Dephasing and the Orthogonality Catastrophe in Tunneling through a Quantum Dot: The "Which Path?" Interferometer

I. L. Aleiner,[1] Ned S. Wingreen,[1] and Yigal Meir[2]
[1]*NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540*
[2]*Physics Department, Ben Gurion University, Beer Sheva, 84105, Israel*
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The "Which Path?" interferometer consists of an Aharonov-Bohm ring with a quantum dot (QD) built in one of its arms, and an additional quantum point contact (QPC) located close to the QD. The transmission coefficient of the QPC depends on the charge state of the QD. Hence the point contact acts as a controllable measurement device for which path an electron takes through the ring. We calculate the suppression of the Aharonov-Bohm oscillations which is caused by both measurement dephasing and the orthogonality catastrophe, i.e., respectively, by real and virtual electron-hole pair creation at the QPC. [S0031-9007(97)04496-7]

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The interference between different trajectories of a particle is one of the central postulates of quantum mechanics. The transition between classical and quantum behavior depends on when and whether this interference is realized. With the advent of mesoscopic conducting structures, it has become possible to study directly the coherence between different trajectories of an electron in a metal or semiconductor Aharonov-Bohm ring. Among the phenomena observed in these systems are Universal conductance fluctuations, weak localization, and inelastic dephasing by electron-electron and electron-phonon scattering [1]. Recently, a set of elegant Aharonov-Bohm ring experiments was performed to detect the phase shift of electrons passing through a quantum dot (QD) built in one arm of the ring [2,3]. These experiments were the first to demonstrate the coherent propagation of electrons through a quantum dot.

The observation of phase coherence in transport through a QD presents an opportunity to study the origins of *decoherence* in mesoscopic structures. Recent work in atomic physics has measured decoherence rates of the electromagnetic field in a cavity [4]. These experiments, however, did not control the rate of dephasing. An Aharonov-Bohm ring with a QD in one of its arms offers the ability not only to measure dephasing rates, but also to directly control these rates by modifying the environment of the quantum system. The proposed experimental set up for this "Which Path?" interferometer [5] is shown in Fig. 1. An electron traversing the ring may follow the upper or the lower arm. In the latter case, the electron must pass through a QD located in the lower arm. In the proposed experiment, an additional wire containing a quantum point contact (QPC) is placed close to the QD. The electrostatic field of an extra electron on the QD changes the transmission coefficient $\mathcal{T}$ of the nearby QPC, and hence changes the conductance of the wire. The change in the current in the wire "measures" which path the electron took around the ring, causes the paths to decohere, and so suppresses the Aharonov-Bohm oscillations. Loss of interference due to the trace left in the environment by an interacting particle was considered in detail in Ref. [7]. Rate equations describing decoherence in multiple dot systems were derived in Ref. [8], however, they are not suitable for the present problem.

To estimate the rate of decoherence induced by the current in the wire, consider the following argument: Adding an electron to the dot changes the conductance of the QPC by $2(e^2/h)\Delta\mathcal{T}$. Detection of this electron requires a time t_d such that the change in the number of electrons crossing the QPC exceeds the typical quantum shot noise,

$$t_d \frac{V}{e} \frac{2e^2}{h} \Delta\mathcal{T} \geq \sqrt{t_d \frac{V}{e} \frac{2e^2}{h} \mathcal{T}(1 - \mathcal{T})}, \qquad (1)$$

where V is the bias voltage in the wire, and the right hand side reflects the quantum shot noise across the QPC [9].

