Infrared Behavior of Gluon and Ghost Propagators in Landau Gauge QCD

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A truncation scheme for the Dyson-Schwinger equations of Euclidean QCD in Landau gauge is presented. It implements the Slavnov-Taylor identities for the three-gluon and ghost-gluon vertices, whereas irreducible four-gluon couplings as well as the gluon-ghost and ghost-ghost scattering kernels are neglected. The infrared behavior of gluon and ghost propagators is obtained analytically: The gluon propagator vanishes for small momenta, whereas the ghost propagator diverges strongly. The numerical solutions are compared with recent lattice results. The running coupling approaches a fixed point, $\alpha_c \simeq 9.5$, in the infrared. [S0031-9007(97)04470-0]

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A theoretical understanding of confinement of quarks and gluons into colorless hadrons could be obtained by proving the failure of the cluster decomposition property for color—nonsinglet gauge—covariant operators. One long established idea in this direction is based on the occurrence of infrared divergences to suppress the emission of colored states from color-singlet states [1]. Such a description of confinement in terms of perturbation theory necessarily has to fail.

Thus, to study the infrared behavior of QCD amplitudes, nonperturbative methods are required, and, since divergences are anticipated, a formulation in the continuum is desirable. Both of these are provided by studies of truncated systems of Dyson-Schwinger equations (DSEs), the equations of motion of QCD Green's functions. Typically, for their truncation, additional sources of information such as the Slavnov-Taylor identities, entailed by gauge invariance, are used to express vertex functions in terms of the elementary two-point functions, i.e., the quark, ghost, and gluon propagators. Those propagators can then be obtained as self-consistent solutions to nonlinear integral equations representing a closed set of truncated DSEs. Some systematic control over the truncating assumptions can be obtained by successively including higher n-point functions in self-consistent calculations, and by assessing their influence on lower n-point functions in this way. At present, even at the level of propagators, no complete solution to truncated DSEs of QCD exists. In particular, even in the absence of quarks, solutions for the gluon propagator in Landau gauge rely on neglecting ghost contributions [2–5]. Ghost-free gauges such as the axial gauge suffer from their own problems [6].

In addition to the prospect of some insight into confinement from studying the infrared behavior of QCD Green's functions, DSEs have proved to be a highly successful tool in developing a hadron phenomenology that interpolates smoothly between the infrared (nonperturbative) and ultraviolet (perturbative) regimes [7]. In particular, a variety of models for the interactions of quarks mediated by gluons exist, which are very well suited for a dynamical description of chiral symmetry breaking from the DSE of the quark propagator [8]. The superficial result of these studies is that for the quark self-energy to reflect a spontaneous breaking of chiral symmetry there has to be some sufficient interaction strength at low energies.

In this Letter we present a simultaneous solution of a truncated set of DSEs for the propagators of gluons and ghosts in Landau gauge. An extension to this self-consistent framework to include quarks dynamically is possible and subject to further studies. The behavior of the solutions in the infrared, implying the existence of a fixed point at a critical coupling $\alpha_c \approx 9.5$, is obtained analytically. The gluon propagator is shown to vanish for small spacelike momenta in the present truncation scheme. This behavior, though in contradiction with previous DSE studies [2–5], can be partially understood from the observation that, in our present calculation, the previously neglected ghost propagator assumes an infrared enhancement similar to what was then obtained for the gluon.

Besides all elementary two-point functions, i.e., the quark, ghost, and gluon propagators, the DSE for the gluon propagator also involves the three- and four-point vertex functions which obey their own DSEs. These equations involve successively higher n-point functions. A first step towards a truncation of the gluon equation is to neglect all terms with four-gluon vertices. These are the momentum independent tadpole term, an irrelevant constant which vanishes perturbatively in Landau gauge, and explicit two-loop contributions to the gluon DSE. The latter are subdominant in the ultraviolet and will thus not affect the behavior of the solutions for asymptotically high momenta. In the infrared it has been argued that the singularity structure of the two-loop terms does not interfere with the one-loop terms [9]. Without contributions from four-gluon vertices (and quarks) the renormalized equation for the inverse gluon propagator in Euclidean momentum space is given by [10]

$$D^{-1}_{\mu\nu}(k) = Z_3 D^{\mathrm{tl}-1}_{\mu\nu}(k) + g^2 N_c Z_1 \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \Gamma^{\mathrm{tl}}_{\mu\rho\alpha}(k,-p,q) D_{\alpha\beta}(q) D_{\rho\sigma}(p) \Gamma_{\beta\sigma\nu}(-q,p,-k)$$
$$- g^2 N_c \tilde{Z}_1 \int \frac{d^4q}{(2\pi)^4} iq_\mu D_G(p) D_G(q) G_\nu(q,p)\,, \tag{1}$$

where $p = k + q$, D^{tl} and Γ^{tl} are the tree level propagator and three-gluon vertex, D_G is the ghost propagator, and Γ and G are the fully dressed three-point vertex functions. The equation for the ghost propagator in Landau gauge QCD, without any truncations, is given by

$$D^{-1}_G(k) = -\tilde{Z}_3 k^2 + g^2 N_c \tilde{Z}_1 \int \frac{d^4q}{(2\pi)^4} ik_\mu D_{\mu\nu}(k-q) G_\nu(k,q) D_G(q)\,. \tag{2}$$

