Supersymmetry and Broken Symmetries at High Temperature

Antonio Riotto[1] and Goran Senjanović[2]
[1]*Fermilab National Accelerator Laboratory, Batavia, Illinois 60510*
[2]*International Center for Theoretical Physics, 34100 Trieste, Italy*
(Received 12 February 1997)

It is generally believed that internal symmetries are necessarily restored at high temperature in supersymmetric theories. We provide simple and natural counterexamples to this no-go theorem for systems having a net background charge. We exemplify our findings on Abelian models, for both cases of global and local symmetries, and discuss their possible implications. [S0031-9007(97)03651-X]

PACS numbers: 11.30.Qc, 11.10.Wx, 11.30.Pb, 98.80.Cq

When heated up, physical systems undergo phase transitions from ordered to less ordered phases. This deep belief, encouraged by everyday life experiences, would tell us that at high temperature spontaneously broken symmetries of high energy physics get restored. This, in fact, is what happens in the standard model (SM) of electroweak interactions. Whether or not this is true in general is an important question in its own right, but it also has a potentially dramatic impact on cosmology. Namely, most of the extensions of SM tend to suggest the existence of the so-called topological defects, and it is known that two types of such defects, i.e., domain walls and monopoles, pose cosmological catastrophe. More precisely, they are supposed to be produced during phase transitions at high temperature T [1], and they simply carry too much energy density to be in accord with the standard big-bang cosmology. One possible way out of this problem could be provided by eliminating phase transitions if possible. In fact, it has been known for a long time [2] that in theories with more than one Higgs field (and the existence of topological defects requires more than one such field in realistic theories) symmetries may remain broken at high T, and even unbroken ones may get broken as the temperature is increased. This offers a simple way out of the domain wall problem [3], whereas the situation regarding the monopole problem is somewhat less clear [4]. Unfortunately, the same mechanism seems to be inoperative in supersymmetric theories. Whereas supersymmetry (SUSY) itself gets broken at high T, internal symmetries, on the other hand, get necessarily restored. This has been proven at the level of renormalizable theories [5], and a recent attempt to evade it using higher dimensional nonrenormalizable operators [6] has been shown not to work [7].

All papers mentioned above have an important assumption in common: The chemical potential is taken to be zero. In other words, they assume the vanishing of any conserved charge. On the other hand, it has been known that in nonsupersymmetric theories the background charge asymmetry may postpone symmetry restoration at high temperature [8], and even more remarkably that it can lead to symmetry breaking of internal symmetries, both in cases of global [9] and local symmetries [10] at arbitrarily high temperatures. This is simply a consequence of the fact that, if the conserved charge stored in the system is larger than a critical value, the charge cannot entirely reside in the thermal excited modes, but it must flow into the vacuum. This is an indication that the expectation value of the charged field is nonzero, i.e., that the symmetry is spontaneously broken. From the work of Affleck and Dine [11] we know that there is nothing unnatural about large densities in SUSY theories. The most natural candidate for the large density of the universe is the lepton number that may reside in the form of neutrinos. In fact, this is precisely what is assumed in [10]. Now, one may fear that the usual washout of the $B + L$ number due to sphalerons will predict the baryon and lepton numbers to be equal, which would be disastrous. However, there is a catch here. If the standard model symmetry is not restored at high temperature, sphaleron effects get exponentially suppressed, and the usually assumed $B + L$ washout becomes ineffective [12]. Thus the large lepton number can coexist with a small baryon number. Furthermore, it should be stressed that a large lepton number is perfectly consistent with the ideas of grand unification. It can be shown that in SO(10) one can naturally arrive at a small baryon number and a large lepton number [13].

It is the purpose of this Letter to demonstrate that reasonably large charge densities (with the chemical potential smaller than temperature) provide a natural mechanism of breaking internal symmetries in SUSY at arbitrary high temperature. We exemplify our findings on simple Abelian models, with both global and local symmetries, and leave the generalization to non-Abelian symmetry for future publication. However, before presenting our results, we wish to make some comments regarding background charge asymmetries.

