Next-to-Leading Order Calculation of Four-Jet Shape Variables

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We present the next-to-leading order calculation of two four-jet event shape variables, the D parameter, and acoplanarity differential distributions. We find large, more than 100% radiative corrections. The theoretical prediction for the D parameter is compared to L3 data obtained at the Z^0 peak and corrected to hadron level. [S0031-9007(97)04436-0]

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In the second phase of the Large Electron-Positron Collider it is an important question how well the characteristics of QCD four-jet events, i.e., events in which an s channel Z^0 or γ^* decays into four quark and gluon jets, are understood at large energies. This question is of interest because W^+W^- events lead to four-jet final states for which the main backgrounds are QCD events and because QCD four-jet events are also the principal source of background for Higgs and other new particle searches. The perturbative description of the QCD four-jet events is also interesting in its own right as a tool for testing perturbation theory in a regime with small hadronization uncertainty and for measuring the QCD color charges [1], or as a means of testing whether experimental data favor or exclude the existence of light gluinos [2].

Recent theoretical developments make possible the next-to-leading order calculation of four-jet quantities. There are now several general methods available for the cancellation of infrared divergences that can be used for setting up a Monte Carlo evaluation of next-to-leading order partonic cross sections [3–5]. The main ingredients of the calculation are the four-parton next-to-leading order and five-parton Born level squared matrix elements. The tree level amplitudes for the processes $e^+e^- \to \bar{q}qggg$ and $e^+e^- \to \bar{q}q\bar{Q}Qg$ from which the latter can be constructed have been known for a long time [6]. Recently Campbell, Glover, and Miller calculated the other vital piece of information, the virtual corrections for the processes $e^+e^- \to \gamma^* \to \bar{q}q\bar{Q}Q$ and $\bar{q}qgg$ [7]. Also, the new techniques developed by Bern, Dixon, and Kosower in the calculation of one-loop multiparton helicity amplitudes [8] made possible the derivation of analytic expressions for the helicity amplitudes of the $e^+e^- \to Z^0, \gamma^* \to \bar{q}q\bar{Q}Q$ process [9] and results for the other subprocess are expected to appear soon. Using these results Dixon and Signer calculated the next-to-leading order corrections for four-jet fractions with various clustering algorithms [10,11].

In this Letter we enlarge the list of four-jet observables that are calculated to next-to-leading order accuracy. We present results of the calculation of QCD radiative corrections to two four-jet shape variable differential distributions—the D parameter and acoplanarity.

We use the matrix elements of Ref. [7] for the loop corrections. In the calculation of these matrix elements all quark and lepton masses are set to zero; thus our results are valid in the massless limit. We note that the results in Ref. [7] do not include the "light-by-glue" virtual contributions which were shown to be negligible [11].

The higher order correction to the leading order partonic cross section $\sigma^{\rm LO}$ is a sum of two integrals, one of the real correction $d\sigma^{\rm R}$ that is an exclusive cross section of five partons in the final state and the other of the virtual correction $d\sigma^{\rm V}$ that is the one-loop correction to the process with four partons in the final state:

$$\sigma^{\rm NLO} \equiv \int d\sigma^{\rm NLO} = \int_5 d\sigma^{\rm R} + \int_4 d\sigma^{\rm V}, \quad (1)$$

where

$$\int_5 d\sigma^{\rm R} = \int d\Gamma^{(5)} \langle |\mathcal{M}_5^{\rm tree}|^2 \rangle J_5 \quad (2)$$

and

$$\int_4 d\sigma^{\rm V} = \int d\Gamma^{(4)} \langle |\mathcal{M}_4^{\rm 1-loop}|^2 \rangle J_4 \,. \quad (3)$$

The two integrals on the right-hand side of Eq. (1) are separately divergent in $d = 4$ dimensions, but their sum is finite provided the jet function J_n defines an infrared safe quantity. Therefore, the separate pieces have to be regularized. We use dimensional regularization in $d = 4 - 2\varepsilon$ dimensions, in which case the divergences are replaced by double and single poles in ε. We assume that ultraviolet renormalization of all Green functions to one-loop order has been carried out, so the poles are of infrared origin.