The renormalized propagators for ghosts and gluons and the renormalized coupling are defined from the respective bare quantities by introducing multiplicative renormalization constants, $\tilde{Z}_3 D_G := D^0_G$, $Z_3 D_{\mu\nu} := D^0_{\mu\nu}$, and $Z_g g := g_0$. Furthermore, $Z_1 = Z_g Z_3^{3/2}$, $\tilde{Z}_1 = Z_g Z_3^{1/2} \tilde{Z}_3$, and we use that $\tilde{Z}_1 = 1$ in Landau gauge [11]. The ghost and gluon propagators are parametrized by their respective renormalization functions G and Z,

$$D_G(k) = -\frac{G(k^2)}{k^2}\,, \qquad D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}\,. \tag{3}$$

In order to arrive at a closed set of equations for the functions G and Z, we use a form for the ghost-gluon vertex which is based on a construction from its Slavnov-Taylor identity (STI) neglecting irreducible four-ghost correlations, in agreement with the present level of truncation [12],

$$G_\mu(p,q) = iq_\mu \frac{G(k^2)}{G(q^2)} + ip_\mu\left(\frac{G(k^2)}{G(p^2)} - 1\right). \tag{4}$$

With this result, we can construct the three-gluon vertex according to general procedures from previous studies [13],

$$\Gamma_{\mu\nu\rho}(p,q,k) = \frac{1}{2} A_+(p^2,q^2;k^2)\delta_{\mu\nu} i(p-q)_\rho + \frac{1}{2} A_-(p^2,q^2;k^2)\delta_{\mu\nu} i(p+q)_\rho + \frac{A_-(p^2,q^2;k^2)}{p^2-q^2}$$
$$\times (\delta_{\mu\nu} pq - p_\nu q_\mu) i(p-q)_\rho + \text{cyclic permutations},$$
$$A_\pm(p^2,q^2;k^2) = \frac{G(k^2)G(q^2)}{G(p^2)Z(p^2)} \pm \frac{G(k^2)G(p^2)}{G(q^2)Z(q^2)}\,. \tag{5}$$

Some additionally possible terms, transverse with respect to all three gluon momenta, cannot be constrained by its STI and are thus disregarded. For the fermion vertex in quantum electrodynamics (QED) as constructed from its Ward-Takahashi identity, it is well known that additional transverse terms, with the further constraint not to introduce kinematic singularities, are essential for multiplicative renormalizability [14]. Based on this requirement, such terms have been obtained explicitly for quenched QED in Ref. [15]. Similar constructions for the vertices in QCD are presently not available. However, the full Bose (exchange) symmetry of the three-gluon vertex alleviates this problem since, combined with the STI, it puts much tighter constraints on this vertex then those obtained for fermion vertices.

Instead of a direct numerical solution of the coupled system of integral equations resulting from the present truncation scheme, we use a one-dimensional approximation: For integration momenta $q^2 < k^2$ we use the angle approximation replacing $G[(k-q)^2] \to G(k^2)$ and $Z[(k-q)^2] \to Z(k^2)$. Since this preserves the limit $q^2 \to 0$, it is suitable for an analytic discussion of the solutions in the infrared. For $q^2 > k^2$ we replace *all* arguments (including the external k^2) by the integration momentum q^2. The justification for this is the weak logarithmic momentum dependence of G and Z at high momenta [16]. The DSEs (1) and (2) then simplify to

$$\frac{1}{Z(k^2)} = Z_3 + Z_1 \frac{g^2}{16\pi^2}\left\{\int_0^{k^2} \frac{dq^2}{k^2}\left(\frac{7}{2}\frac{q^4}{k^4} - \frac{17}{2}\frac{q^2}{k^2} - \frac{9}{8}\right) Z(q^2)G(q^2) + \int_{k^2}^{\Lambda^2_{\mathrm{UV}}} \frac{dq^2}{q^2}\left(\frac{7}{8}\frac{k^2}{q^2} - 7\right) Z(q^2)G(q^2)\right\}$$
$$+ \frac{g^2}{16\pi^2}\left\{\int_0^{k^2} \frac{dq^2}{k^2}\frac{3}{2}\frac{q^2}{k^2} G(k^2)G(q^2) - \frac{1}{3}G^2(k^2) + \frac{1}{2}\int_{k^2}^{\Lambda^2_{\mathrm{UV}}} \frac{dq^2}{q^2} G^2(q^2)\right\}, \tag{6}$$

$$\frac{1}{G(k^2)} = \tilde{Z}_3 - \frac{g^2}{16\pi^2}\frac{9}{4}\left\{\frac{1}{2} Z(k^2)G(k^2) + \int_{k^2}^{\Lambda^2_{\mathrm{UV}}} \frac{dq^2}{q^2} Z(q^2)G(q^2)\right\}. \tag{7}$$

