

Limit on the Two-Photon Production of the Glueball Candidate $f_J(2220)$ at the Cornell Electron Storage Ring

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We use the CLEO detector at the Cornell e^+e^- storage ring, CESR, to search for the two-photon production of the glueball candidate $f_J(2220)$ in its decay to $K_s K_s$. We present a restrictive upper limit on the product of the two-photon partial width and the $K_s K_s$ branching fraction, $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$. We use this limit to calculate a lower limit on the stickiness, which is a measure of the two-gluon coupling relative to the two-photon coupling. This limit on stickiness indicates that the $f_J(2220)$ has substantial glueball content. [S0031-9007(97)04293-2]

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The two-photon width of a resonance is a probe of the electric charge of its constituents, so the magnitude of the two-photon coupling can serve to distinguish quark-dominated resonances from glue-dominated resonances (henceforth simply called “glueballs”). The $f_J(2220)$, sometimes referred to as the $\xi(2230)$, was first reported by the Mark III Collaboration [1]. This resonance is a glueball candidate due to its narrow width [1,2], its observation in production modes consistent with those expected for glueballs [1–5], and its proximity in mass to lattice QCD predictions of the tensor glueball [6,7].

In this Letter we report on a search for the $f_J(2220)$ in two-photon interactions at CLEO and set an upper limit on the product of its two-photon partial width and branching fraction to $K_s K_s$ [8], improving on a previous limit set by ARGUS [9] using the $K^+ K^-$ decay mode. Using our measurement, we calculate the stickiness, a useful glueball figure of merit defined in Ref. [10], of the $f_J(2220)$ resonance.

CLEO II is a general purpose detector [11] using the e^+e^- storage ring, CESR [12], operating at $\sqrt{s} \sim 10.6$ GeV. CLEO II contains three concentric wire chambers that detect charged particles over 95% of the solid angle. A superconducting solenoid provides a magnetic field of 1.5 T, giving a momentum resolution of $\sigma_p/p \approx 0.5\%$ for $p = 1$ GeV/ c . Outside of the wire chambers and a time of flight system, but inside the solenoid, is a CsI electromagnetic calorimeter, consisting of 7800 crystals arranged as two end caps and a barrel region. For a 100 MeV electromagnetic shower in the barrel, the calorimeter achieves an energy resolution of $\sigma_E/E \approx 4\%$.

In two-photon events, the initial state photons are approximately real and tend to have a large fraction of their momenta along the beam line. The electron and positron rarely have enough transverse momentum to be observed. As the two photons generally have unequal momentum, the $\gamma\gamma$ center of mass tends to be boosted along the beam axis. We detect those events in which the decay products have sufficient transverse momentum to be observed in CLEO.

We search for the two-photon production of $f_J(2220)$ in its decay to $K_s K_s$ with each K_s decaying into $\pi^+ \pi^-$:

$$\gamma\gamma \rightarrow f_J(2220) \rightarrow K_{s,1} K_{s,2} \begin{array}{l} \cdot \\ \downarrow \\ \begin{array}{l} \downarrow (\pi^+ \pi^-)_2 \\ \downarrow (\pi^+ \pi^-)_1 \end{array} \end{array}$$

In our analysis of 3.0 fb^{-1} of data, we use the following selection criteria to minimize background. We select events with four tracks, and we require that the sum of charges is zero. To select two-photon events we require that the event energy is less than 6.0 GeV, and that the transverse component of the vector sum of the track momenta is less than 0.2 GeV/ c . To suppress $\gamma\gamma \rightarrow 4\pi$, where the four pions do not result from K_s decays, we require two $\pi^+ \pi^-$ pairs to form K_s vertices separated in the $r - \phi$ plane by more than 5 mm. The vertex separation resolution is 1 mm. Finally, we evaluate the π^\pm track parameters at the respective vertices, and select events in which $[m(\pi^+ \pi^-)_1, m(\pi^+ \pi^-)_2]$ lies within a circle of radius 10 MeV about the point $[m_{K_s}, m_{K_s}]$. The detector K_s mass resolution is ~ 3.3 MeV.

The distribution of $m(\pi^+ \pi^-)_1$ versus $m(\pi^+ \pi^-)_2$ observed in data is displayed in Fig. 1 with all selection criteria applied except the mass circle requirement. There is a strong enhancement near the $[m_{K_s}, m_{K_s}]$ point in the $[m(\pi^+ \pi^-)_1, m(\pi^+ \pi^-)_2]$ mass plane. After applying the 10 MeV mass circle criterion, there is less than 5% non- K_s background.