A. *Chemical potential: generalities.*—Let us assume that associated with some unbroken Abelian symmetry, either global or local, there exists a nonvanishing net background charge $Q = (n - \bar{n})V$, where V is the physical volume of the system and n and $\bar{n}$ are the particle and the antiparticle density distributions

$$n = \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{(E-\mu)/T} \pm 1} \qquad (1)$$

and $\bar{n}(\mu) = n(-\mu)$. In the above, $\pm$ refers to fermions and bosons, respectively, and μ is the chemical potential associated with the charge Q. It is easy to show that for fermions (which is all we need here) $j_0 \equiv \frac{Q}{V} = \frac{g_*}{6\pi^2}\mu^3(\pi^2\mu T^2)$ for $\mu \gg T$ ($\mu \ll T$), where g_* denotes the number of degrees of freedom. Now, if these particles are in equilibrium with the photons, then $n - \bar{n} < n + \bar{n} \simeq n_\gamma \simeq T^3$, and thus $(n - \bar{n})/n_\gamma < 1$ implies $\mu < T$. We shall stick to this reasonable physical assumption. In general, μ has a temperature dependence dictated by the charge conservation constraint. In an expanding Universe $V \sim T^{-3}$, and thus the fact that Q remains constant during the expansion implies $\mu/T =$ const. This simple, but important observation plays a crucial role in what follows.

B. *Global Abelian charge.*—The simplest supersymmetric model with a global U(1) symmetry is provided by a chiral superfield Φ and a superpotential

$$W = \frac{\lambda}{3}\Phi^3. \tag{2}$$

It has a global U(1) R symmetry under which fields transform as $\phi \to e^{i\alpha}\phi$ and $\psi \to e^{-i\alpha/2}\psi$, where ϕ and ψ are the scalar and the fermionic component of the superfield Φ. Thus the fermionic and bosonic charges are related by $2Q_\psi + Q_\phi = 0$, or, in other words, the chemical potentials satisfy the relation $\mu \equiv \mu_\phi = -2\mu_\psi$.

Now, the main point here is that the presence of a nonvanishing net R charge at *tree level* already leads to a mass term for the scalar field with a "wrong" (negative) sign after canonical momenta have been integrated out in the path integral [8,9]. The one-loop high temperature effective potential receives dominant contributions from μ and T for a small Yukawa coupling $\lambda \ll 1$ and reads (for $\mu < T$) [8,9]

$$V_T(\phi) = \left(-\mu^2 + \frac{1}{2}\lambda^2T^2\right)\phi^\dagger\phi + \lambda^2(\phi^\dagger\phi)^2. \tag{3}$$

The above result is not new; it has been found before in the case of charged scalar fields [8,9]. The important new ingredient here is that it holds in the case of supersymmetry, since we may safely ignore the one-loop fermionic contribution in the chemical potential term, being of the order $\lambda^2\mu^2\phi^\dagger\phi$ and suppressed by small λ. Of course, the fermions enter in the T^2 mass term, and the coefficient 1/2 reflects that. Obviously, for $\mu^2 > \lambda^2T^2/2$ the symmetry is spontaneously broken at high temperature, and the field ϕ gets a vacuum expectation value (VEV)

$$\langle\phi\rangle^2 = \frac{\mu^2 - \frac{\lambda^2}{2}T^2}{\lambda^2}. \tag{4}$$

This result is valid as long as the chemical potential μ is smaller than the scalar mass in the $\langle\phi\rangle$ background, i.e., $\mu^2 < m_\phi^2 = 2\lambda^2\langle\phi\rangle^2$ [9]. This in turn implies $\mu > \lambda T$. In short, for a perfectly reasonable range

$$\lambda T < \mu < T. \tag{5}$$

The original $U(1)_R$ global symmetry is spontaneously broken, and this is valid at arbitrarily high temperatures (as long as the approximation of λ small holds true).

Notice that, in all the above, we have assumed unbroken supersymmetry. When supersymmetry is softly broken, $U(1)_R$ also gets explicitly broken because of the presence of soft trilinear scalar couplings in the Lagrangian. Therefore, the associated net charge vanishes, and the reader might be worried about the validity of our result. However, the typical rate for $U(1)_R$-symmetry-breaking effects is given by $\Gamma \sim \tilde{m}^2/T$, where we have indicated by $\tilde{m} \sim 10^2$ GeV the typical soft SUSY breaking mass term. Since the expansion rate of the Universe is given by $H \sim 30T^2/M_{P\ell}$, $M_{P\ell}$ being the Planck mass, one finds that $U(1)_R$-symmetry-breaking effects are in equilibrium, and the net charge must vanish only at temperatures *smaller* than $T_{SS} \sim \tilde{m}^{2/3}M_{P\ell}^{1/3} \sim 10^7$ GeV. Therefore, it is perfectly legitimate to consider the presence of a nonvanishing R charge at very high temperatures even in the case of softly broken SUSY.