We introduced an $O(4)$-invariant momentum cutoff $\Lambda_{\rm UV}$ to account for logarithmic ultraviolet divergences which are absorbed by the renormalization constants Z_3 and $\tilde{Z}_3$. Z_1 has to be ultraviolet finite [17]. This is inconsistent with gauge invariance implying $Z_1 = Z_3/\tilde{Z}_3$. While this problem, appearing at order g^4 in a perturbative expansion, is quite natural for a truncation scheme neglecting explicit four-gluon couplings at the same order, its remedy could provide information on purely transverse terms in the three-gluon vertex. For details of the renormalization and the numerical procedure, see Ref. [17].

To deduce the infrared behavior of the propagators we make the Ansatz so that for $x := k^2 \to 0$ the product $Z(x)G(x) \to cx^\kappa$ with $\kappa \neq 0$ and some constant c. The special case $\kappa = 0$ leads to a logarithmic singularity in Eq. (7) for $x \to 0$ which precludes the possibility of a self-consistent solution. In order to obtain a positive definite function $G(x)$ for positive x from an equally positive $Z(x)$, as $x \to 0$, we obtain the further restriction $0 < \kappa < 2$. Equation (7) then yields

$$G(x) \to \left[g^2 \gamma_0^G \left(\frac{1}{\kappa} - \frac{1}{2} \right) \right]^{-1} c^{-1} x^{-\kappa} \Rightarrow \qquad (8)$$

$$Z(x) \to \left[g^2 \gamma_0^G \left(\frac{1}{\kappa} - \frac{1}{2} \right) \right] c^2 x^{2\kappa}, \qquad (9)$$

where $\gamma_0^G = 9/(64\pi^2)$ is the leading perturbative coefficient of the anomalous dimension of the ghost field. Using (8) and (9) in Eq. (6), we find that the three-gluon loop contributes terms $\sim x^\kappa$ to the gluon equation for $x \to 0$, while the dominant (infrared singular) contribution $\sim x^{-2\kappa}$ arises from the ghost loop, i.e.,

$$Z(x) \to g^2 \gamma_0^G \, \frac{9}{4} \left(\frac{1}{\kappa} - \frac{1}{2} \right)^2 \times \left(\frac{3}{2} \, \frac{1}{2-\kappa} - \frac{1}{3} + \frac{1}{4\kappa} \right)^{-1} c^2 x^{2\kappa}.$$

Comparing this to (9) we obtain a quadratic equation with a unique solution $\kappa = (61 - \sqrt{1897})/19 \simeq 0.92$ for the exponent $\kappa < 2$. The leading behavior of the gluon and ghost renormalization functions is entirely due to ghost contributions. The details of the approximations to the three-gluon loop have no influence on these considerations. In particular, additional transverse terms of the three-gluon vertex, free of kinematical singularities, will yield contributions that are even further suppressed in the infrared. Compared to the Mandelstam approximation, in which the three-gluon loop alone determines the infrared behavior of the gluon propagator and the running coupling in Landau gauge [2–5], this shows the importance of ghosts. The result presented here implies an infrared stable fixed point in the nonperturbative running coupling of our subtraction scheme, defined by

$$\alpha_S(s) = \frac{g^2}{4\pi} Z(s) G^2(s) \to \frac{16\pi}{9} \left(\frac{1}{\kappa} - \frac{1}{2} \right)^{-1} \approx 9.5\,, \qquad (10)$$

for $s \to 0$. This is qualitatively different from the infrared singular coupling of the Mandelstam approximation [5].

The momentum scale in our calculations is fixed from the phenomenological value $\alpha_S(M_Z) = 0.118$ at the mass of the Z boson [18]. The ratio of the Z to the τ mass, $M_Z/M_\tau \simeq 51.5$, then yields $\alpha_S(M_\tau) = 0.38$ which is in encouraging agreement with the experimental value.

It is interesting to compare our solutions to recent lattice results using implementations of the Landau gauge condition [19–21]. In Fig. 1 we compare our solution for the gluon propagator to data from Ref. [20]. We normalized the gluon propagator according to $Z(x=1) \approx 11.3$ to account for the units used in Ref. [20] (with $x = k^2 a^2$ in units of the inverse lattice spacing). According to the authors of Ref. [20], the arrow indicates a bound below which finite size effects become considerable.

