Has a Josephson-Plasma Mode Been Observed in Layered Superconductors?

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The plasma mode in layered superconductors is analyzed using the continuous approach for an anisotropic superconductor and the Lawrence-Doniach model. The analysis predicts, for the plasma frequency, a magnetic field dependence different from that of magnetoabsorption resonances recently observed in various materials and conditions. This puts in doubt their Josephson-plasma-mode interpretation commonly accepted by experimentalists. [S0031-9007(97)04517-1]

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Collective modes of vortex arrays have been studied from the 1960s when the works of Hall on rotating superfluid ^{4}He [1] and of de Gennes and Matricon on type-II superconductors [2] were published. In superfluid ^{4}He the vortex modes have been observed as comparatively narrow resonances [3], whereas in superfluid ^{3}He they are overdamped and have been identified as long relaxation processes after modulation of rotation speed [4].

Until recently vortex modes in superconductors remained a topic of the theory: large viscous losses in low-T_c superconductors prevented the observation of these modes. Therefore observation of the magnetoabsorption resonances in high-T_c Bi compounds by Tsui *et al.* [5] has attracted great attention and has given an impetus to a great number of experimental works [6–10] in which they observed these resonances in different conditions and materials. It seemed quite natural to interpret them as slow collective vortex modes (Kopnin *et al.* [11]). But later the experimentalists *unanimously* interpreted the resonances as plasma oscillations in Josephson junctions between CuO superconducting layers.

The original plasma-mode interpretation of the magnetoabsorption resonances was suggested by Bulaevskii *et al.* [12,13] using the idea of the Josephson-plasma edge with frequency $\omega = \omega_J\sqrt{\langle\cos\varphi\rangle}$ where ω_J is the frequency in zero magnetic field and $\langle\cos\varphi\rangle$ is the spatial average for the phase difference φ between two layers. In order to agree with experiment, $\langle\cos\varphi\rangle$ had to be small for any direction of the magnetic field. However, it was known that either in a wide Josephson junction [14] or a layered superconductor [15] there is no plasma edge for *any finite density* of vortices (fluxons); i.e., the plasma frequency ceases to be a lower bound on the oscillation frequency since the soft Goldstone mode related to the vortex-array translation arises. Moreover, in parallel fields considered in Ref. [12], there is no plasma mode which could be observable by microwaves. Thus the plasma-edge idea could not be a basis for interpretation of magnetoabsorption resonances [16].

Later Bulaevskii *et al.* [17–19] have revised their theory relating now the resonances observed in parallel field with the Goldstone mode mentioned above which they call "sliding mode." The latter is, in fact, the same vortex mode branch suggested in Ref. [11] for explanation of resonances in a perpendicular and a tilted field, but extrapolated now to the parallel field where it has new properties (e.g., vortex mass becomes essential). But Bulaevskii *et al.* [19] still insist that the Josephson-plasma modes have been experimentally observed, and this interpretation is accepted by experimentalists [6–10].

In the present Letter, I show that properties of the plasma mode in layered superconductors are different from those observed for magnetoabsorption resonances, and therefore their interpretation in terms of the Josephson plasma mode looks at least very unlikely. The most serious drawbacks for this interpretation are: (i) The experimental resonance in a tilted magnetic field is governed by the field component normal to layers while the plasma mode is governed mostly by the in-plane one (parallel to layers), and (ii) the experimental resonance frequency goes to zero when the vortices become parallel to the layers [9] while the plasma-mode frequency remains finite in this limit. These drawbacks are known to proponents of plasma-mode interpretation, and they have suggested scenarios to overcome them, but I shall show that these scenarios are not persuasive. The presented analysis refers to temperatures and fields below the irreversibility line where layers are not decoupled, though its results might be important for the vortex-liquid phase also.

