Nonlinear Kinetic Fluid Equations

Nathan Mattor[1] and Scott E. Parker[2]
[1]*Lawrence Livermore National Laboratory, Livermore, California 94550*
[2]*Department of Physics, University of Colorado, Boulder, Colorado 80309*
(Received 1 August 1997)

Fluid equations are derived that describe wave-particle resonances, which usually require kinetic theory. Unlike previous such efforts the closure is not linearized, and so retains many nonlinear wave-particle effects never before described by fluid equations. To demonstrate utility, we show that the new equations can retain trapping, time reversibility, and rectify a discrepancy between full kinetic theory and linear kinetic fluid closures. [S0031-9007(97)04396-2]

PACS numbers: 52.25.Dg, 52.35.Ra, 52.65.Kj

In plasmas, wave-particle resonances produce numerous effects. This includes linear Landau damping, nonlinear effects such as trapping, scattering, plasma echoes [1], and probably many others which are yet unknown. Studies of wave-particle resonances have traditionally employed *kinetic* equations, which describe the one-particle phase space distribution, $f(\mathbf{x}, \mathbf{v}, t)$. There have been efforts to simplify this by incorporating such effects in fluid equations. "Landau-fluid" equations [2,3] model linear Landau damping by adding *linear* terms to fluid equations. But this approach can miss *nonlinear* wave-particle effects, such as listed above, and so can miscalculate nonlinear phenomena like saturation amplitudes and transport [4–6]. To correct this, Ref. [7] introduced a *nonlinear* kinetic fluid closure, deriving a renormalized term to describe wave-particle resonances in Langmuir turbulence. This present paper makes a major advance, showing that moment equations can be closed with a simple integration trick, skipping most approximations of prior approaches. This suggests that fluid equations may be able to include *many* resonance effects previously thought to require kinetic theory.

Kinetic theory addresses resonant interactions by following particle velocities. However, this may be unnecessary because resonant particles just follow the phase velocity of the wave, which fluid equations already retain. This paper shows that the particle velocity can be integrated out of the kinetic equations without sacrificing resonances. Thus the only required knowledge is the phase velocities and the initial distribution function. This may make great simplification possible.

The first part of this paper gives an exact kinetic fluid closure for the drift kinetic equation. The method can apply to a number of other systems as well. The second part demonstrates the utility of this approach, showing that the closure rectifies a known discrepancy between kinetic solutions and linear kinetic fluid closures [5].

We begin with the drift kinetic equation, describing the evolution of a plasma in a strong magnetic field,

$$\partial_t f + (\mathbf{v}_\parallel + \mathbf{v}_E) \cdot \nabla f + E_\parallel \partial_{v_\parallel} f = 0 , \qquad (1)$$

where $f(\mathbf{x}, v_\parallel, t)$ is the distribution function for some species, $\mathbf{v}_\parallel$ is the kinetic velocity parallel to the magnetic field $\mathbf{B}$, $\mathbf{v}_E$ is the $\mathbf{E} \times \mathbf{B}$ velocity, and $E_\parallel$ is the electric field parallel to $\mathbf{B}$. Taking $f = F_0(x, v_\parallel) + \tilde{f}(\mathbf{x}, v_\parallel, t)$, and for simplicity assuming straight constant magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$, electrostatic field $\mathbf{E} = -\nabla\tilde{\phi}$, Maxwellian F_0 with density and temperature gradients in $\hat{\mathbf{x}}$, $F_0(x, v_\parallel) = n_0(2\pi T_0/m)^{-1/2} \exp[-m v_\parallel^2/2T_0]$, neglecting nonlinear parallel acceleration [8], and Fourier-Laplace transforming in $\mathbf{x}$ and t

$$-i(\omega - v_\parallel k_\parallel)\tilde{f}_\mathbf{k} + \sum_{\mathbf{k}'} \tilde{\mathbf{v}}_{E,\mathbf{k}''} \cdot i\mathbf{k}'\tilde{f}_{\mathbf{k}'} + i[k_\parallel v_\parallel - \omega_* + \omega_*^T(\tfrac{1}{2} - \tfrac{1}{2}v_\parallel^2)]F_{M\parallel}\tilde{\phi}_\mathbf{k} = 0 , \qquad (2)$$