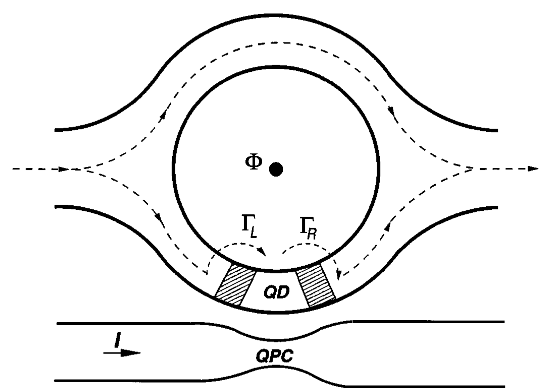

FIG. 1. Schematic view of the "Which Path?" interferometer [5]. The quantum dot (QD) is built in the lower arm of an Aharonov-Bohm ring, as shown. The transmission coefficient of the nearby quantum point contact (QPC) depends on the occupation number of the dot because of electrostatic interactions. (Four-terminal measurement is implied, so that closed orbits in the ring are not important.)

The decoherence rate, therefore, depends on both the bias across the QPC and its transmission coefficient:

$$\frac{1}{t_d} \approx \frac{eV}{h} \frac{(\Delta \mathcal{T})^2}{\mathcal{T}(1 - \mathcal{T})}. \tag{2}$$

In this paper, we calculate nonperturbatively the suppression of the Aharonov-Bohm oscillations in a ring with a QD due to the close proximity of a wire containing a QPC. Our results support the simple argument given above, and explicitly show that $1/t_d$ is the rate of real electron-hole pair creation in the wire. The simple estimate (2), however, neglects the effect of virtual electron-hole pairs. The latter do not directly cause decoherence, but they decrease the transmission amplitude through the QD. These virtual processes result in *power-law suppression* of the Aharonov-Bohm oscillations. This is an example of the orthogonality catastrophe [10,11], and is an inevitable consequence of "measurement" by local interaction with a many-body system. (We neglect the additional orthogonality catastrophe due to ring electrons [12] because it cannot be externally controlled.)

In the proposed experiment, the transmission coefficient across the ring $\mathcal{T}_{\rm ring}$ can be obtained from the appropriate combination of measurements in a multiprobe geometry [3]. According to the Aharonov-Bohm effect, i.e., the phase difference of $2\pi\Phi/\Phi_0$ between electron trajectories which encompass a magnetic flux Φ, one has

$$\mathcal{T}_{\rm ring} = \mathcal{T}_{\rm ring}^{(0)} + \mathrm{Re}\{t^* t_{\rm QD} e^{2\pi i\Phi/\Phi_0}\} + \ldots, \tag{3}$$

where the dots indicate higher harmonics of Φ, and $\Phi_0 = hc/e$ is the flux quantum. The magnetic-flux independent term $\mathcal{T}_{\rm ring}^{(0)}$ and the amplitude t^* are sensitive to the geometry of the system (e.g., the structure of the leads, lengths of the arms, etc.). The amplitude $t_{\rm QD}$ for coherent transmission through the dot reflects only the properties of the dot and its immediate environment; this quantity will be discussed in the remainder of this paper.

We are interested in the Aharonov-Bohm oscillations in the vicinity of Coulomb blockade peaks, i.e., near the charge degeneracy point of the QD. This means that only two charging states of the dot, N and $N + 1$, are relevant to transport [13]. We neglect energy dependence of the phase from propagation down the arms of the ring [14], so that $t_{\rm QD} = \int d\epsilon (-\partial f/\partial \epsilon)\, t_{\rm QD}(\epsilon)$, where $f(\epsilon)$ is the Fermi distribution function (all energies are counted from the Fermi level) and $t_{\rm QD}(\epsilon)$ is the transmission amplitude for an electron with energy ϵ through the QD.

In the Coulomb-blockade regime the broadening of levels is smaller than the level spacing in the dot [13]. Thus, it is natural to consider only a single resonant level in the dot. The amplitude $t_{\rm QD}(\epsilon)$ can then be expressed in terms of the exact retarded Green's function of this level:

$$t_{\rm QD}(\epsilon) = -i\sqrt{4\Gamma_L\Gamma_R} \int dt\, e^{i\epsilon t} G_{\rm QD}^r(t), \tag{4}$$

where $\Gamma_{L,R}$ are the half-widths of the level with respect to tunneling to the left or to the right. The retarded Green's function is defined as $G_{\rm QD}^r(t) = -i\theta(t)\langle \hat{c}(t)\hat{c}^\dagger(0) + \hat{c}^\dagger(0)\hat{c}(t)\rangle$, where $\hat{c}(t)$ is the Heisenberg operator which removes an electron from the resonant level (we put $\hbar = 1$).

