses. Figure 2(a) shows an input Stokes pulse that is time delayed with respect to the pump. Once again, a Stokes clone is formed at the expense of the pump, and on comparing with Fig. 1(b) we note that the Stokes clone has now been dragged to a different temporal location; i.e., the clone is now peaked at $\tau = 350$ instead of $\tau = 290$. We have also inquired into the effect of the input Stokes pulse leading the pump soliton. This *counterintuitive sequence* is the

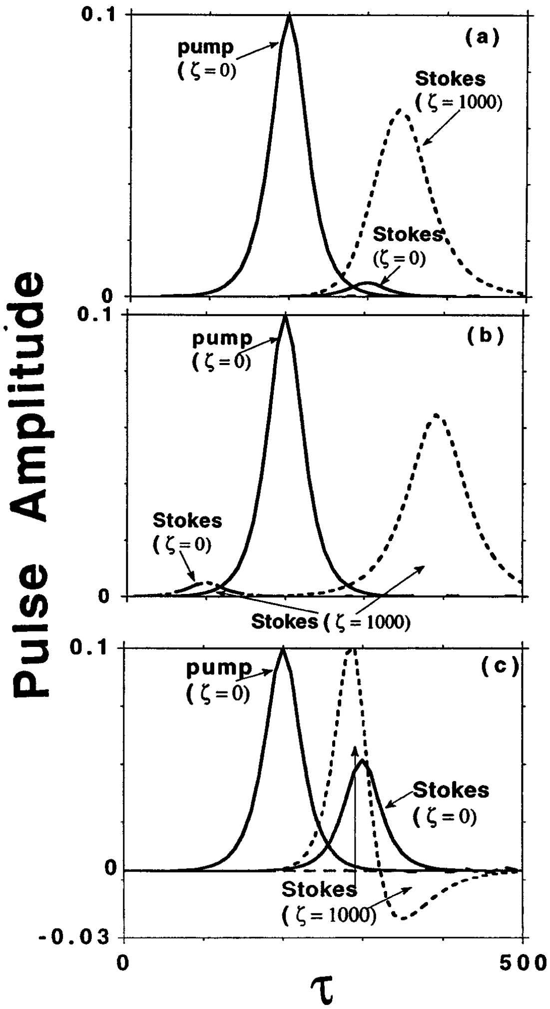

FIG. 2. Temporal profiles of the pulses at $\zeta = 0$ (solid) and $\zeta = 1000$ (dashed). $\sigma = 20$, $\Omega_p = 0.1$, $\Omega_s^0 = 0.05\Omega_p^0$, $\mu_s = \mu_p/1.5$, and $\tau_0 = 200$ for pump pulse. (a) $\tau_0 = 300$ for the input Stokes pulse; (b) $\tau_0 = 100$ for the input Stokes pulse. Note that the input Stokes pulse here is indistinguishable from the leading part of the final dual pulse; (c) as in (a) with $\Omega_s^0 = 0.5\Omega_p^0$.

key to realizing some coherence and interference effects in driven atoms [10]. In the context of soliton control, we find the surprising result that when the Stokes pulse initially leads the pump, as seen in Fig. 2(b), one finally obtains a dual pulse at the Stokes frequency. The origin of this dual pulse is interesting—the leading pulse is the initial Stokes pulse which propagates *unchanged*, whereas the trailing pulse arises from the conversion of the pump soliton into a Stokes clone. Thus, only the trailing pulse has an area of 2π. This dual pulse solution is distinct from the two-soliton solution that results when a 4π pulse at a two-level transition splits into two 2π pulses.

Varying the temporal overlap, and the ratio of Ω_s^0 to Ω_p^0, of the input pulses not only enables dual-pulse generation at the Stokes transition, but also permits a control over the relative phases of these pulses. This is illustrated in Fig. 2(c), where $\Omega_s^0 = 0.5\Omega_p^0$, and the input Stokes pulse lags the pump, leading to dual pulses that are *out of phase* with each other. Such phase control can be important, for example, in experiments where atoms and molecules need to be prepared in prescribed quantum states. We note that if the Stokes leads the pump at the input to the medium, one gets a dual pulse as in Fig. 2(a).

Parametric population transfer.—Our numerical calculations reveal that, despite the use of resonant pulses, soliton cloning and dragging are accompanied by a parametric transfer of population to the initial state, $|3\rangle$. To understand this analytically, we look back at Eq. (1), and derive conservation laws for the energy and excitation number as

$$\frac{\partial}{\partial\zeta}(|\Omega_p|^2 + |\Omega_s|^2) + 4\mu_0\frac{\partial}{\partial\tau}\times [\omega_p|c_1|^2 + (\omega_p - \omega_s)|c_2|^2] = 0 \tag{3a}$$

and

$$\frac{\partial}{\partial\zeta}\left(\frac{|\Omega_p|^2}{\omega_p} + \frac{|\Omega_s|^2}{\omega_s}\right) + 4\mu_0\frac{\partial}{\partial\tau}|c_1|^2 = 0, \tag{3b}$$

respectively, where $\mu_p(\mu_s) = \mu_0\omega_p(\omega_s)$, $\mu_0 = 4\pi \times N|d|^2/\hbar c$, N is the number of atoms, and d is the dipole moment. From Eq. (3a), one obtains

$$\frac{\partial}{\partial\zeta}\int_0^\tau(|\Omega_p|^2 + |\Omega_s|^2)\,d\tau + 4\mu_0\omega_p\times [|c_1|^2 + (1 - \omega_s/\omega_p)|c_2|^2] = 0. \tag{4a}$$

The numerics show that for $\zeta \to \infty$, the integral in Eq. (4a) vanishes, and one gets

$$\lim_{\zeta\to\infty}|c_1|_\tau^2 + (1 - \omega_s/\omega_p)|c_2|_\tau^2 = 0. \tag{4b}$$

Similarly, from Eq. (3b) one has

$$\lim_{\zeta\to\infty}|c_1|_\tau^2 = 0, \tag{4c}$$

and so Eqs. (4b) and (4c) imply that, as $\zeta \to \infty$, the populations in levels $|1\rangle$ and $|2\rangle$ become zero and the entire population settles in $|3\rangle$.