There are several ways of exposing the cancellation of infrared singularities directly at the integrand level [3–5]. The method used in the calculation presented in this Letter is a slightly modified version of the dipole formalism of Catani and Seymour [5] that is based on the subtraction method. The general idea of the subtraction method for writing a general-purpose Monte Carlo program is to use the identity

$$\sigma^{\rm NLO} = \int_5 [d\sigma^{\rm R} - d\sigma^{\rm A}] + \int_4 \left[d\sigma^{\rm V} + \int_1 d\sigma^{\rm A} \right], \quad (4)$$

where $d\sigma^{\rm A}$ in the dipole formalism is a proper approximation of $d\sigma^{\rm R}$ in the kinematically degenerate (soft and collinear) region so that it has the same pointwise singular behavior (in d dimensions) as $d\sigma^{\rm R}$ itself. As a result, $d\sigma^{\rm A}$ acts as a local counterterm for $d\sigma^{\rm R}$; that is, $(d\sigma^{\rm R} - d\sigma^{\rm A})$ is integrable in four dimensions by definition. The approximate cross section is constructed in such a way that it can be integrated analytically over the exactly factorized one-parton subspace leading to ε poles, that can be combined with those in $d\sigma^{\rm V}$. The ε poles are guaranteed to cancel for infrared safe observables (Kinoshita-Lee-Nauenberg theorem). These quantities have to be experimentally (theoretically) defined in such a way that their actual value is independent of the number of soft and collinear hadrons (partons) produced in the final state. In particular, this value has to be the same in a given four-parton configuration and in all five-parton configurations that are kinematically degenerate with it (i.e., that are obtained from the four-parton configuration by adding a soft parton or replacing a parton with a pair of collinear partons carrying the same total momentum). This property can be simply restated in a formal way. If the function J_n gives the value of a certain jet observable in terms of the momenta of the n final-state partons, we should have

$$J_5 \longrightarrow J_4\,, \tag{5}$$

in any case where the five-parton and the four-parton configurations are kinematically degenerate. It is easy to prove that the observables considered in this Letter fulfill this property. When the requirement of infrared safety, relation (5) is fulfilled the second integral in Eq. (4) is also finite in $d = 4$ dimensions and $\sigma^{\rm NLO}$ can be easily implemented in a "partonic Monte Carlo" program that generates appropriately weighted partonic events with five final-state partons and events with four partons.

For the precise definition of the approximate cross section in the dipole formalism, we refer to the original work of Catani and Seymour [5]. The distinct feature of this formalism as compared to other subtraction methods [4] is the exact factorization of the five-particle phase space into a four-particle and a one-particle phase space, and that the approximate cross section provides a single and smooth approximation of the real cross section in all of its singular limits. These features lead to a well-converging partonic Monte Carlo program.

In this Letter we consider two classic four-jet event shape variables. The D parameter [12] is derived from the eigenvalues of the infrared safe momentum tensor

$$\theta^{ij} = \sum_a \frac{p_a^i p_a^j}{|\vec{p}_a|} \Big/ \sum_a |\vec{p}_a|\,, \tag{6}$$

where the sum on a runs over all final state hadrons and p_a^i is the ith component of the three-momentum $\vec{p}_a$ of hadron a in the c.m. system. The tensor θ is normalized to have unit trace. In terms of the eigenvalues λ_i of the 3×3 matrix θ, the global shape parameter D is defined as

$$D = 27\lambda_1\lambda_2\lambda_3\,. \tag{7}$$

The second observable is acoplanarity [13] defined as

$$A = 4\min\left(\frac{\sum_a |\vec{p}_a^{\,\rm out}|}{\sum_a |\vec{p}_a|}\right)^2, \tag{8}$$

where the sum runs over all particles in an event, and $\vec{p}_a^{\,\rm out}$ is measured perpendicular to a plane chosen to minimize A.

In the case of the shape variable differential distributions for observable O the jet function J_n is actually a functional,

$$J_n = \delta(O - O^{(n)})\,, \tag{9}$$

where $D^{(n)}$ is given by Eq. (7) and $A^{(n)}$ is given by Eq. (8).

Once the integrations in Eq. (4) are carried out, the next-to-leading order differential cross section for the four-jet observable O_4 takes the general form

$$\frac{1}{\sigma_0}\,O_4\,\frac{d\sigma}{dO_4}(O_4) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 B_{O_4}(O_4) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^3 \times \left[B_{O_4}(O_4)\beta_0 \ln\frac{\mu^2}{s} + C_{O_4}(O_4)\right]. \tag{10}$$

In this equation σ_0 denotes the Born cross section for the process $e^+e^- \rightarrow \bar{q}q$, $\beta_0 = (\frac{11}{3}C_A - \frac{4}{3}T_R N_f)$ with the normalization $T_R = \frac{1}{2}$ in ${\rm Tr}(T^a T^{\dagger b}) = T_R\delta^{ab}$, s is the total c.m. energy squared, μ is the normalization

TABLE I. Comparison of the four-jet fractions calculated by the two partonic Monte Carlo programs MENLO PARC and DEBRECEN (this work).