where $-i\omega$ is the Laplace time conjugate, $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$, $k_\parallel = \mathbf{k} \cdot \mathbf{B}/B$, $\omega_* = k_y/L_n$, $\omega_*^T = -k_y/L_T$, $L_n^{-1} = d\ln n_0/dx$, $L_T^{-1} = d\ln T_0/dx$, $F_{M\parallel} = n_0 \exp(-v_\parallel^2/2)/\sqrt{2\pi}$, and units are so that $eB/mc = T_0 = m = 1$, and $v_{\text{th}}^2 = T_0/m = 1$. We regard Eq. (2) as exact for the purpose of deriving the fluid equations. A number of other kinetic equations could also serve as a starting point; Eq. (2) is chosen for definiteness.

Fluid equations come from velocity moments, $\int dv_\parallel\, v_\parallel^n$, of Eq. (2), for $n = 0$, 1, and 2,

$$-i\omega\tilde{n}_\mathbf{k} + \sum_{\mathbf{k}'} \tilde{\mathbf{v}}_{E,\mathbf{k}''} \cdot i\mathbf{k}'\tilde{n}_{\mathbf{k}'} + i\omega_*\tilde{\phi}_\mathbf{k} + ik_\parallel\tilde{V}_{\parallel\mathbf{k}} = 0 , \qquad (3)$$

$$-i\omega\tilde{V}_{\parallel\mathbf{k}} + \sum_{\mathbf{k}'} \tilde{\mathbf{v}}_{E,\mathbf{k}''} \cdot i\mathbf{k}'\tilde{V}_{\parallel\mathbf{k}'} + ik_\parallel\tilde{\phi}_\mathbf{k} + ik_\parallel\tilde{p}_{\parallel\mathbf{k}} = 0 , \qquad (4)$$

$$-i\omega\tilde{T}_{\parallel\mathbf{k}} + \sum_{\mathbf{k}'} \tilde{\mathbf{v}}_{E,\mathbf{k}''} \cdot i\mathbf{k}'\tilde{T}_{\parallel\mathbf{k}'} - i\omega_*^T\tilde{\phi}_\mathbf{k} - ik_\parallel\tilde{V}_{\parallel\mathbf{k}} + ik_\parallel \int dv_\parallel\, v_\parallel^3\tilde{f}_\mathbf{k}/n_0 = 0 , \qquad (5)$$

where $\tilde{n}_\mathbf{k} = \int dv_\parallel\, \tilde{f}_\mathbf{k}$, $n_0\tilde{V}_{\parallel\mathbf{k}} = \int dv_\parallel v_\parallel \tilde{f}_\mathbf{k}$, $\tilde{p}_{\parallel\mathbf{k}} = \int dv_\parallel v_\parallel^2\tilde{f}_\mathbf{k}$, and $p_\parallel = nT_\parallel$. Closure is now needed, to express $\int dv_\parallel v_\parallel^3\tilde{f}_\mathbf{k}$ in terms of lower moments.

The closure proceeds as follows. First, we streamline notation by representing Eq. (2) in matrix form, as

$$(\Omega - \Gamma - v_\parallel\mathcal{K}_\parallel)\vec{f} = \Omega_*(v_\parallel)F_{M\parallel}\vec{\phi} . \qquad (6)$$