Thus, we have provided a simple and natural counterexample to the theorem of the restoration of internal symmetries in supersymmetry [5]. The situation here is completely analogous to what happens in the nonsupersymmetric scalar field theory with potential $V = m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ described in [9]. The presence of a nonvanishing chemical potential for the associated U(1) global charge makes it possible for the global symmetry to be broken at arbitrarily high temperatures even in the case $m^2 > 0$. This is a consequence of the fact that the charge cannot be stored in the thermal excited modes, but it must reside in the vacuum, and this is an indication that the expectation value of the charged field is nonzero, i.e., that the symmetry is spontaneously broken.

C. *Local gauge charge.*—It has been known for a long time [10] that a background charge asymmetry tends to increase symmetry breaking in the case of a local gauge symmetry. In his work, Linde has shown how a large fermion number density would prevent symmetry restoration at high temperature in both Abelian [10] and non-Abelian theories [14]. The essential point is that the external charge leads to the condensation of the gauge field which in turn implies the nonvanishing VEV of the Higgs field. This phenomenon may be easily understood if one recalls that an increase of an external fermion current $\mathbf{j}$ leads to symmetry restoration in the superconductivity theory [15]. In gauge theories, symmetry breaking is necessarily a function of $j^2 = j_0^2 - \mathbf{j}^2$, where j_0 is the charge density of fermions. An increase of j_0 is therefore accompanied by an increase of symmetry breaking [10]. We now demonstrate that this phenomenon persists in supersymmetric theories, at least in the case of Abelian symmetry.

The simplest model is based on U(1) supersymmetric local gauge symmetry. The minimal anomaly free matter content consists of two chiral superfields Φ^+ and Φ^- with opposite gauge charges, and the most general renormalizable superpotential takes the form

$$W = m\Phi^+\Phi^- . \tag{6}$$

Notice that the symmetry is *not* spontaneously broken at zero temperature. Since there is no Yukawa interaction, there is also a global U(1) R symmetry, under which the bosons have, say, the same charge and the fermions are invariant. Furthermore, at very high temperature, $T > m^{2/3}M_{P\ell}^{1/3}$, the fermion mass can be neglected, and we also get a chiral U(1) symmetry under which the bosons are invariant. We may now suppose for simplicity that there is a net background charge density j_0, with the zero current density and that it lies entirely in the fermionic sector. In other words, we assume the background charge to be in the form of the chiral fermionic charge. Thus only the fermions have a nonvanishing chemical potential. Equally important, we assume that the gauge charge of the Universe is zero, just as in [10]. In the realistic version of this example, one would imagine the gauge charge to be the electromagnetic one and the chiral fermionic charge to be, say, the lepton charge in the minimal supersymmetric standard model (MSSM). We know from observation that the electromagnetic charge of the Universe vanishes to a good precision. Thus we have to minimize the action with the constraint that the electric field is zero. What will happen is that some amount of bosonic charge will get stored into the vacuum in order to compensate for the fermionic one and achieve the vanishing of the electric field. In this way, the total U(1) charge density of the system, including the charge of the condensate, is equal to zero even if symmetry is broken and the gauge forces are short-range ones. This is the principal reason behind the resulting spontaneous symmetry breaking of the local gauge symmetry, as we show below. Thus it is crucial to have some nonvanishing external background charge, i.e., the model should have some extra global symmetry as provided by our chiral symmetry. We can obviously take $A_i = 0$ in the vacuum and treat A_0 on the same footing with the scalar fields $\phi^\pm$ (due to the net charge, A_0 cannot vanish in the vacuum). If we now integrate out A_0 using its equation of motion, assuming the electric field to be zero, we can then compute the high temperature potential for the scalar fields in question at high temperature and large charge density with the following result:

$$\begin{aligned} V_{\rm eff}(T) = {} & \frac{g^2}{2} T^2(|\phi^+|^2 + |\phi^-|^2) \\ & + \frac{g^2}{2}(|\phi^+|^2 - |\phi^-|^2)^2 \\ & + \frac{1}{2}\frac{j_0^2}{2(|\phi^+|^2 + |\phi^-|^2) + T^2}, \end{aligned} \tag{7}$$

