Hysteretic Depinning of Anisotropic Charge Density Waves

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We investigate the depinning transition in a dirty periodic medium considering a model of layered charge density waves as a prototype system. We find that depinning from strong disorder occurs via a two stage process where, first, the pinned system experiences a continuous transition into a plastically sliding state and undergoes a second sharp hysteretic transition into a coherently moving 3D state at higher drives. In the weakly disordered system the depinning into a coherently sliding state remains continuous. [S0031-9007(97)04405-0]

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Depinning of periodic structures such as charge density waves (CDW), vortex and domain wall lattices, and Wigner crystal from a random pinning potential under the influence of an external driving force is one of the paradigms of condensed matter physics. All of these systems share one thing in common: many elastically coupled degrees of freedom interacting with a quenched random environment. In this Letter we report analytic results on the nature of the depinning transition in dirty periodic media using CDW dynamics as a prototype model and discuss possible extensions to other systems.

Two types of depinning have been observed: a smooth nonhysteretic transition with a unique pinning threshold and transport switching characterized by an abrupt hysteretic transition into a sliding state [1,2]. Smooth depinning described in terms of critical behavior [3] follows from the description of the above systems as a classical field associated with the distortions of the system. Much of the switching behavior is explained by the possibility of plastic deformations allowing the amplitude of CDW to vanish along certain surfaces within the system [4,5]. Recent experimental and numerical studies [6–9] of vortex transport in high temperature superconductors (HTS) have demonstrated that the depinning of the vortex lattice can also be accompanied by plastic effects.

These latter findings suggest that plastic effects can play an essential role in the depinning and that the nonequilibrium steady state near the transition resembles fluidlike motion. However, at very high velocities well above the depinning transition the influence of disorder on the dynamics is suppressed, and one can expect coherent motion of an almost perfect solid periodic system. A problem of separation of these two different driven regimes has been addressed in [9] in the context of vortex transport. It was proposed that the driven periodic medium subject to sufficiently strong disorder undergoes a sharp hysteretic *dynamic* transition from coherent motion with almost perfect structure to fluidlike plastic dynamics upon decreasing the drive at a second critical force F_f *well above the pinning threshold* F_T. This was called *dynamic freezing*. Upon increasing the driving force from the pinned state, the vortex lattice starts to slide at $F = F_T$, this depinning being followed by the multiple plastic effects, and elastic motion recovers at $F = F_f > F_T$. This concept received strong support both from earlier observations of plastic effects [6] and from the subsequent transport measurements on MoGe superconducting films [10].

The prediction of the possibility of a dynamical phase transition in the driven state [9] was later expanded by nice scaling arguments onto three-dimensional CDWs [11]. The properties of the driven coherent phase were examined and discussed in [12–16]. Yet a number of unresolved problems remains. The fundamental issue is the nature of the depinning transition (continuous vs switching), and the question is under what conditions either type occurs. How would dynamic freezing evolve with decreasing strength of the disorder? What are the conditions for the existence of the plastic flow regime as the state intermediate between the pinned and coherently moving states? Is it possible to have depinning directly to a coherently moving state?

In this Letter we address these questions using CDW transport as an example system. We consider a model of an anisotropic CDW [17] and develop a nonperturbative self-consistent description of the dynamic transitions in driven dirty systems. We find that, if disorder is sufficiently strong, depinning occurs in two stages: First the CDW depins in a driven decoupled state, where 2D CDWs in each layer slide independently; and, second, upon further increasing the driving force, the system experiences a second transition into a coupled coherently moving 3D phase. This "sequential depinning" corresponds to a dynamic freezing transition scenario proposed in [9], where the periodic system first depins into a plastically moving state and then, upon further increase of the drive, experiences a transition into a coherently moving dynamic state. In the system with weak disorder, decoupling does not occur.

This Letter is organized as follows. First we describe the model and derive a self-consistent equation for the shear modulus. Then we analyze the cases of weak and strong pinning, and determine the disorder induced dynamic decoupling (the instability point) as the current at which the onset/disappearance of the coupling occurs. In conclusion, we construct the general dynamic phase diagram for periodic structures driven through quenched disorder.