The electrons in the dot interact with the electrons in the wire. Only the local scattering potential of the QPC is significantly affected by this electrostatic interaction. We use the standard description of a QPC as a 1D noninteracting electron system, and choose the basis of scattering eigenstates corresponding to the potential in the QPC when exactly N electrons occupy the QD:

$$\hat{H}_N = \int \frac{dk}{2\pi}\, k[\psi_{\mathcal{L}}^\dagger(k)\psi_{\mathcal{L}}(k) + \psi_{\mathcal{R}}^\dagger(k)\psi_{\mathcal{R}}(k)]. \tag{5}$$

$\psi_{\mathcal{L},\mathcal{R}}$ are the fermionic operators for the scattering states moving from the left and right, respectively, with summation over spin indices implied. We linearize the spectrum and put the Fermi velocity in the wire $v_F = 1$. The electrostatic field of an additional (N + 1st) electron on the QD changes the wire Hamiltonian to $\hat{H}_{N+1} = \hat{H}_N + \hat{V}$:

$$\begin{aligned} \hat{V}(t) &= \hat{V}_{\mathcal{LL}}(t) + \hat{V}_{\mathcal{RR}}(t) + \hat{V}_{\mathcal{LR}}(t); \\ \hat{V}_{\mathcal{LL}(\mathcal{RR})}(t) &= \lambda \int \frac{dk_1 dk_2}{2\pi}\, \psi_{\mathcal{L}(\mathcal{R})}^\dagger(k_1,t)\psi_{\mathcal{L}(\mathcal{R})}(k_2,t), \\ \hat{V}_{\mathcal{LR}}(t) &= \lambda_{\mathcal{LR}} \int \frac{dk_1 dk_2}{2\pi} \\ &\quad \times [\psi_{\mathcal{L}}^\dagger(k_1,t)\psi_{\mathcal{R}}(k_2,t)e^{ieVt} + \mathrm{H.c.}], \end{aligned} \tag{6}$$

where the $\hat{\psi}(t) = e^{i\hat{H}_0 t}\hat{\psi} e^{-i\hat{H}_0 t}$ are electron operators in the interaction representation, and λ and $\lambda_{\mathcal{LR}}$ are scattering matrix elements. The operators $\hat{V}_{\mathcal{LL}}(t)$ and $\hat{V}_{\mathcal{RR}}(t)$ each mix scattering states propagating in a single direction, and only produce a change in the phase of the transmission amplitude of the QPC. The mixing between scattering states which are incident from opposite directions is given by $\hat{V}_{\mathcal{LR}}(t)$, and corresponds to a change in the transmission coefficient $\mathcal{T}$ of the QPC. The explicit oscillatory time dependence of $\hat{V}_{\mathcal{LR}}(t)$ describes a finite bias in the wire, i.e., eV corresponds to the chemical potential difference between $\mathcal{L}$ and $\mathcal{R}$ scattering states.

The Green's function of the resonant level in the dot interacting with the wire can be approximated as

$$G_{\rm QD}^r(t) = -i\theta(t)e^{-i\epsilon_0 t - \Gamma t}[P_{N+1}A_-(t) + P_N A_+(t)], \tag{7}$$

where ϵ_0 is the single-electron energy of the level, and P_n is the probability of the corresponding charging state of the dot, $P_N + P_{N+1} = 1$. The total tunneling half-width Γ of the level is given by $\Gamma = \Gamma_L + \Gamma_R$, and the coherence factors $A_\pm(t)$ account for the response of the wire to the addition (removal) of an electron from the dot,

$$A_+(t) = \langle e^{i\hat{H}_N t} e^{-i\hat{H}_{N+1} t}\rangle_{H_N}, \tag{8a}$$

$$A_-(t) = \langle e^{i\hat{H}_N t} e^{-i\hat{H}_{N+1} t}\rangle_{H_{N+1}}. \tag{8b}$$