We use a Monte Carlo simulation to determine our sensitivity to the two-photon production of the $f_J(2220)$. The two-photon Monte Carlo events were generated using a program based on the Budnev-Ginzburg-Meledin-Serbo formalism [13]. For the simulation we assume the value $J = 2$ for the total angular momentum. We use a mass and width determined by combining [14] the Mark III [1] and BES [2] results, giving $m_{f_J} = 2234 \pm 6$ MeV and $\Gamma_{f_J} = 19 \pm 11$ MeV. The simulation of the transport and decay of the final state particles through the CLEO detector is performed by a GEANT-based detector simulator [15]. From the detector simulation we find a

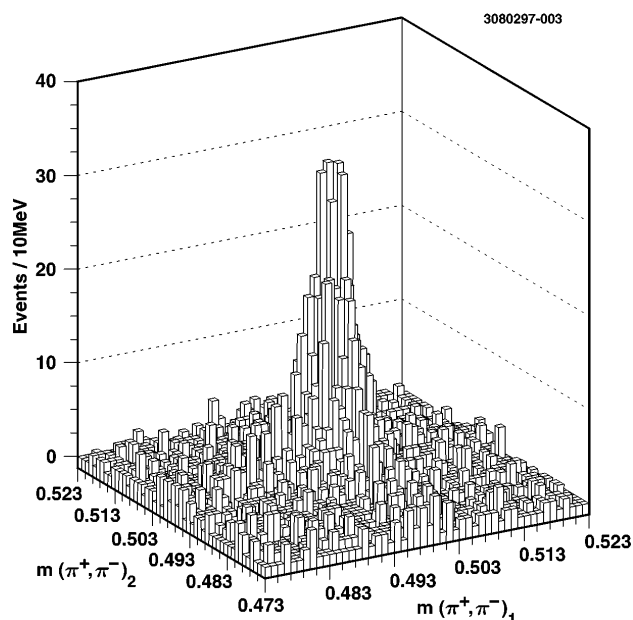


FIG. 1. $m(\pi^+\pi^-)_1$ versus $m(\pi^+\pi^-)_2$ for data. Each event has two entries corresponding to transposition of the labels $1 \leftrightarrow 2$.

$K_s K_s$ mass resolution, $\sigma_{K_s K_s}$, of 9 MeV for $m_{K_s K_s}$ near 2.23 GeV. The net selection efficiencies are 0.07 and 0.15 for pure helicity 0 and pure helicity ± 2 , respectively [16].

We construct a $K_s K_s$ mass distribution for those events that satisfy all of the selection criteria. In Fig. 2, we display the data for the $K_s K_s$ mass region of interest. No enhancement at the $f_J(2220)$ mass is observed.

To determine the number of $\gamma\gamma \rightarrow f_J(2220)$ events, we count the number of events within a region that has been optimized based on the line shape of the $f_J(2220)$.

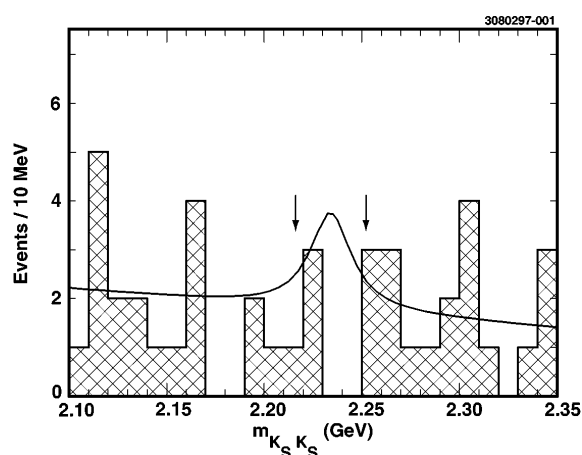


FIG. 2. $K_s K_s$ mass distribution (GeV) observed in data near the $f_J(2220)$ mass. The vertical arrows delineate the signal region in which events are counted. The solid line is the sum of a fit to the background and the signal line shape for central values of the resonance parameters, $m_{f_J} = 2.234$ GeV and $\Gamma_{f_J} = 19$ MeV, corresponding to the observed 95% C.L. upper limit of 4.9 signal events.