Let us start from a phenomenological approach. If the magnetic field $\vec{B}$ is along the axis c (the axis z), the plasma mode is described by equations for z components of the electric field and the current,

$$\frac{1}{c}\frac{\partial E_z}{\partial t} = -\frac{4\pi}{c} j_z\,, \tag{1}$$

$$\frac{\partial j_z}{\partial t} = \frac{e^2 n_c}{m} E_z\,, \tag{2}$$

where n_c is the superfluid electron density for the currents normal to the layers. It determines the plasma frequency $\omega_c = \sqrt{4\pi e^2 n_c/m}$ and the penetration depth $\lambda_c = c/\omega_c$.

The vortex mode is related with in-plane motion and is completely decoupled from the plasma mode.

In the opposite limit of $\vec{B}$ in the plane ab ($B = B_y$) Eq. (2) is replaced by

$$\frac{\partial j_z}{\partial t} = \frac{\omega_c^2}{4\pi}\left(E_z + B_y \frac{v_L}{c}\right), \tag{3}$$

where $v_L = du/dt$ is the velocity and u is the displacement of the vortex lines along the axis x which is determined from the equation

$$-M \frac{\partial^2 u}{\partial t^2} + K \frac{\partial^2 u}{\partial z^2} = \frac{\Phi_0}{c} j_z . \tag{4}$$

One sees the Lorentz force on the right-hand side, but there is no Magnus force: the latter is in the c direction in which vortices cannot move because of intrinsic pinning. Then one must take into account the vortex mass M. Also the shear elasticity of the vortex lattice given by the elastic modulus K is important.

For a plane wave $\sim\exp(ikz - i\omega t)$ the dispersion equation is

$$(\omega^2 - \omega_c^2)(\omega^2 - c_v^2 k^2) - \Gamma\omega^2\omega_c^2 = 0, \tag{5}$$

where $c_v^2 = K/M$, and $\Gamma = B_y\Phi_0/4\pi Mc^2$. In the limit of $k \to 0$ this dispersion equation yields the plasma mode with renormalized gap $\omega_c\sqrt{1+\Gamma}$ and the soft soundlike vortex mode $\omega = c_v k/\sqrt{1+\Gamma}$.

More general equations for an arbitrary direction of $\vec{B}$ must include the Magnus force which becomes more important than the vortex inertia force. But the final conclusion of the phenomenological theory is that one can easily discern the plasma mode with a gap originated from the Coulomb interaction, and the soft vortex mode which is gapless in the uniform case, but has a gap in the presence of pinning. The plasma gap depends only on the in-plane component B_y.

However, even though B_z has no direct effect on the plasma frequency, it can affect it via the superfluid density n_c. The latter is constant outside the vortex cores (the London region), but is suppressed inside the cores. Thus B_z diminishes the average $\langle n_c\rangle$ and the plasma frequency $\omega_c \propto \sqrt{\langle n_c\rangle}$. In continuous superconductors the effect is roughly proportional to B/H_{c2} which is the ratio of the core area to the area of the vortex unit cell. In layered superconductors the core area must be found from the Lawrence-Doniach model. In this model the current is $j_z = j_c \sin\varphi$, where j_c is the critical current for the Josephson coupling between two layers. Suppose that a smooth phase modulation $\varphi' \approx s\partial\varphi/\partial z$ is superimposed on the ground-state phase pattern. Here s is the period of the layered structure. Then the average current is $j_z = j_c\langle\cos\varphi\rangle\varphi'$ where now φ is the phase in the ground state. Comparison of this expression with that for a continuous superconductor, $j_z = en_c(B)(\hbar/m)\partial\varphi/\partial z$, yields that $n_c(B_z) = n_c(0)\langle\cos\varphi\rangle$ where $n_c(0) = j_c sm/e\hbar$ is the superfluid density without magnetic field. Since the zero-field plasma frequency $\omega_c(0)$ coincides with the Josephson-plasma frequency ω_J, this gives an expression for the plasma frequency $\omega_c(B) = \omega_J\sqrt{\langle\cos\varphi\rangle}$ suggested by Bulaevskii *et al.* [12] and used by all experimentalists for comparison with their results. Thus the whole B_z dependence is presented by the factor $\langle\cos\varphi\rangle$, and in order to explain the experimental dependence this factor must be essentially smaller than unity. Let us estimate how small it could be.