The overdamped dynamics of an anisotropic layered CDW are governed by the equation (CDW moves along the x direction),

$$\lambda\dot{\phi}_i(\mathbf{x},t) = \gamma\nabla^2\phi_i(\mathbf{x},t) + \gamma\mu_0[\sin(\phi_{i+1}-\phi_i) + \sin(\phi_{i-1}-\phi_i)] + F + \gamma V\sin[\phi_i - \alpha_i(\mathbf{x})]. \quad (1)$$

Here ϕ is a CDW phase (displacement field), λ is a friction coefficient, v is the average velocity of CDW, γ is the elastic constant, μ_0 is the anisotropy parameter characterizing layer coupling, V is the strength of the random potential, $\alpha(\mathbf{x})$ is a random phase, i is a layer index, and $\mathbf{x}$ is a D-dimensional vector in the layer. In the CDW models, α comes from the backscattering part of the impurity potential: $\alpha_i(\mathbf{x}) = 2\mathbf{k}_F(\mathbf{x} + \mathbf{z}_0 ia)$, where $\mathbf{k}_F$ is the Fermi vector and $\mathbf{r} = (\mathbf{x}, \mathbf{z}_0 ia)$ is the coordinate of the impurity ($\mathbf{z}_0$ is the unit vector in the z direction). Since the in-plane pinning correlation length exceeds the impurity spacing the random phase α is commonly considered as a random variable homogeneously distributed in the interval $[0, 2\pi]$ [1].

If the anisotropy parameter μ_0 is large one recovers the continuous limit, then $i \to z$ and the term in brackets becomes simply $\partial^2\phi/\partial z^2$. The related Hamiltonian has the form,

$$\mathcal{H} = \gamma\int d^D x\,\frac{dz}{a} \times \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\mu_0(\partial_z\phi)^2 + V(\mathbf{x},z,\phi)\right], \quad (2)$$

where we set $a = 1$. This $(D+1)$-dimensional system experiences a continuous depinning transition at critical force $F_{c,d} = \gamma(V^4/\mu_0)^{1/(4-d)}$, $\mu_0 \gg 1$, and the pinning correlation length is $\xi_{c,d} = (V^4/\mu_0)^{-1/2(4-d)}$, $d = D + 1$. If $\mu_0 \to 0$ the decoupling transition occurs in a static 3D system at $\mu_0 < \mu_{\min} = V^2$. Note that in a decoupled limit the equation of motion (1) becomes isotropic and $\mu_0 \to 1$, $d \to D$.

To obtain a coarse-grained description in terms of a slowly varying $\phi^<$ part of the phase, we integrate out its fast component $\overline{\phi} = \frac{1}{t_0}\int_0^{t_0}\phi(t)dt$, $t_0 = l/v$; l is the CDW period (hereafter we will drop the bar). The coarse-graining procedure is straightforward in case of weak (as compared to coupling) disorder, $\mu_0 \gg \sqrt{V}$, where phase variations from layer to layer are small and $\sin(\phi_{i+1}-\phi_i) \approx (\phi_{i+1}-\phi_i)$. Going over to a continuous description, one arrives at the coarse-grained equation of motion for the slowly varying part of the phase,

$$\lambda\dot{\phi}(\mathbf{x},z) = \gamma(\nabla^2 + \mu\partial_z^2)\phi - \lambda v\partial_x\phi + F_p(\mathbf{x},z), \quad (3)$$

where $\mu \approx \mu_0$ and F_p is the effective random force (random mobilitylike term) originating from the random sin field (analogous to [18]) with the correlator defined as $\langle F_p(\mathbf{x},z)F_p(\mathbf{x}',z')\rangle = \Delta(v)\delta(\mathbf{x}-\mathbf{x}')\delta(z-z')$ with [15,16]

$$\Delta(v) = \begin{cases} \frac{(V\gamma)^4}{(\lambda v)^2}, & v \gg \gamma\sqrt{\mu}/\lambda, \\ \frac{V^4(\lambda v)^2}{\mu^2}, & v \ll \gamma\sqrt{\mu}/\lambda. \end{cases} \quad (4)$$

We omitted the coarse-grained pinning potential term $f_p(\phi)$ and the disorder-induced KPZ term: It can be shown [18] that in the perturbative high velocity limit those terms are irrelevant.