We convolve a detector resolution function with a Breit-Wigner resonance to determine the expected shape. This line shape is used to determine the signal region size that maximizes ε^2/b , to maximize the sensitivity to observe the resonance, where ε is the fraction of the area under the signal line shape that falls within the region, and b is the estimated number of background events determined as described below. For $\sigma_{K_s K_s} = 9$ MeV and $\Gamma_{f_J} = 19$ MeV this window is ± 18 MeV, for which $\varepsilon = 70\%$.

To obtain a background shape, we fit the $m_{K_s K_s}$ distribution with a linear function from 2.05 to 2.35 GeV, excluding a ± 40 MeV region centered on the expected mass. From this we extract an average background of 1.8 ± 0.3 events per 10 MeV for $m_{f_J} = 2.234$ GeV. Within the signal region determined for the central values of the resonance parameters, we count four events. Having observed four events while expecting 6.5 from background, we use the standard Particle Data Group (PDG) technique of extracting an upper limit for a Poisson distribution with background [17] to extract an upper limit of 4.9 signal events at the 95% C.L.

To determine the value of $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$, we assume that $f_J(2220)$ is produced incoherently with the background. We scale the branching fraction and partial width used in the Monte Carlo generator by the ratio of the upper limit on the number of data events to the number of selected Monte Carlo events, and by the ratio of Monte Carlo to data luminosities,

$$\Gamma_{\gamma\gamma}^{\text{data}} \mathcal{B}_{K_s K_s}^{\text{data}} = \frac{n^{\text{data}}}{n^{\text{MC}}} \frac{L^{\text{MC}}}{L^{\text{data}}} \Gamma_{\gamma\gamma}^{\text{MC}} \mathcal{B}_{K_s K_s}^{\text{MC}}. \quad (1)$$

The two-photon partial width, $\Gamma_{\gamma\gamma}$, can be expressed as the sum of two components, $\Gamma_{\gamma\gamma}^{2,0}$ and $\Gamma_{\gamma\gamma}^{2,2}$, the two-photon partial widths associated with helicity 0 and helicity ± 2 projections, respectively. We must differentiate between the two partial widths because the detection efficiencies for the two allowed helicity projections are not the same due to their different final state angular distributions. Under the assumption that the ratio of $\Gamma_{\gamma\gamma}^{2,2} \cdot \Gamma_{\gamma\gamma}^{2,0}$ is 6:1 [18,19] based on Clebsch-Gordon coefficients, we obtain the result

$$(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)} \leq 1.4 \text{ eV}, \quad 95\% \text{ C.L.} \quad (2)$$

The limit is slightly stronger, 1.3 eV, if $J = 4$ is assumed for the resonance [20]. The limits include uncertainties associated with systematics which will be discussed below.

Without making any assumption about the ratio of partial widths of the two helicity projections, we can set a 95% C.L. functional limit for $J = 2$,

$$(0.52\Gamma_{\gamma\gamma}^{2,0} + 1.08\Gamma_{\gamma\gamma}^{2,2}) \mathcal{B}_{K_s K_s} \leq 1.4 \text{ eV}, \quad 95\% \text{ C.L.} \quad (3)$$

The ratio of the partial width coefficients in Eq. (3) is given by the ratio of efficiencies for helicity 0 to helicity ± 2 . The overall normalization is set to be consistent with Eq. (2).

Systematic uncertainties have been included in determining these upper limits using a Monte Carlo program.

We estimate the following systematic uncertainties in the overall detector efficiency: 8% due to triggering, 7% due to tracking, and 7% due to simulation of selection criteria. The total systematic uncertainty associated with efficiency is 13%. We estimate the systematic uncertainty in the background normalization to be 16%. The uncertainty associated with the $f_J(2220)$ resonance parameters is also included by statistically sampling measurement results when varying these resonance parameters up to ± 2.5 standard deviations [21]. The window selection size is optimized for each case of varied resonance parameters. For a variation of -1 and $+1$ standard deviations in the resonance width the window sizes are ± 13 and ± 26 MeV, respectively.

We have verified our technique by using the same Monte Carlo simulation and analysis approach to measure the two-photon partial width of the $f_2'(1525)$. The $f_2'(1525)$ measurement is a sound test as the $f_2'(1525)$ produces a prominent peak in the $K_s K_s$ mass distribution and has quantum numbers consistent with those expected for the $f_J(2220)$. We measure a value for the product of the partial width and the $K_s K_s$ branching fraction that is within 1 standard deviation [22] of the PDG central value.