By definition, in the London region the superfluid density is close to its value far from the vortex line, i.e., $\cos\varphi \approx 1$. Then the core area must be defined as an area where the phase φ is not small compared to unity. In the Lawrence-Doniach model the vortex line, which is directed at the tilt angle $\vartheta = \arcsin B_z/B$ to the ab plane in average, consists of pancakes in superconducting CuO layers connected by Josephson strings in interlayer spacings. The phase φ is large (i.e., $\cos\varphi$ may be small) only inside a Josephson string where φ varies in the 2π interval. The length of the Josephson string is $L_J = s/\tan\vartheta$. The width of the string also cannot exceed this value, as pointed out by Clem [20]. Thus the core area in the ab plane does not exceed $\sim L_J^2$. A more accurate estimation shows that the effective core area differs from this value by a logarithm factor, but it is not important for our rough estimation. A strong inequality $\langle\cos\varphi\rangle \ll 1$ takes place if cores occupy the whole ab plane, i.e., the core area $\sim L_J^2 = s^2/\tan^2\vartheta$ is on the order or more than the area Φ_0/B_z per one vortex. It is possible if $\tan\vartheta \approx \vartheta \ll B/H_s$ where $H_s = \Phi_0/s^2$ is about 1000 T while fields relevant for experiment are less than 10 T. Therefore an essential suppression of the plasma frequency by the magnetic field is possible for tilt angles of about 1°.

In order to conciliate the plasma-mode theory with experiment at fields perpendicular to the ab plane, Bulaevskii *et al.* [13] suggested that even for a field normal to layers in average, the local direction of vortex line strongly fluctuates because of pinning of pancakes. As a result, there appears a great number of Josephson strings which, as they believe, can effectively decrease the factor $\langle\cos\varphi\rangle$. But they strongly overestimated this effect using the relation $\langle\cos\varphi\rangle = \exp(-\langle\varphi^2\rangle/2)$ which is incorrect if $\langle\varphi^2\rangle$ is large, i.e., if $\langle\cos\varphi\rangle$ is small. Our estimation has shown that in order to essentially decrease $\langle\cos\varphi\rangle$, the deviations from the direction normal to layer must be *everywhere* about 90°. It is difficult to imagine a structure in which random directions of the vortex line are kept so close to the plane ab: In the presence of numerous pinning sites pancakes can always choose those which do not require a vortex line to deviate from its average direction so strongly which would cost a higher energy. But in the case that such an exotic structure was realized, nevertheless, one would expect a strong effect of the factor $\langle\cos\varphi\rangle$ also on the field-dependent penetration depth

$\lambda_c(B) = c/\omega_c(B)$. Such huge enhancement of λ_c could not be unnoticed in direct magnetic measurements on λ_c.

Our phenomenology assumed existence of the London region. But for a parallel field this region may be eliminated as shown above. Therefore one must check this case using the Lawrence-Doniach model which deals with equations for the gauge-invariant interlayer phase difference $\varphi_{n,n+1} = \varphi_{n+1} - \varphi_n$ between the layers n and $n + 1$ [see Eq. (11) of [21], dissipation is neglected],

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}[(2+\alpha)\varphi_{n,n+1} - \varphi_{n+1,n+2} - \varphi_{n-1,n}] = \frac{\partial^2\varphi_{n,n+1}}{\partial x^2} - \frac{1}{\lambda_J^2}[(2+\alpha)\sin\varphi_{n,n+1} - \sin\varphi_{n+1,n+2} - \sin\varphi_{n-1,n}]. \quad (6)$$

Here $\alpha = s^2/\lambda_{ab}^2$, $\lambda_J = s\gamma$, $\gamma = \omega_{ab}/\omega_c = \lambda_c/\lambda_{ab}$, ω_{ab}, and λ_{ab} are the plasma frequency and the penetration depth for currents in the ab plane, and $c_0 = c\sqrt{\alpha}$ is the analog of the Swihart velocity. Without magnetic field all phase differences vanish in the ground state: $\varphi_{n,n+1}^{(0)}(x) = 0$. But the phase differences can oscillate with the plasma frequency $\omega_J = c_0/\lambda_J = c/\lambda_c$.