We are interested, however, in the strong disorder regime where the static CDW is decoupled. In this case, the coarse-graining procedure is more subtle. To carry it out, note that in the high-velocity case, $v \gg \gamma\sqrt{\mu}/\lambda$, $\Delta(v)$ does not depend on the elastic constants γ and the anisotropy parameter μ (provided the strength of disorder γV is fixed). This reflects that fact that at high velocities $\Delta(v)$ is controlled by the dissipative part $1/i\lambda v$ of the response function and enables us to carry out the coarse-graining procedure also in the case of weak anisotropy $\mu_0 \ll \sqrt{V}$ [i.e., when a replacement of $\sin(\phi_{i+1}-\phi_i)$ by $\phi_{i+1}-\phi_i$ is not possible]. The coarse-grained equation of motion reads ($v \gg \gamma\sqrt{\mu}/\lambda$)

$$\lambda\dot{\phi}_i(\mathbf{x}) = \gamma\nabla^2\phi_i(\mathbf{x}) + \gamma\mu_0[\sin(\phi_{i+1}-\phi_i) + \sin(\phi_{i-1}-\phi_i)] - \lambda v\partial_x\phi_i + F_{p,i}(\mathbf{x}). \quad (5)$$

It is convenient to rewrite Eq. (5) in the form,

$$\sum_j G_{ij}^{-1}\phi_j = \gamma\mu_0[\sin(\phi_{i+1}-\phi_i) + \sin(\phi_{i-1}-\phi_i)] - \gamma\mu[\phi_{i+1} + \phi_{i-1} - 2\phi_i] + F_{p,i}(\mathbf{x}) + \varepsilon_i(\mathbf{x}) \quad (6)$$

with

$$G_{ij}^{-1}(t) = (\lambda\partial_t + \lambda v\partial_x - \gamma\nabla^2 + 2\gamma\mu)\delta_{ij} - \gamma\mu(\delta_{i,i+1} + \delta_{i,i-1}),$$

where $\varepsilon_i(\mathbf{x})$ denotes a source term, which will be sent to zero at the end of the calculations. One can now solve Eq. (6) iteratively, generating an infinite number of tree diagrams with either $F_{p,i}(\mathbf{x})$ or $\varepsilon_i(\mathbf{x})$ at the ends of each branch and averaging subsequently over the random force, keeping only the linear in $\varepsilon_i(\mathbf{x})$ terms. To find the self-consistent equation for μ we note that the self-consistency condition requires that the phase-containing terms on the right-hand side of Eq. (6) cancel each other. After that, we obtain in the lowest order in μ_0 the following:

$$\mu = \mu_0\exp[-\tfrac{1}{2}\langle(\phi(\mathbf{x},z,t) - \phi(\mathbf{x},z+a,t))^2\rangle]. \quad (7)$$

Note also that since the correlations of $F_{p,i}(\mathbf{x})$ are Gaussian, the higher order cumulants do not appear within this scheme.

The correlation function in the exponent is calculated with the Hamiltonian from (2) with $a = 1$:

$$C(\tilde{v}, \mu, \Delta(v)) = \langle (\phi(\mathbf{x}, z, t) - \phi(\mathbf{x}, z + a, t))^2 \rangle = \frac{1}{\gamma^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\Delta(v)(1 - \cos k_z a)}{(k_x^2 + k_y^2 + \mu k_z^2)^2 + \lambda^2 v^2 k_x^2/\gamma^2}. \quad (8)$$