The small value of the $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$ upper limit obtained from this analysis supports the identification of the $f_J(2220)$ as a glueball. We can make a more quantitative statement by calculating the stickiness of the resonance. Stickiness is a useful glueball figure of merit that is a measure of color charge relative to electric charge. The definition of stickiness is [10]

$$S_X \equiv N_l \left(\frac{m_X}{k_{\psi \rightarrow \gamma X}} \right)^{2l+1} \frac{\Gamma(J/\psi \rightarrow \gamma X)}{\Gamma(X \rightarrow \gamma\gamma)} \sim \frac{|\langle X | gg \rangle|^2}{|\langle X | \gamma\gamma \rangle|^2}. \quad (4)$$

The parameter $k_{\psi \rightarrow \gamma X} = (m_\psi^2 - m_X^2)/(2m_\psi)$ is the energy of the photon from a radiative decay of the J/ψ at rest. The phase-space term removes the mass dependence. The quantum number l indicates the angular momentum between the initial state gauge bosons. N_l is a normalization parameter defined so that the stickinesses of the $f_2(1270)$ ($l = 0$) is 1. To determine the value of N_l we use the mass, two-photon width, and radiative J/ψ decay branching fraction of the $f_2(1270)$ given by the PDG [17].

To calculate a lower limit on the stickiness, we combine our upper limit for $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$ from Eq. (2) with a value for $\Gamma(J/\psi \rightarrow \gamma f_J(2220)) \mathcal{B}(f_J(2220) \rightarrow K_s K_s)$ obtained by averaging [23] results from Mark III [1] and BES [2]. The $\mathcal{B}(J/\psi \rightarrow \gamma f_J(2220)) \mathcal{B}(f_J(2220) \rightarrow K_s K_s)$ branching fraction so determined is $(2.2 \pm 0.6) \times 10^{-5}$. From this we calculate a lower limit on stickiness of 76 at the 95% C.L. for the $f_J(2220)$. The statistical and systematic uncertainties of the inputs, including the uncertainty on the J/ψ branching fraction, and the $f_J(2220)$ resonance parameters [24], are incorporated into this limit through a Monte Carlo program.

The observation of the $f_J(2220)$ in production modes consistent with those expected for glueballs has made it a

glueball candidate. With the limit on $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$ presented here we are able to make a much stronger statement. In particular, it is difficult to explain how a $q\bar{q}$ meson, even pure $s\bar{s}$, could have such a large stickiness. In general, explanations that give small two-photon partial widths give small radiative J/ψ decay branching fractions. Radial and angular excitations fall into this category. A $J = 4$ resonance is not ruled out experimentally. However, under the assumption $J = 4$, the limit on the product of the two-photon width and $K_s K_s$ branching fraction is slightly more stringent, and the phase-space term to which stickiness is proportional becomes very large. A small two-photon width due to a cancellation involving specific values of the singlet-octet mixing and the ratio of matrix elements is possible but unlikely.

In this Letter we have presented the results of the search for $f_J(2220)$ production in two-photon interactions. We have reported a very small upper limit for $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$. The minimum stickiness obtained from the two-photon width upper limit is difficult to understand in the context of a $q\bar{q}$ resonance, and should be considered as strong evidence that the $f_J(2220)$ is a glueball.

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- [20] Although the efficiency for a $J = 4$ resonance is lower than for a $J = 2$ resonance, the production rate is proportional to $2J + 1$. Consequently, the limit on the product of the two-photon width and branching fraction is more stringent for $J = 4$.
- [21] The limit on $(\Gamma_{\gamma\gamma} \mathcal{B}_{K_s K_s})_{f_J(2220)}$ is 1.3 eV if evaluated at the most likely values of the resonance parameters.
- [22] We measure the two-photon width of the $f_2'(1525)$ to be (84 ± 8) eV (statistical uncertainty only) as compared to the PDG value of (97 ± 16) eV [17]. We have not performed a study of systematic uncertainties for this measurement.
- [23] We form an average $K_s K_s$ branching fraction by combining the $K_s K_s$ with the $K^+ K^-$ measurements. We treat systematic uncertainties as 100% correlated within an experiment and uncorrelated between experiments.
- [24] When evaluated at the most likely values of the resonance parameters, we obtain a stickiness of 82.