In the limit of high magnetic fields $B_y = B \gg H_0 = \Phi_0/s\lambda_J$ vortices fill all interlayer spacings forming periodic chains with period $a = \Phi_0/sB \ll \lambda_J$. In neighboring interlayer spacings the vortex chains are shifted by $a/2$ forming a triangular lattice [22]. The equilibrium stationary configuration $\varphi_{n,n+1}^{(0)}(x)$ satisfies Eq. (6) without time derivatives and may be found treating the sine terms as weak perturbations,

$$\varphi_{n,n+1}^{(0)}(x) = kx + \pi n - (-1)^n\frac{4+\alpha}{k^2\lambda_J^2}\sin kx, \quad (7)$$

where $k = 2\pi/a$. Then one might expect the plasma mode with frequency

$$\omega_c^2 = \omega_J^2\langle\cos\varphi_{n,n+1}^{(0)}\rangle = \frac{(4+\alpha)c_0^2}{2k^2\lambda_J^4} = \omega_J^2\frac{(4+\alpha)H_0^2}{8\pi^2B^2}. \quad (8)$$

Let us try to find it solving Eq. (6) linearized with respect to small phase modulation $\varphi_n' = \varphi_{n,n+1} - \varphi_{n,n+1}^{(0)}$. Equations for $\varphi_n'(x)$ are Mathieu equations which one may solve using the perturbation theory with respect to the periodical potential. For modes propagating normally to the layers, $\varphi_n'(x,t) = \phi(x)\exp(iqn - i\omega t)$ and the phase modulation is periodical: $\phi(x+a) = \phi(x)$. We obtain that the frequency of these modes is given by

$$\omega^2 = \frac{c_0^2}{k^2\lambda_J^4}(1-\cos q) = \omega_J^2\frac{H_0^2}{4\pi^2B^2}(1-\cos q). \quad (9)$$

This is a gapless mode which agrees with the spectrum derived earlier by Volkov [15] [see his Eq. (17) for $k = 0$].

The mode at the boundary of the first Brillouin zone $q = \pm\pi$ has a maximum frequency $\omega^2 = 2c_0^2/k^2\lambda_J^4$ which is roughly equal to the plasma frequency given by Eq. (8) (α is small), but in this mode the average current j_z and the average electric field E_z vanish (they have opposite signs in neighboring layers). Therefore this mode is not coupled with a practically uniform electric field generated in microwave experiments.

Thus according to the Lawrence-Doniach model one cannot observe the plasma mode in a strong parallel field. But the soft vortex mode predicted by phenomenology is observable and corresponds to Eq. (9) in the limit of small $q = ks$. Indeed, in this limit one may present an oscillation in terms of vortex displacements u assuming that $\varphi_n'(x) = -u\partial\varphi_{n,n+1}^{(0)}(x)/\partial x$. Then it is evident that the mode under consideration is the transverse sound in the lattice of vortices which have a mass.

Note that our triangular lattice has a *single vortex* in the elementary cell. Therefore the structure cannot sustain any optical mode. The optical mode obtained in Ref. [17] results from an improper choice of the elementary cell with two vortices in neighboring layers. Formally one may choose the unit cell containing two vortices. But this yields a wrong physical picture: the optical branch in such a picture must be continuously connected with acoustic branch at the boundary of the first Brillouin zone for a two-site cell, i.e., belongs to the same branch, in fact. Discontinuity between "acoustic" and "optical" modes at the Brillouin-zone boundary obtained in Ref. [17] contradicts to symmetry of the vortex lattice.