where we have taken $T \gg m$, and we have included both scalar and fermionic loop contributions in the T^2 mass term for A_0. Now, except for the D term, the rest of the potential depends only on the sum $\phi^2 \equiv |\phi^+|^2 + |\phi^-|^2$, and thus the energy is minimized for the vanishing of the D-term potential, i.e., for $|\phi^+|^2 = |\phi^-|^2$. It is easy to see that, in this case, the effective potential has two extrema:

$$\phi = 0 \quad \text{and} \quad \phi^2 = \frac{j_0}{\sqrt{2}\, gT} - \frac{T^2}{2}. \tag{8}$$

The second extremum obviously exists only for

$$j_0 > \frac{gT^3}{\sqrt{2}}. \tag{9}$$

Moreover, in that case, it is an absolute minimum, while $\phi = 0$ is a maximum.

Now, we can rephrase the above condition in the language of the chemical potential (using $g_* = 4$): $\mu > gT$. For $g \ll 1$, which is of our interest, μ easily satisfies the condition $\mu < T$. As noted above, since Yukawa interactions are absent, the role of the external charge may have been played by the R charge in the scalar sector, the two scalars being equally charged under this symmetry. In such a case, the analysis requires careful handling because of issues related to gauge invariance, and we will extensively explore this in a future publication. We only mention here that the addition of the term $-\mu(\phi_+^\dagger \overleftrightarrow{\partial}_0 \phi_+ + \phi_-^\dagger \overleftrightarrow{\partial}_0 \phi_-)$ to the Lagrangian requires a simultaneous addition of the term $2A_0\mu(|\phi_+|^2 - |\phi_-|^2)$ to conserve gauge invariance. It is straightforward to show that, for large enough chemical potential μ, the effective potential at high temperature is again minimized for vanishing D term, which results in $A_0 = 0$, and that $|\phi|^4$ terms induced at one loop are crucial for the existence of global minimum which breaks local gauge symmetry. For the physical realization of this situation the existence of an extra source is necessary and, in our case, the latter is provided by the chiral fermionic charge. This situation is quite different from what happens in a simple nonsupersymmetric model with local Abelian symmetry and only one scalar field, where only the combination $(A_0 + \mu)$ appears in the computation, and the absence of an extra source imposes the condition that this quantity is equal to zero [16].

Thus we have also provided a supersymmetric example of a local gauge symmetry being broken at high temperature in the presence of a background charge density.

D. *Summary and outlook.*—The main point of our paper is that internal symmetries in supersymmetric theories, contrary to the general belief, may be broken at high temperature, as long as the system has a nonvanishing background charge. The examples we have provided here, based on both global and local Abelian symmetries, are natural and simple and should be viewed as prototypes of more realistic theories. The necessary requirement for the phenomenon to take place is that the chemical potential be bigger than a fraction of temperature on the order of (1–10)%. Notice that this is by no means unnatural. In the expanding Universe, as we have stressed before, the chemical potential is proportional to temperature, and, thus, unless zero for some reason, μ/T is naturally expected to be of order one. More important, this chemical potential could be zero today; all that is needed is that it is nonvanishing at high temperature. We have seen how soft supersymmetry breaking may naturally provide such a scenario if there is some nonvanishing external charge.

Now, it is well known that in suspersymmetry the existence of flat directions may lead to large baryon and lepton number densities at very high temperature [11]. In any case, we wish to be even more open minded and simply allow for a charge density without worrying about its origin. It is noteworthy that a large neutrino number density may persist all the way through nucleosynthesis up to today [17]. This has been used by Linde [14] in order to argue that, even in SM, the $SU(2)_L \otimes U(1)_Y$ symmetry may not be restored at high temperature. Since SUSY, as we have seen, does not spoil the possibility of large chemical potentials allowing symmetry breaking at high T, it is important to see if our results remain valid in the case of non-Abelian global and local symmetries. This work requires particular attention due to the issues related to gauge invariance, and is now in progress. We only wish to observe here that, if some conserved charge is present in the system, e.g., the lepton charge in the minimal supersymmetric extension of the SM, it will be automatically shared among fermions and scalars of the same supersymmetric multiplet. However, in the realistic situation in which supersymmetry is softly broken, sfermions acquire a mass $\tilde{m}$ and are much heavier than their fermionic partners. Hence, for temperatures T smaller than $\tilde{m}$, the number density of sfermions in the thermal bath is drastically reduced by the Boltzmann suppression factor, and the external conserved charge will reside in the fermionic sector. This might render the analysis easier and make it similar to the one performed in Section C.