The main contribution comes from the maximal k_z; therefore, we can replace $k_z \to \pi/a$ in the integrand. Then in the 3D case, one arrives at

$$C(\tilde{v}, \mu, \Delta(v)) = \frac{\Delta(v)}{(4\pi)^3 \gamma^2 \mu^{3/2}} [(\tilde{v}^2 + 4\pi^2\mu)^{1/2} - \tilde{v}], \quad (9)$$

where $\tilde{v} = \lambda v/\gamma$. Outside the critical region $\tilde{v} \approx F/\gamma$, and within the critical region $\tilde{v} \simeq (F - F_c)^{1-(4-d)/6}/\gamma$ [19]. Note that, in the limit of very large velocities, Eq. (8) provides the expected behavior $\mu \to \mu_0$, and the system always remains coupled.

Now comes the central point of our discussion: solving the self-consistent equation for μ. The disappearance of the solution to Eq. (7) implies decoupling of the system. To capture the transition we will be seeking for the moment of the first disappearance of the solution. For the sake of simplicity we can replace the expression in square brackets in (9) by $2\pi^2\mu/\sqrt{\tilde{v}^2 + 4\pi^2\mu}$, which gives the same asymptotics in the limit of small and large velocities. As we will shortly see, in the case of strong disorder the decoupling transition occurs at large velocities, whereas in the weak disorder case there is no decoupling.

Strong disorder $V^2 > \mu_0$.—In the limit of large velocities Eq. (7) assumes the form,

$$\mu = \mu_0 \exp\left(-\frac{V^4}{32\pi\tilde{v}^3\mu^{1/2}}\right). \quad (10)$$

To find the point μ_c of the disappearance of the solution we derivate both sides of (10), obtaining the condition $V^4 = 64\pi\tilde{v}^3\mu^{1/2}$, and find

$$\mu_c = \mu_0/e^2, \qquad \tilde{v}_c = \left(\frac{e}{64\pi} \frac{V^4}{\sqrt{\mu_0}}\right)^{1/3}. \quad (11)$$

The last task to complete our calculation is to verify that the critical decoupling velocity indeed falls into a large velocity interval. To this end, note that the large velocity condition $\tilde{v}_c \gg \sqrt{\mu}$ reduces at the instability point to $V^2 \gg 8\sqrt{\pi}\,\mu_0/e^2$, which is just the condition of the strong disorder assumed, and therefore our assumption is justified (one has to bear in mind that, in our dimensionless units, $\mu \ll V^2 \ll 1$). The critical velocity (11) agrees with the result of [11] suggested by nice scaling arguments if one substitutes $\Delta(v)$ from (4) instead of the unspecified mean squared pinning strength g of [11].

To understand the meaning of the obtained result let us construct the F-v dependence starting with the ascending branch. Below the 2D critical force F_c^{2D} the *decoupled* system remains pinned. At $F = F_c^{2D}$ the system undergoes a smooth depinning transition into the plastically moving state in which the system remains decoupled and each layer moves independently. Upon further increase of the drive, the mean velocity of the CDW reaches the critical value, and the system gets coupled into a 3D moving state. Note that since, in the 3D regime pinning force experienced by the moving CDW, $F_p^{3D}(v)$ is less then the corresponding pinning force in the 2D regime, $F_p^{3D}(v) \simeq F_c^{2D}(v)/\sqrt{L_\perp(v)} < F_p^{2D}(v)$, where $L_\perp(v)$ is the (velocity dependent) correlation length across the layers, the pinning correction to the 3D velocity is *smaller* than the corresponding correction to the 2D velocity. As a result, the 3D branch of the F-v dependence lies *above* the $v_{2D}(F)$ curve, and the transition from the plastic to the coupled elastic motion at $v = v_c$ upon increasing drive acquires an *abrupt switching character* (see Fig. 1). Going down from the high velocities, the system follows first the elastic 3D behavior with $\mu \approx \mu_0$ and then, as velocity decreases to $v = v_c$, the system decouples and jumps down to the 2D branch corresponding to the plastic motion (see Fig. 1) at $F = F_{c(\mathrm{down})} < F_{c(\mathrm{up})}$. Therefore in the limit of strong pinning the transition from plastic to elastic motion is a switching hysteretic transition.