The expectation values are taken with respect to an equilibrium ensemble in the wire with the Hamiltonian, H_N

or H_{N+1}, indicated as a subscript. It is easy to see that Eq. (7) is exact in two important limiting cases. In the absence of the interaction $A_\pm(t) = 1$ and Eq. (7) reduces to the retarded Green's function for a noninteracting resonant level, and Eq. (4) becomes a simple Breit-Wigner formula. Also, in the absence of tunneling, $\Gamma = 0$, Eqs. (7) and (8) are exact expressions for an isolated level coupled to the wire. For the intermediate regime $\Gamma > 0$, Eq. (7) is not exact. Physically, it neglects interaction induced correlations between consecutive tunneling events of different electrons into the dot. However, such events are rare in the case of weak tunneling, and Eq. (7) is expected to be a good approximation even for $\Gamma \neq 0$.

Let us now turn to the calculation of the coherence factors $A_\pm(t)$. For zero current in the wire, Eq. (8) corresponds to the well-known "orthogonality catastrophe" [10], i.e., the response of an equilibrium noninteracting electron system to a sudden perturbation. Exact results for this problem were first obtained in Ref. [15]. The long-time behavior ($eVt \gg 1$) of the nonequilibrium orthogonality catastrophe was recently considered by Ng [16]. In order to find the dependence of $t_{\mathrm{QD}}(\epsilon)$ on bias eV, we need to know $A_\pm(t)$ at all times. For the case of nonequilibrium in the wire we were not able to obtain exact results for arbitrary constants $\lambda, \lambda_{\mathcal{LR}}$. Instead, we restrict ourselves to the case where the mixing between scattering states is small, $\lambda_{\mathcal{LR}} \ll 1$, but λ is arbitrary.

We begin by rewriting the coherence factor $A+(t)$ as

$$A_+(t) = \left\langle T_t e^{-i\int_0^t \hat{V}(t_1)\,dt_1} \right\rangle_{H_N} = A(t) A_{\mathcal{LR}}(t), \quad (9)$$

where $A(t)$ describes the orthogonality catastrophe in the absence of mixing between the scattering states:

$$A(t) = \left\langle T_t e^{-i\int_0^t dt_1 [\hat{V}_{\mathcal{LL}}(t_1) + \hat{V}_{\mathcal{RR}}(t_1)]} \right\rangle_{H_N}, \quad (10)$$

and can be evaluated exactly. The results for the coherence factor (10) are well known [15]. One has

$$A(t) = \left(\frac{i\pi T}{\xi_0 \sinh \pi T t} \right)^{4(\delta/\pi)^2}, \qquad \delta = \arctan \pi\lambda, \quad (11)$$

where ξ_0 is the high-energy cutoff, the smaller of the Fermi energy in the wire or the inverse rise time of the perturbation of the QPC. The factor of 4 in the exponent in (11) corresponds to the number of affected channels (two scattering states multiplied by the spin degeneracy in the wire). Equation (11) is identical to the expression describing the "shake up" effect in the x-ray absorption spectra in metals [15], which results in power-law suppression $\epsilon^{4(\delta/\pi)^2}$ of the absorption at low energies.

The factor $A_{\mathcal{LR}}(t)$ in (9) describes the mixing of the scattering states in the wire and we evaluate it in the linked-cluster approximation, keeping terms to order $\lambda_{\mathcal{LR}}^2$:

$$A_{\mathcal{LR}}(t) = e^{-2\lambda_{\mathcal{LR}}^2 \int\int_0^t dt_1 dt_2 \cos[eV(t_1 - t_2)] g(t_1,t_2) g(t_2,t_1)}, \quad (12)$$

where the Green's function $g(t_1, t_2)$ is defined as

$$g(t_1, t_2) = -iA(t)^{-1} \int \frac{dk_1 dk_2}{2\pi} \times \langle T_t \psi_1(k_1, t_1) \psi_1^\dagger(k_2, t_2) \times e^{-i\int_0^t dt [\hat{V}_{\mathcal{LL}}(t) + \hat{V}_{\mathcal{RR}}(t)]} \rangle . \quad (13)$$