Here $\vec{f}$ and $\vec{\phi}$ are column vectors of $\tilde{f}_{\mathbf{k}}$ and $\tilde{\phi}_{\mathbf{k}}$ for different $\mathbf{k}$, Ω and $\mathcal{K}_\parallel$ are diagonal matrices of ω and $k_\parallel$, Γ is a matrix of nonlinear coupling coefficients, $\Gamma_{\mathbf{k},\mathbf{k}'} \equiv \hat{\mathbf{v}}_{E,\mathbf{k}''} \cdot \mathbf{k}'$, and $\Omega_*(v_\parallel)$ is the diagonal matrix of $k_\parallel v_\parallel - \omega_* + \omega_*^T(\frac{1}{2} - \frac{1}{2}v_\parallel^2)$. All matrices are diagonal except Γ. A number of other kinetic equations can be cast in this form, and so the following derivation, up to Eq. (10), is fairly general. Inverting the evolution operator in Eq. (6) gives

$$\vec{f} = -(v_\parallel - \mathcal{W})^{-1}\mathcal{K}_\parallel^{-1}\Omega_*(v_\parallel)F_{M\parallel}\vec{\phi}\,, \qquad (7)$$

where $\mathcal{W} \equiv \mathcal{K}_\parallel^{-1}[\Omega - \Gamma]$ is the *nonlinear phase velocity matrix*, a central object in this theory, and a generalization of the $\omega/k_\parallel$ appearing in linear kinetic theory. Now we take two particular moments of Eq. (7), obtaining

$$\int dv_\parallel (v_\parallel^3 - 3v_\parallel)\vec{f} = \partial_\alpha^3\left[\int_{C_L} dv_\parallel \frac{F_{M\parallel}}{v_\parallel - \alpha - \mathcal{W}}\right]_{\alpha=0} \times \mathcal{K}_\parallel^{-1}\Omega_*(\mathcal{W})\vec{\phi}\,, \qquad (8)$$

$$\int dv_\parallel (v_\parallel^2 - 1)\vec{f} = -\partial_\alpha^2\left[\int_{C_L} dv_\parallel \frac{F_{M\parallel}}{v_\parallel - \alpha - \mathcal{W}}\right]_{\alpha=0} \times \mathcal{K}_\parallel^{-1}\Omega_*(\mathcal{W})\vec{\phi}\,, \qquad (9)$$

where C_L is the Landau contour, giving analytic continuation as discussed below. To derive Eqs. (8) and (9), one uses $(v^3 - 3v)F_M(v) = -\partial_v^3 F_M(v)$ and $(v^2 - 1)F_M(v) = \partial_v^2 F_M(v)$, a special property of Maxwellian distributions that relates moments and derivatives. This allows integration by parts of the right side, after which we have applied the identity

$$\partial_v^m \frac{v^n}{v - \mathcal{W}} = (-\partial_\alpha)^m \frac{\mathcal{W}^n}{v - \alpha - \mathcal{W}}\bigg|_{\alpha=0} \quad (m \geq n)\,.$$

This identity shows that closing the fluid equations at the $n = 2$ moment is related to the appearance of $v_\parallel^2$ as the highest power in $\Omega_*(v_\parallel)$. Solving Eqs. (8) and (9) for the $v_\parallel^3$ moment, applying definitions of the fluid moments, and substituting into Eq. (5) yields

$$-i\omega\tilde{T}_{\parallel\mathbf{k}} + \sum_{\mathbf{k}'}\tilde{\mathbf{v}}_{E,\mathbf{k}''} \cdot i\mathbf{k}'\tilde{T}_{\parallel\mathbf{k}'} - i\omega_*^T\tilde{\phi}_{\mathbf{k}} + 2ik_\parallel\tilde{V}_{\parallel\mathbf{k}} + ik_\parallel\sum_{\mathbf{k}'}B_{\mathbf{k},\mathbf{k}'}\tilde{T}_{\parallel\mathbf{k}'} = 0\,, \qquad (10)$$

where we have defined

$$B(\mathcal{W}) \equiv -Z_0^{[3]}(\mathcal{W}/\sqrt{2})[\sqrt{2}\,Z_0^{[2]}(\mathcal{W}/\sqrt{2})]^{-1},$$

Z_0 is the analytically continued plasma dispersion function

$$Z_0(\mathcal{W}/\sqrt{2}) \equiv \frac{1}{\sqrt{\pi}}\int_{C_L} dv_\parallel \frac{e^{-v_\parallel^2/2}}{v_\parallel - \mathcal{W}}\,,$$

the superscripts $^{[3]}$ and $^{[2]}$ denote derivatives evaluated with α as in Eqs. (8) and (9), $B_{\mathbf{k},\mathbf{k}'} \equiv \vec{e}_{\mathbf{k}}^{\,\dagger}B(\mathcal{W})\vec{e}_{\mathbf{k}'}$, and $\vec{e}_{\mathbf{k}}^{\,\dagger}$ and $\vec{e}_{\mathbf{k}'}$ are unit row and column vectors.