Algorithm	$y_{\rm cut}$	MENLO PARC	DEBRECEN
Durham	0.005	$(1.04 \pm 0.02) \times 10^{-1}$	$(1.05 \pm 0.004) \times 10^{-1}$
	0.01	$(4.70 \pm 0.06) \times 10^{-2}$	$(4.66 \pm 0.02) \times 10^{-2}$
	0.03	$(6.82 \pm 0.08) \times 10^{-3}$	$(6.87 \pm 0.04) \times 10^{-3}$
Geneva	0.02	$(2.56 \pm 0.06) \times 10^{-1}$	$(2.63 \pm 0.06) \times 10^{-1}$
	0.03	$(1.71 \pm 0.03) \times 10^{-1}$	$(1.75 \pm 0.03) \times 10^{-1}$
	0.05	$(8.58 \pm 0.15) \times 10^{-2}$	$(8.27 \pm 0.08) \times 10^{-2}$
E0	0.005	$(3.79 \pm 0.08) \times 10^{-1}$	$(3.88 \pm 0.07) \times 10^{-1}$
	0.01	$(1.88 \pm 0.03) \times 10^{-1}$	$(1.92 \pm 0.01) \times 10^{-1}$
	0.03	$(3.46 \pm 0.05) \times 10^{-2}$	$(3.37 \pm 0.01) \times 10^{-2}$

TABLE II. The Born level and next-to-leading order scale independent functions B_D and C_D.

D	B_D	C_D
0.00	$(6.60 \pm 0.02) \times 10^2$	$(1.08 \pm 0.06) \times 10^4$
0.04	$(2.32 \pm 0.01) \times 10^2$	$(1.24 \pm 0.02) \times 10^4$
0.08	$(1.45 \pm 0.01) \times 10^2$	$(8.59 \pm 0.12) \times 10^3$
0.12	$(1.03 \pm 0.01) \times 10^2$	$(6.24 \pm 0.12) \times 10^3$
0.16	$(7.74 \pm 0.05) \times 10^1$	$(4.99 \pm 0.11) \times 10^3$
0.20	$(5.97 \pm 0.04) \times 10^1$	$(3.85 \pm 0.06) \times 10^3$
0.24	$(4.69 \pm 0.03) \times 10^1$	$(2.98 \pm 0.05) \times 10^3$
0.28	$(3.77 \pm 0.03) \times 10^1$	$(2.52 \pm 0.05) \times 10^3$
0.32	$(3.01 \pm 0.02) \times 10^1$	$(1.94 \pm 0.05) \times 10^3$
0.36	$(2.41 \pm 0.02) \times 10^1$	$(1.59 \pm 0.04) \times 10^3$
0.40	$(1.98 \pm 0.02) \times 10^1$	$(1.37 \pm 0.03) \times 10^3$
0.44	$(1.61 \pm 0.02) \times 10^1$	$(1.06 \pm 0.03) \times 10^3$
0.48	$(1.30 \pm 0.01) \times 10^1$	$(8.72 \pm 0.19) \times 10^2$
0.52	$(1.07 \pm 0.01) \times 10^1$	$(7.11 \pm 0.16) \times 10^2$
0.56	$(8.48 \pm 0.10) \times 10^0$	$(5.68 \pm 0.14) \times 10^2$
0.60	$(6.70 \pm 0.09) \times 10^0$	$(4.46 \pm 0.21) \times 10^2$
0.64	$(5.33 \pm 0.08) \times 10^0$	$(3.52 \pm 0.11) \times 10^2$
0.68	$(4.10 \pm 0.07) \times 10^0$	$(2.74 \pm 0.09) \times 10^2$
0.72	$(3.11 \pm 0.06) \times 10^0$	$(2.08 \pm 0.08) \times 10^2$
0.76	$(2.24 \pm 0.05) \times 10^0$	$(1.54 \pm 0.06) \times 10^2$
0.80	$(1.52 \pm 0.04) \times 10^0$	$(1.03 \pm 0.04) \times 10^2$
0.84	$(9.95 \pm 0.30) \times 10^{-1}$	$(6.66 \pm 0.31) \times 10^1$
0.88	$(5.74 \pm 0.22) \times 10^{-1}$	$(3.89 \pm 0.20) \times 10^1$
0.92	$(2.68 \pm 0.15) \times 10^{-1}$	$(1.71 \pm 0.19) \times 10^1$
0.96	$(5.16 \pm 0.61) \times 10^{-2}$	$(2.60 \pm 1.30) \times 10^0$

scale, while B_{O_4} and C_{O_4} are scale independent functions, B_{O_4} is the Born approximation, and C_{O_4} is the radiative correction.