The factor of 2 in the exponent in Eq. (12) comes from the summation over spin directions in the wire. The Green's function is given by [15]

$$g(t_1, t_2) = \left(\frac{\sinh \pi T(t - t_1)}{\sinh \pi T(t - t_2)} \frac{\sinh \pi T t_2}{\sinh \pi T t_1} \right)^{\delta/\pi} \times \left\{ P \frac{\pi T \cos^2 \delta}{\sinh \pi T (t_2 - t_1)} - \frac{\pi}{2} \delta(t_1 - t_2) \sin 2\delta \right\}, \quad (14)$$

where P stands for the principal value, and $0 \leq t_{1,2} \leq t$.

Substituting $g(t_1, t_2)$ from Eq. (14) into Eq. (12), we obtain with the help of Eq. (9)

$$A_+(t) = \left(\frac{i\pi T}{\xi_0 \sinh \pi T t} \right)^{\alpha + \gamma} e^{-\Gamma_d t + \gamma h(t,T,eV)}, \quad (15)$$

where the exponents are related to the scattering constants $\lambda, \lambda_{\mathcal{LR}}$ from Eq. (6) by

$$\alpha = 4\left(\frac{\delta}{\pi} \right)^2, \qquad \gamma = 4\lambda_{\mathcal{LR}}^2 \cos^4 \delta, \quad (16)$$

and the dephasing rate is given by

$$\Gamma_d = \pi\gamma |eV|. \quad (17)$$

The crossover function h in Eq. (15) is

$$h(t, T, eV) = \int_0^t d\tau\, \tau (1 - \cos eV\tau) \frac{\pi^2 T^2}{\sinh^2 \pi T \tau}.$$

Let us now reexpress the exponents (16) in terms of the physical characteristics of the QPC: the transmission probability $\mathcal{T}$ and the phase of the transmission amplitude θ. In order to do so, we notice that switching on the perturbation (6) by adding an electron to the dot corresponds to changing the phase shifts $\delta_{e,o}$ for the even (e) and odd (o) channels in the wire:

$$\delta_{e,o}^{(N+1)} = \delta_{e,o}^{(N)} + \Delta\delta_{e,o}, \quad \Delta\delta_{e,o} = \arctan \pi(\lambda \pm \lambda_{\mathcal{LR}}).$$

The transmission probability of the QPC is related to these phase shifts by $\mathcal{T} = \cos^2(\delta_e - \delta_o)$, and the phase of the transmission amplitude is given by $\theta = \delta_e + \delta_o$. We obtain from Eq. (16)

$$\alpha = \left(\frac{\Delta\theta}{\pi} \right)^2 + O(\lambda_{\mathcal{LR}}^2), \qquad \gamma = \frac{(\Delta\mathcal{T})^2}{8\pi^2 \mathcal{T}(1 - \mathcal{T})}. \quad (18)$$

The dephasing rate $\Gamma_d = |eV|(\Delta\mathcal{T})^2/[8\pi\hbar\mathcal{T}(1 - \mathcal{T})]$ given by Eqs. (17) and (18) agrees up to a constant factor with the estimate for $1/t_d$ obtained earlier in Eq. (2).

The physical meaning of the dephasing rate Γ_d deserves some additional discussion. Indeed, Γ_d reflects the efficiency with which the QPC measures the charge state of the quantum dot. One can rigorously define this measurement using the basis of scattering eigenstates of the wire before an electron is added to the dot. If the added electron creates a single excitation in this basis, the passage of the electron through the QD is "detected" and interference with the other, remote path through the Aharonov-Bohm ring is destroyed. The dephasing rate Γ_d is the rate at which such excitations are created. Using the golden rule, for the simplest case $\delta = 0$, we obtain

$$\Gamma_d = 2\pi\lambda_{\mathcal{LR}}^2 \times 2\int_{-\infty}^{0} dk_i \int_{0}^{\infty} dk_f\, \delta(k_i - k_f - |eV|) = 4\pi\lambda_{\mathcal{LR}}^2 |eV|\,,$$

which agrees with (17), and which can easily be generalized to $\delta \neq 0$.