Equation (10) is the central result of this paper, representing a fluid closure which fully retains nonlinear resonances. It reduces to the Chang-Callen closure [3] when $\Gamma = 0$ in $\mathcal{W}$, and the Hammet-Perkins closure [2] when $\Omega = \Gamma = 0$ in $\mathcal{W}$. There are a number of directions to go from this point. Here we will discuss some general features and then demonstrate the utility with the specific example of a three-mode ITG problem.

One might think that such a closure is impossible, that a description of wave-particle interactions must retain some aspect of particle motion. In reality, we have not really eliminated any complication; we have only integrated $v_\parallel$ out of the equation. The details of the kinetic interactions still exist in B, which is a complicated function of the frequency spectrum. What this procedure has done is to replace the usual complication in *particle* velocity by complication in *phase* velocity. Since there are typically many fewer waves than particles, and since fluid equations track the wave velocities anyway, this can simplify computation. However, Eq. (10) is clearly still not useful, as several issues must be addressed to make it tractable, among them handling the function of a matrix, $B(\mathcal{W})$, handling the analytic continuation of Z_0, and evolving a complicated function of frequency.

As a scalar function, $B(w)$ is mostly a smooth function of w, with only one pole, at $w \to \infty$. $B(w)$ has a rapid jump across the lines $w_i < -|w_r|$ in the complex w plane [9]. As a matrix function, $B(\mathcal{W})$ requires some explanation. $B(\mathcal{W})\vec{T}$ can be reduced to a scalar function by dividing $\vec{T}$ into *eigenvectors*. More specifically, if $\{\vec{\xi}_j\}$ is a complete set of eigenvectors of $\mathcal{W}$, such that $\mathcal{W}\vec{\xi}_j = w_j\vec{\xi}_j$, and $\vec{T} = \sum_j T_j\vec{\xi}_j$, then

$$B(\mathcal{W})\vec{T} = \sum_j B(w_j)T_j\vec{\xi}_j\,. \qquad (11)$$

So the solution involves an eigenvector problem. For low order systems, such as the three-mode problem below, eigenvectors can be found analytically. For systems with a large number of modes, perturbation methods or numerics can be used. We briefly mention two possibilities. The first is to divide $\mathcal{W} = \mathcal{V} - \varepsilon\mathcal{U}$, where $\mathcal{V} = \mathcal{K}_\parallel^{-1}\Omega$ is the diagonal phase velocity, $\mathcal{U}$ is the nonlinear coupling, and $\varepsilon \ll 1$. In this case, the eigenvalue problem becomes

$$[\mathcal{V} - w_j]\vec{\xi}_j = \varepsilon\mathcal{U}\vec{\xi}_j\,.$$

The left side is simple and the right side is small, which is the form needed to apply the "Fredholm perturbation formula." Reference [10] calculates this to several orders. The second method, relevant numerically, is an iterative procedure where eigenvectors are calculated at each time step as a small perturbation on the previous values.