In Fig. 2 we compared our infrared enhanced ghost propagator to the results of Ref. [21]. It is quite amazing to observe that our solution fits the lattice data at low momenta significantly better than the fit to an infrared singular form $D_G(k^2) = c/k^2 + d/k^4$ given in Ref. [21]. We therefore conclude that the present lattice calculations confirm the existence of an infrared enhanced ghost propagator of the form $D_G \sim 1/(k^2)^{1+\kappa}$ with $0 < \kappa < 1$. This is an interesting result for yet another reason: In Ref. [21] the Landau gauge condition was supplemented by an algorithm to select gauge field configurations from the fundamental modular region which is to avoid Gribov copies. Thus, our results suggest that the existence of such copies of gauge configurations might have little effect on the solutions to Landau gauge DSEs [22].

The Euclidean gluon correlation function presented here can be shown to violate reflection positivity [17], which is a necessary and sufficient condition for the existence of a Lehmann representation [23]. We interpret this as representing confined gluons. In order to understand how these correlations can give rise to confinement of quarks, it

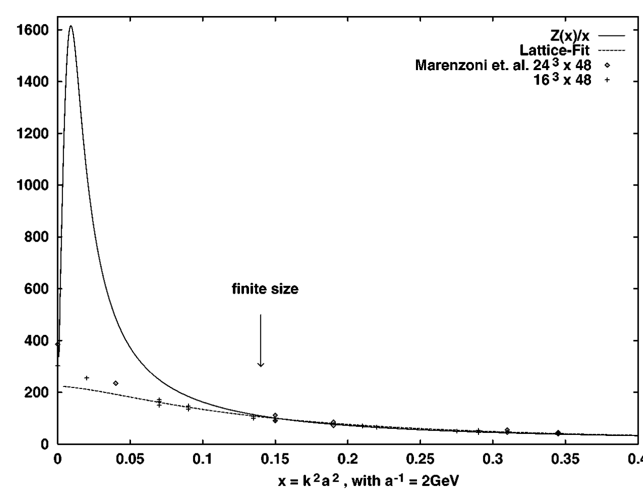

FIG. 1. The numerical result for the gluon propagator from Dyson-Schwinger equations (solid line) compared to lattice data from Fig. 3 in Ref. [20].

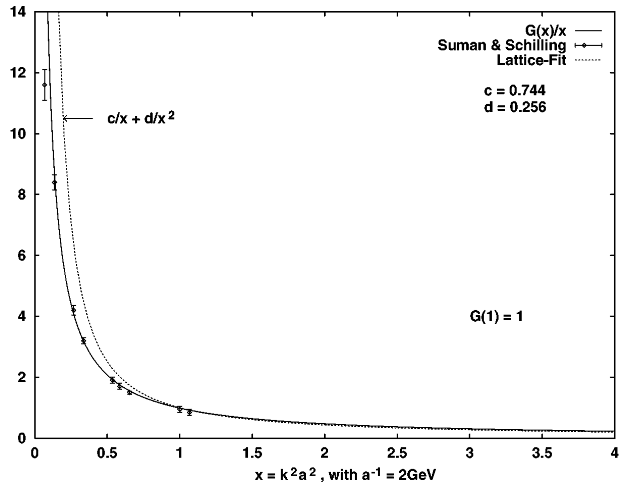

FIG. 2. The numerical result for the ghost propagator (solid line) compared to data from Fig. 1 in Ref. [21] for the 24^4 lattice up to $x \approx 1$, and a fit as obtained in Ref. [21].

will be necessary to include the quark propagator. The size of the coupling at the fixed point, $\alpha_c \approx 9.5$, is, however, a good indication that dynamical chiral symmetry breaking will be generated in the quark DSE.

In summary, we have presented a solution to a truncated set of coupled Dyson-Schwinger equations for gluons and ghosts in Landau gauge. The infrared behavior of this solution, obtained analytically, represents a strongly infrared enhanced ghost propagator and an infrared vanishing gluon propagator. Our results, in particular for the ghost propagator, compare favorably with recent lattice calculations [20,21]. Since the lattice implementations of the Landau gauge are such that configurations are restricted to the fundamental modular region, this might indicate that Gribov copies have little influence on solutions to the DSEs in Landau gauge. The absence of a Lehmann representation for the gluon propagator can be interpreted as a signal for confined gluons. The existence of an infrared fixed point is in qualitative disagreement with previous studies of the gluon DSE neglecting ghost contributions in Landau gauge [2–5]. This shows that ghosts are important, in particular, at low energy scales relevant to hadronic observables.

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