Note the symmetry in the expressions for γ and Γ_d between the transmission probability $\mathcal{T}$ and the reflection probability $1 - \mathcal{T}$ in the wire. An extra electron transmitted through a normally reflecting QPC provides the same measurement of the charge state of the QD as an extra electron reflected by a normally transmitting point contact. For the case of a parabolic potential barrier in the QPC, $\Delta\mathcal{T} \sim \mathcal{T}(1 - \mathcal{T})\Delta V_{\mathrm{QPC}}$, where ΔV_{QPC} is the change in the height of the potential caused by adding an electron to the dot. One then finds $\Gamma_d \propto \mathcal{T}(1 - \mathcal{T})$, with the maximum dephasing rate at $\mathcal{T} = 1/2$.

The calculation of the coherence factor $A_-(t)$ from Eq. (8) is performed analogously, starting from the diagonalization of the Hamiltonian $\hat{H}_{N+1}(t)$ in the basis of scattering states. The result is $A_-(t) = A_+(t)^*$. Because $A_-(t) \neq A_+(t)$, the probability P_N for the occupation of the dot does not cancel from the result. For the general position of the level ϵ_0, the probability P_N can be found from the thermodynamic formula $P_N = -\int (d\epsilon/\pi) f(\epsilon)\, \mathrm{Im}\, G_{\mathrm{QD}}^r(\epsilon)$. However, at the peak of the Coulomb blockade, $\epsilon_0 = 0$, it is obvious that $P_N = P_{N+1} = 1/2$. The total transmission amplitude through the quantum dot can then be obtained (4) as a Fourier transform of $G_{\mathrm{QD}}^r(t)$. We find that the result can be well approximated by the simple formula

$$t_{\mathrm{QD}} \simeq \frac{2\sqrt{\Gamma_L \Gamma_R}}{4T/\pi + \Gamma_{\mathrm{tot}}} \left(\frac{T + \Gamma_{\mathrm{tot}}}{\xi_0}\right)^{\alpha} \left(\frac{T + \Gamma_{\mathrm{tot}} + |eV|}{\xi_0}\right)^{\gamma}, \tag{19}$$

where the total half-width is given by $\Gamma_{\mathrm{tot}} = \Gamma_L + \Gamma_R + \Gamma_d$. Equation (19) is the central result of our study. It describes the anomalous scaling of the amplitude of the Aharonov-Bohm oscillations with the temperature or with the current flowing through the quantum wire. While in general observation of the *coherent* transmission amplitude t_{QD} requires the Aharonov-Bohm geometry, the ordinary conductance through a quantum dot is $\sim |t_{\mathrm{QD}}|^2$ at $T = V = 0$. Hence we expect a suppression of conductance by the orthogonality catastrophe even without the Aharonov-Bohm geometry, though only the coherent part of the transmission will be suppressed by dephasing as V is increased.

In conclusion, we have analyzed theoretically electron transport through the "Which Path?" interferometer [5]: an Aharonov-Bohm ring with a quantum dot in one arm, and an additional wire containing a quantum point contact located close to the dot. The presence of the wire suppresses the Aharonov-Bohm oscillations in the ring in two ways. First, real electron-hole-pair creation in the wire measures which path the electron took around the ring, and so causes the paths to decohere. Second, virtual electron-hole-pair creation in the wire decreases the transmission amplitude through the QD, leading to power-law dependence of the Aharonov-Bohm oscillations on the temperature or the current through the wire. Unfortunately, in the experimental setup in Ref. [6] the exponents in Eq. (19) appear to be rather small.

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Note added.—After this paper was submitted, Levinson [17] independently obtained our Eq. (17) using a different approach. However, he neglected the orthogonality catastrophe which leads to scaling in Eq. (19).

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