The first complete results obtained for four-jet observables at next-to-leading order accuracy [11] are four-jet rates for three clustering algorithms: the Durham [14], the Geneva [15], and the E0 [16] schemes calculated for three colors, five massless flavors, and with $\alpha_s(M_Z) = 0.118$.

TABLE III. The Born level and next-to-leading order scale independent functions B_A and C_A.

A	B_A	C_A
0.00	$(3.34 \pm 0.01) \times 10^2$	$(1.56 \pm 0.01) \times 10^4$
0.04	$(7.39 \pm 0.03) \times 10^1$	$(5.17 \pm 0.08) \times 10^3$
0.08	$(3.63 \pm 0.02) \times 10^1$	$(2.69 \pm 0.06) \times 10^3$
0.12	$(2.05 \pm 0.01) \times 10^1$	$(1.56 \pm 0.03) \times 10^3$
0.16	$(1.23 \pm 0.01) \times 10^1$	$(9.59 \pm 0.22) \times 10^2$
0.20	$(7.63 \pm 0.07) \times 10^0$	$(6.12 \pm 0.15) \times 10^2$
0.24	$(4.81 \pm 0.05) \times 10^0$	$(3.97 \pm 0.12) \times 10^2$
0.28	$(3.02 \pm 0.04) \times 10^0$	$(2.57 \pm 0.09) \times 10^2$
0.32	$(1.78 \pm 0.03) \times 10^0$	$(1.59 \pm 0.08) \times 10^2$
0.36	$(1.08 \pm 0.02) \times 10^0$	$(1.10 \pm 0.06) \times 10^2$
0.40	$(5.99 \pm 0.17) \times 10^{-1}$	$(5.99 \pm 0.33) \times 10^1$
0.44	$(3.19 \pm 0.12) \times 10^{-1}$	$(3.67 \pm 0.25) \times 10^1$
0.48	$(1.51 \pm 0.09) \times 10^{-1}$	$(1.90 \pm 0.99) \times 10^1$
0.52	$(5.91 \pm 0.52) \times 10^{-2}$	$(8.45 \pm 0.99) \times 10^0$
0.56	$(1.55 \pm 0.26) \times 10^{-2}$	$(2.84 \pm 0.42) \times 10^0$
0.60	$(1.33 \pm 0.84) \times 10^{-3}$	$(7.64 \pm 1.77) \times 10^{-1}$
0.64	$(0.00 \pm 0.00) \times 10^0$	$(5.55 \pm 2.66) \times 10^{-2}$

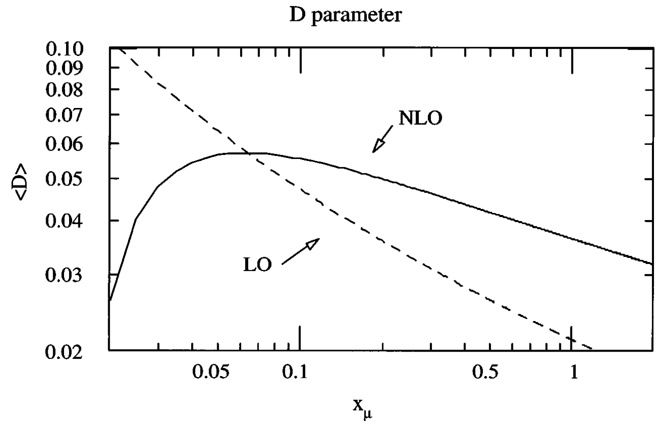

FIG. 1. Renormalization scale dependence of the average value of the D parameter. $x_\mu = \mu/\sqrt{s}$.

We also calculated these observables and compared the results of the two calculations in Table I. There is a very good agreement between the two calculations apart from the 3% discrepancy in the Geneva scheme result at $y_{\rm cut} = 0.05$.

We list the numerical values for B_D, C_D in Table II and those for B_A and C_A in Table III. Our program generates four and five parton events with an appropriate weight. In order to obtain the B_{O_4} and C_{O_4} functions we calculated the O_4 observable of each event, multiplied each weight by O_4, and added to the appropriate bin O_4.