The optical mode appears, however, if vortices do not fill all interlayer spacings. According to Bulaevskii and Clem [22] the vortex lattice with the vortex chains in any layer transforms at some magnetic field of order H_0 to the lattice with the double period along the axis c, in which any second interlayer spacing is free from vortices. We assume that the even spacings with the phases $\varphi_{2n,2n+1}$ are filled with vortices (vortex layers), whereas the odd ones with the phases $\varphi_{2n+1,2n+2}$ are vortex free (Meissner layers). This structure also has only one vortex in the unit cell. However, the oscillating phase in the Meissner layer is an independent variable. So the unit cell contains two sites: one is occupied by a vortex, another is vortex free.

Using the perturbation theory for the high magnetic field again, we obtain for the static triangular structure that $\varphi_{2n+1,2n+2}^{(0)}(x) = 0$, i.e., $\cos\varphi_{2n+1,2n+2}^{(0)} = 1$ and

$$\varphi_{2n,2n+1}^{(0)}(x) = kx + \pi n - (-1)^n\frac{2+\alpha}{k^2\lambda_J^2}\sin kx. \quad (10)$$

Now we introduce small deviations from the equilibrium in vortex layers, $u_n = \varphi_{2n,2n+1} - \varphi_{2n,2n+1}^{(0)}$, and in Meissner layers, $v_n = \varphi_{2n+1,2n+2}$. Taking the periodic potential into account by the perturbation theory as before, the equations for x-independent u_n and v_n are

$$(2+\alpha)\frac{\omega^2}{c_0^2}u_n - \left(\frac{\omega^2}{c_0^2} - \frac{1}{\lambda_J^2}\right)(v_{n-1} + v_n) = 0,$$
$$(2+\alpha)\left(\frac{\omega^2}{c_0^2} - \frac{1}{\lambda_J^2}\right)v_n - \frac{\omega^2}{c_0^2}(u_n + u_{n+1}) = 0. \quad (11)$$

We restrict ourselves by a case $q = 0$. Then u_n and v_n do not depend on the layer number n and the equations yield two modes: the soft vortex mode with $\omega = 0$ (acoustic mode), and the plasma mode with frequency close to ω_J (optical mode). Note that the plasma frequency is not suppressed by the magnetic field since the current and the electric field are confined in the Meissner layers where $\cos\varphi \sim 1$. Therefore the expression $\omega_c(B)^2 = \omega_J^2\langle\cos\phi\rangle$ does not predict a correct value of the plasma frequency, since $\langle\cos\phi\rangle \approx 1/2$ in the present case.

On the basis of these calculations one may conclude when the plasma mode is observable by microwaves and when it is not. It is definitely observable if there is an essential London region, where there is no fast phase variation and $\cos\varphi \approx 1$. In the second example the vortex-free interlayer spacings play a role of such a region. I argue that magnetic-field tilting is also able to make the plasma mode observable. At any finite tilt angle $\vartheta \sim B_z/B_y$, however small, there is no infinite-length Josephson vortices anymore. Instead there are finite-length Josephson strings stretched between pancakes. Only infinite-length strings are able to eliminate an observable plasma mode. This may be illustrated by an analysis in the low-field limit, when in the plane *ab* the transverse size λ_J of vortices (fluxons) is much smaller than the intervortex distance. Infinite Josephson strings cut the whole plane onto disconnected strips with phases $2\pi n$. The mode with plasma frequency still exists, but it has nodes along fluxon lines, so that the current and the electric field have opposite directions on the two sides of the fluxon. As a result, they vanish in average and the mode cannot be observed. Roughly speaking, even one infinite-length string is enough for it. But a finite-length Josephson string *cannot* disconnect the area with the same phase (the London region); i.e., this area remains *single connected*: the current (field) direction must be the same far from the string, so the average current (field) does not vanish.