An immediate reservation regarding the proposed scenario is in order. The above picture suggests that the real *unique* F-v characteristic describing the dynamic behavior of the system is the S-shape-like curve, and, accordingly, up and down switchings occur at the instability points where $|dv/dF| = \infty$. To derive rigorously this kind of $v(F)$ dependence, a careful analysis in the vicinity of the critical point v_c, accounting for the interaction between

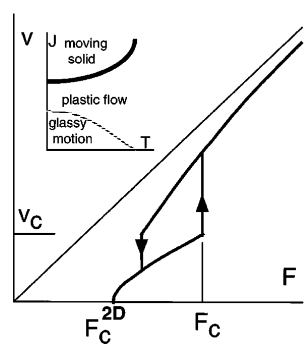

FIG. 1. v-F transport characteristic for the strong pinning at $T = 0$. The straight diagonal line displays viscous behavior in the absence of pinning. Inset: dynamic phase diagram for periodic medium driven through strong disorder. The dashed line denotes the critical depinning force $j_c(T)$ at which crossover from creep plastic flow takes place. The solid line denotes the switchinglike freezing transition at $j_f(T)$.

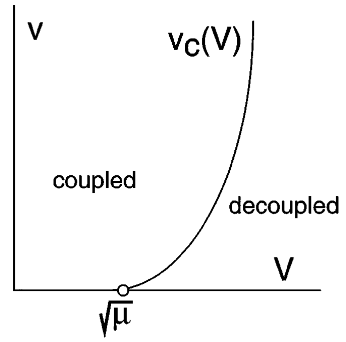

FIG. 2. v-V phase diagram: the system recovers coupling at $v > v_c(V)$, the large velocity dependence of $v_c(V)$ is given by Eq. (11).

the 2D and 3D modes and nonlinear pinning effects, is needed. Leaving the detailed derivation for a forthcoming publication, we point out that the related scenario, where the critical points merge and $v(F)$ retains only the inflection point near $v = v_c$, is also possible. In this degenerate case the plastic-elastic dynamic transition becomes nonhysteretic and can be detected by the position of the inflection point in $v(F)$ dependence.

Turning now to the case of *weak disorder*, $V < \mu^2$, one can easily verify that in this case there will always be a nonzero solution for μ (note that, for weak disorder, $F_c^{3D} = V^4/\mu < F_c^{2D} = V^2$). Combining this observation with Eq. (11), we conjecture the v-V diagram shown in Fig. 2.

Our discussion was restricted to $T = 0$, but the form of the F-v curve (S shape, for example) is determined by the intrinsic dynamic properties of the system and is stable with respect to thermal effects (as long as the latter leave the periodic structure intact). Another point is that, although our consideration was focused on an anisotropic CDW model, the form of the coarse-grained random force we used is not specific to CDWs but seems to be generic for any periodic structure subject to quenched disorder [16,18]. We expect therefore that the above ideas apply to a general case of periodic media driven through quenched disorder at finite temperatures. In particular, one can view the "strong depinning" switching scenario as the "zero-temperature projection" of the sequential depinning or nonequilibrium freezing transition in the vortex system subject to strong disorder as proposed in [9]. The instability force transcribes into a freezing force introduced for a moving vortex lattice, and the independent layered motion of 2D CDWs maps onto a regime of plastic flow at the intermediate currents $j_c < j < j_f$ (see inset in Fig. 1). The details of the finite temperature behavior, as well as the quantitative transcription of the ideas developed for the anisotropic CDW onto a general case of periodic driven media, will be presented elsewhere.

In conclusion, we described the depinning behavior in the driven periodic structures using the model of the layered CDW as a prototype system. We have found that in the case of strong disorder where the static state is decoupled the depinning occurs via a two stage process. First, the pinned system experiences a continuous depinning into a plastically sliding state and at higher drives undergoes a second sharp hysteretic transition into a coherently moving 3D state. The second transition is identified with the freezing transition proposed in [9]. For weak disorder, depinning occurs in a coupled state.

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