We should stress that there is more than a sole academic interest to the issue discussed in this paper. If symmetries remain broken at high temperature, there may be no domain wall and monopole problems at all. Furthermore, it is well known that, in SUSY, grand unified theories (GUTs) symmetry restoration at high temperature prevents the system from leaving the false vacuum and finding itself in the broken phase at low T. This is a direct consequence of the vacuum degeneracy characteristic of supersymmetry which says that at zero temperature, the SM vacuum and the unbroken GUT symmetry have the same (zero) energy. If the symmetry is restored at high T, one would start with the unbroken symmetry in the early universe and would thus get caught in this state forever. Obviously, if our ideas hold true in realistic grand unified theories, this problem would not arise in the first place.

In conclusion, our Abelian examples, based on both global and local gauge symmetries, show that it is perfectly consistent to have a spontaneously broken internal symmetry in supersymmetric theories. For this to happen, though, an important condition must be satisfied: The system must possess a non-negligible chemical potential, $\mu \leq T$, or, in other words, a net background charge. This is the novel and main point of our paper. The previous works, it should be stressed, indicating the role of chemical potential in symmetry nonrestoration at high temperature referred only to nonsupersymmetric theories. Now, in nonsupersymmetric theories, in many cases it is possible to achieve high temperature symmetry breaking even without resorting to charge asymmetries, whereas it is impossible in SUSY. Generalization of our results to supersymmetric non-Abelian theories is now in progress.

We are deeply grateful to Borut Bajc for his keen interest in our work, for many useful discussions, for checking some of our computations, and for the careful reading of the manuscript. We also wish to acknowledge discussions with Graciela Gelmini, Andrei Linde, Alejandra Melfo, and, in particular, Scott Dodeldson. A. R. is supported by the DOE and NASA under Grant No. NAG5-2788.

[1] T. W. Kibble, J. Phys. A **9**, 1987 (1976); Phys. Rep. **67**, 183 (1980).
[2] S. Weinberg, Phys. Rev. D **9**, 3357 (1974); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **42**, 1651 (1979); Phys. Rev. D **20**, 3390 (1979).
[3] G. Dvali and G. Senjanović, Phys. Rev. Lett. **74**, 5178 (1995).
[4] G. Dvali, A. Melfo, and G. Senjanović, Phys. Rev. Lett. **75**, 4559 (1995); G. Bimonte and L. Lozano, Phys. Lett. B **366**, 248 (1996); Nucl. Phys. **B460**, 155 (1996).
[5] H. Haber, Phys. Rev. D **26**, 1317 (1982); M. Mangano, Phys. Lett. **147B**, 307 (1984).
[6] G. Dvali and K. Tamvakis, Phys. Lett. B **378**, 141 (1996).
[7] B. Bajc, A. Melfo, and G. Senjanović, Phys. Lett. B **387**, 796 (1996).
[8] H. E. Haber and H. A. Weldon, Phys. Rev. D **25**, 502 (1982).
[9] K. M. Benson, J. Bernstein, and S. Dodelson, Phys. Rev. D **44**, 2480 (1991).
[10] A. D. Linde, Phys. Rev. D **14**, 3345 (1976).
[11] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[12] J. Liu and G. Segrè, Phys. Lett. B **338**, 259 (1994).
[13] J. Harvey and E. Kolb, Phys. Rev. D **24**, 2090 (1981).
[14] A. D. Linde, Phys. Lett. **86B**, 39 (1979).
[15] See, for instance, P. G. De Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
[16] D. A. Kirzhnits and A. D. Linde, Ann. Phys. (N.Y.) **101**, 195 (1976).
[17] Large neutrino chemical potentials $|\mu|$ for the three neutrino species affect the energy density in the neutrino species, and thereby the expansion rate of the Universe, dangerously increasing the primordial production of ^{4}He. However, it is possible to dial in the desired ^{4}He abundance by the appropriate sign of the neutrino chemical potential. See, for example, E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990), p. 107.