All this leads to the conclusion that the plasma mode is not observable only in a rather strong magnetic field *strictly* parallel to the *ab* plane. But its frequency never vanishes. Therefore extrapolation of the plasma frequency to zero B_z must always give a *finite* frequency. In contrast to it, recent measurements of the resonance frequency for small tilt angles [9] yielded the dependence $\omega \propto \sqrt{H_z}/H$. Here $\vec{H}$ is the external field, but its difference from the magnetic induction $\vec{B}$ is not important for frequency extrapolation. Thus this observation does disprove, but not prove, the plasma-mode interpretation.

In summary, our analysis of the plasma mode in the layered superconductor shows that its magnetic-field dependence is essentially different from that of observed magnetoabsorption resonances. Except for quite small tilt angles between the field and layers, the factor $\langle\cos\varphi\rangle$ is about unity and the magnetic-field dependence of the plasma frequency $\omega_c = \omega_J\sqrt{\langle\cos\varphi\rangle}$ is negligible compared to that observed. At small tilt angles extrapolation of the plasma frequency to zero angle must yield a finite observable value, in contrast to zero value obtained in the recent experiment [9].

This conclusion, which is negative for plasma-mode interpretation, does not mean that the vortex-mode interpretation must automatically replace it. It is evident now that even if experimentalists really observed the vortex mode, its properties are different from those suggested in Ref. [11]. In particular, the mode must be governed by pinning in the bulk, but not only on the surface. Other interpretations different from "either plasma, or vortex mode" also must not be ruled out. This problem will be addressed elsewhere. Additional experiments, which could help to resolve the problem, were extensions of measurements to lower magnetic field, where the difference between vortex and plasma modes became more pronounced. Since the penetration depth λ_c is also affected by the factor $\langle\cos\varphi\rangle$, comparison with measurements of λ_c in magnetic fields would also be useful.

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[1] H. E. Hall, Adv. Phys. **9**, 89 (1960).
[2] P. G. de Gennes and J. Matricon, Rev. Mod. Phys. **36**, 45 (1964).
[3] E. B. Sonin, Rev. Mod. Phys. **59**, 87 (1987).
[4] M. Krusius *et al.*, Phys. Rev. B **47**, 15 113 (1993).
[5] O. K. C. Tsui *et al.*, Phys. Rev. Lett. **73**, 724 (1994).
[6] Y. Matsuda *et al.*, Phys. Rev. Lett. **75**, 4512 (1995).
[7] O. K. C. Tsui, N. P. Ong, and J. B. Peterson, Phys. Rev. Lett. **76**, 819 (1996).
[8] Y. Matsuda *et al.*, Phys. Rev. Lett. **78**, 1972 (1997).
[9] Y. Matsuda *et al.*, Phys. Rev. B **55**, R8685 (1997).
[10] T. Hanaguri *et al.*, Phys. Rev. Lett. **78**, 3177 (1997).
[11] N. B. Kopnin *et al.*, Phys. Rev. Lett. **74**, 4527 (1995).
[12] L. N. Bulaevskii *et al.*, Phys. Rev. B **53**, 6634 (1996).
[13] L. N. Bulaevskii, V. L. Pokrovskii, and M. P. Maley, Phys. Rev. Lett. **76**, 1719 (1996).
[14] A. L. Fetter and M. J. Stephen, Phys. Rev. **168**, 475 (1968).
[15] A. F. Volkov, Phys. Lett. A **138**, 213 (1989).
[16] E. B. Sonin, in Proceedings of the Workshop on Vortex Matter, Shoresh, Israel, 1996 (unpublished).
[17] L. N. Bulaevskii *et al.*, Phys. Rev. B **53**, 14 601 (1996).
[18] L. N. Bulaevskii *et al.*, Phys. Rev. B **54**, 7521 (1996).
[19] L. N. Bulaevskii *et al.*, Phys. Rev. B **55**, 8482 (1997).
[20] J. R. Clem, Physica (Amsterdam) **200A**, 118 (1993).
[21] L. N. Bulaevskii *et al.*, Phys. Rev. B **50**, 12 831 (1994).
[22] L. N. Bulaevskii and J. R. Clem, Phys. Rev. B **44**, 10 234 (1991).