We define the average value of these shape variables as

$$\langle O_4 \rangle = \frac{1}{\sigma} \int_0^1 dO_4 \, O_4 \, \frac{d\sigma}{dO_4} \, . \qquad (11)$$

We studied the dependence of the average value of the D parameter on the renormalization scale in Fig. 1. The strong dependence found at leading order is decreased at next-to-leading order. However, there still remains substantial scale dependence showing that the uncalculated

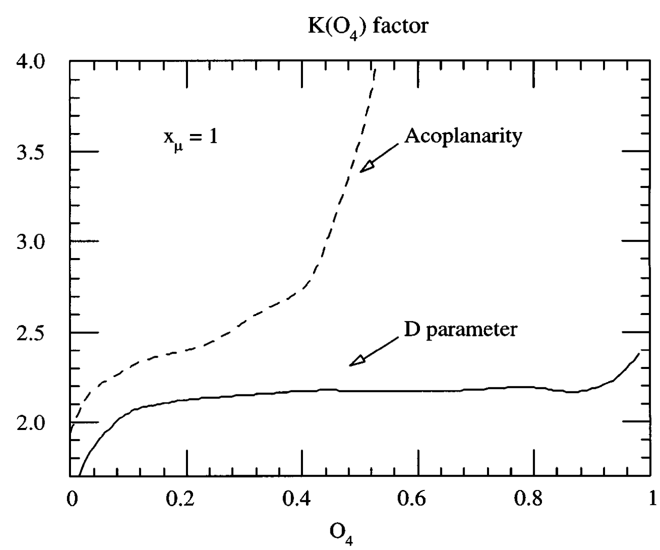

FIG. 2. K factor of the D parameter and acoplanarity.

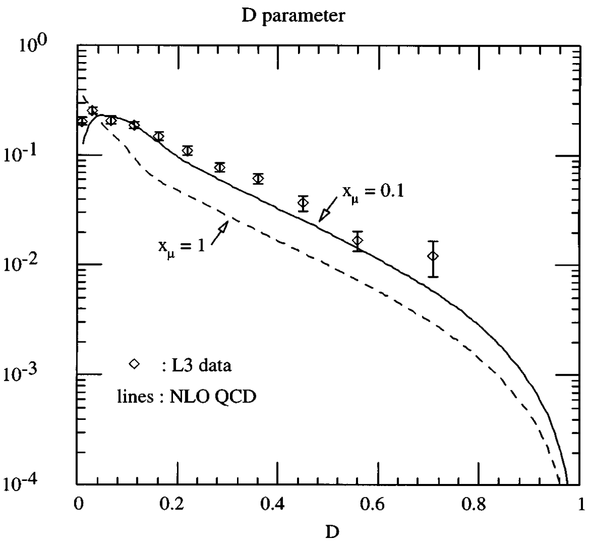

FIG. 3. Comparison of the next-to-leading order QCD prediction for the D parameter differential distribution, $\frac{D}{\sigma}\frac{d\sigma}{dD}$ to L3 data obtained at the Z^0 peak and corrected to hadron level. The upper edge of the theoretical band is obtained with renormalization scale $x_\mu = 0.1$, while the lower edge at $x_\mu = 1$.

higher order corrections are presumably large. The feature is similar in the case of acoplanarity, but the residual scale dependence is even larger.

The same conclusion is drawn if we look at the dependence of the K factors on the observables as depicted in Fig. 2. In case of the D parameter the K factor is slightly above 2 for the whole range, while for acoplanarity it is even larger and increases for larger values of A. This suggests that A cannot be reliable calculated in perturbation theory.

Finally, in Fig. 3 we compare the next-to-leading order QCD prediction for the D parameter to L3 data obtained at the Z^0 peak [17] and corrected to hadron level. The inclusion of the higher order correction decreases the discrepancy between the next-to-leading order QCD prediction and the data. However, there still remains significant discrepancy. This difference may come in part from hadronization effects, and also from the uncalculated even higher order contributions.

In this Letter we presented for the first time a next-to-leading order calculation of the differential cross section of two classic four-jet shape variables, the D parameter, and acoplanarity. We gave explicit results for the radiative corrections to the leading order cross sections. The corrections are large indicating that the uncalculated even higher order terms are important. This feature is especially dramatic in the case of acoplanarity suggesting that this observable cannot be reliable calculated in perturbation theory. We also compared the four-jet rates obtained by our program to the results of Dixon and Signer [11] and found agreement.

These results were produced by a partonic Monte Carlo program that can be used for the calculation of QCD radiative corrections to the differential cross section of any kind of four-jet observable in electron-positron annihilation.

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Note added:—After the completion of this work, the helicity amplitudes of the $e^+e^- \to Z^0, \gamma^* \to \bar{q}qgg$ process have been published [18], and the agreement with the results of Ref. [7] in the γ^* channel has been established. Also, the authors of Ref. [11] pointed out a slight error in our binning procedure in the case of the Geneva algorithm. Correcting this error we find $R_4(y_{\rm cut} = 0.05) = (8.37 \pm 0.12) \times 10^{-2}$ that agrees with the result of Ref. [11].

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