The numerical results also show that for small ζ (relative to $1/\mu_p\sigma$) it is possible to transfer the entire population to $|1\rangle$, and that this transfer reduces progressively with increasing ζ. The value of ζ at which all population returns to $|3\rangle$ coincides with that at which the pump is fully depleted, and so once in $|3\rangle$, population cannot be moved from there. Thus, the cloned soliton, once formed, propagates unaltered at the speed of light.

We now demonstrate that the pulse shaping effects described here are a consequence of our model and cannot be reproduced by other processes, such as stimulated Raman scattering (SRS) [11]. Note that, unlike SRS, our model utilizes resonant fields and neglects the dephasing between levels $|2\rangle$ and $|3\rangle$. In Fig. 3(a), we show a typical result when the detunings of the initial pump and Stokes pulses are equal to Ω_p^0. Clearly, even at $\zeta = 2000$, the pump is not fully extinguished, and we find that the Stokes pulse does not fit a sech profile. For larger detunings,

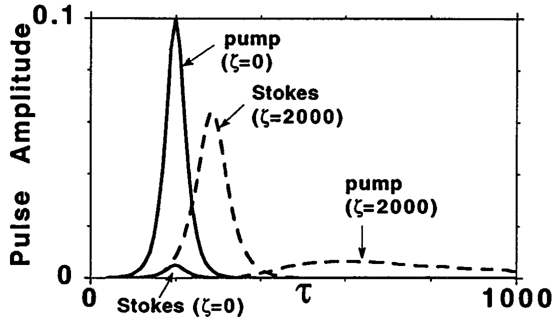

FIG. 3. Temporal profiles of the pulses at $\zeta = 0$ (solid) and $\zeta = 1000$ (dashed) for nonzero detuning of input pulses. $\sigma = 20$, $\Omega_p = 0.1$, $\Omega_s^0 = 0.05\Omega_p^0$, $\mu_s = \mu_p/1.5$, and $\tau_0 = 100$ for pump pulse. $\Delta_1 = \Delta_2 = 0.1$.

the two input pulses propagate unchanged through the medium, and so once again the pulse shaping effects are lost (results not shown). This unaltered propagation of the pulses is a consequence of coherent population trapping (CPT). Conventional CPT occurs when relaxation from the excited state is very large, and so that state can be adiabatically eliminated. In our model, though the relaxation of the upper state is neglected, the large detuning effectively allows elimination of the upper state, thereby leading to CPT. In contrast, the usual SRS is dominated by dephasing effects and so no CPT occurs. For even larger detunings, the interaction between atoms and pulses becomes less effective, and once again cloning does not occur. Thus, we emphasize that the pulse shaping effects described in this Letter are a consequence of our model. We have also experimented with unequal detunings for the two fields, and find that our conclusions are still valid.

We now contrast our results with other previous work. In electromagnetically induced transparency [6,7], the attenuation of a weak pulse, at the pump transition of an absorbing medium, is retarded by the application of a strong pulse at the Stokes transition. Our physical problem, in contrast, shows that the presence of a weak Stokes pulse leads to an attenuation of the pump pulse. A soliton at the pump transition would ordinarily propagate invariantly, and we rely upon energy transfer from the pump to the Stokes pulse to produce a soliton at a different frequency. Finally, all previous work on soliton dragging has been in the context of optical fibers [2–4], which are dispersive media with a large $\chi^{(3)}$, while we utilize a resonant medium in which the nonlinearities are included to all orders.

Altering the initial conditions on Eq. (1) can lead to a variety of interesting phenomena; e.g., if the initial population is coherently distributed between $|2\rangle$ and $|3\rangle$ [8], both the pump and Stokes pulses become identical in amplitude and shape, and then propagate unaltered. This suggests a technique for producing twin pulses of different frequencies.

In summary, we have shown that in a coherently driven, nonabsorbing Λ system [12], a soliton at one transition can transform a weak field of arbitrary profile at the other transition into a clone of the soliton. We have demonstrated that this system acts as a parametric amplifier, and that the location of the Stokes clone can be controlled through the temporal overlap of the input pulses and the ratio, Ω_s/Ω_p. Finally, the overlap of the pulses can also be used to produce dual pulses, with control over their relative phases. We suggest using the $5P_{3/2}$ and $5S_{1/2}$ $(F = 1, 2)$ levels in atomic rubidium as a medium for testing our predictions. The upper state relaxation is about 6 MHz, and so the use of picosecond pulses will enable Rabi frequencies that satisfy the requirements of our model.

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*Electronic address: gvemuri@indyvax.iupui.edu

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[12] Though our results are derived for a Λ system, it is anticipated that one would find a parameter regime for ladder and V systems where similar results will hold.