We next discuss analytical continuation of $B(\mathcal{W})$. For a scalar argument, say $w = \omega/k$, this is standard [11]; Z_0 becomes a weak function of two variables, with $Z_0(w/\sqrt{2}, k) = Z(w/\sqrt{2})$ for $k > 0$, and $Z_0(w/\sqrt{2}, k) = -Z(-w/\sqrt{2})$ for $k < 0$, where Z is the usual scalar argument plasma dispersion function. The function $B(w)$ becomes $B(w, k) \equiv$

$B_+(w)$ for $k > 0$ and $B(w,k) \equiv B_-(w)$ for $k < 0$. These satisfy the symmetries $B_+(w) = -B_+^*(-w^*) = B_-^*(w^*)$. This can be generalized to matrix arguments, $Z_0(\mathcal{W})$, by first decomposing into eigenvalues, $Z_0(w_j)$, as in the above paragraph. Following analytic continuation, then if w_j is continuously connected to a linear-limit eigenvalue with $k > 0$, then $Z_0(w_j) = Z(w_j)$; if $k < 0$, then $Z_0(w_j) = -Z(-w_j)$. If fluctuations grow from linear amplitude, one can trace the eigenmodes continuously and maintain the same branch of Z_0. Difficulty arises when a $k > 0$ and a $k < 0$ eigenvector become degenerate. This is discussed a bit in the example below.

A final general comment about Eq. (10) concerns the utility of an equation that is such a complicated function of frequency. More specifically, with ω inextricably embedded in $B(\mathcal{W})$, it seems hopeless to transform Eq. (10) back to the time domain. One might try solving entirely in frequency space instead of time, but what is really desired is a time domain equation, soluble with the usual partial differential equation algorithms. Currently we favor the ansatz of transforming the fluid part of the temperature equation back to the time domain, which is routine, and in B evaluating ω in eikonal fashion, as

$$i\omega_{\mathbf{k}} = \partial_t \ln \tilde{\phi}_{\mathbf{k}} . \tag{12}$$

For linear normal modes this method is exact. Nonlinearly the frequency spectrum broadens, and Eq. (12) obtains some sample of this spectrum. Keeping time histories could incorporate fuller spectra if more accuracy is needed. Choosing $\tilde{\phi}_{\mathbf{k}}$ in Eq. (12) is motivated by three considerations. Mathematically, in Eq. (7) ω originates essentially from a time-history integral over $\tilde{\phi}$. Physically, it is the contours of the potential $\tilde{\phi}$ (not $\tilde{n}$, $\tilde{V}$, or $\tilde{T}$) that resonate with the particle. With Eq. (12) $B(\mathcal{W})$ becomes purely a function of $\tilde{\phi}$. Practically, we find that $\partial_t \ln \tilde{\phi}_{\mathbf{k}}$ gives a more continuous solution than $\partial_t \ln \tilde{T}_{\mathbf{k}}$, which can have abrupt losses of roots.

We now demonstrate how this approach is an improvement over linear models with a specific calculation. We consider the "3-mode ion temperature gradient driven (ITG) system," similar to that of Lee and Tang [12] except bounded. This is one of the simplest systems with linear agreement but nonlinear disagreement between kinetic and Landau fluid models [5]. The equilibrium has a straight tilted magnetic field $\mathbf{B} = B(\hat{z} + \Theta\hat{y})$, and a Maxwellian ion distribution with density and temperature gradients along $\hat{\mathbf{x}}$. There are perturbations in $\tilde{f}$ and $\tilde{\phi}$, which are periodic in y, vanish at $x = 0, L_x$, and have $k_z = 0$ so that $k_\parallel = \Theta k_y$. The spectrum is truncated at three modes, with wave numbers $\mathbf{k} = (k_x, k_y), (k_x, -k_y), (2k_x, 0)$, denoted $+$, $-$, and 0, respectively, and their conjugates. Vanishing at $x = 0, L_x$ gives the relations $\tilde{f}_- = -\tilde{f}_+^*$ and $\tilde{\phi}_- = -\tilde{\phi}_+^*$, reducing these to two independent modes, and $\mathrm{Re}(\tilde{f}_0) = \mathrm{Re}(\omega_0) = 0$. We also take $\tilde{\phi}_0 \equiv 0$. Electrons are adiabatic and quasineutral, with $T_e = T_i$, so $\tilde{\phi} = \tilde{n}_e = \tilde{n}_i$. The drift kinetic equation for this system, Eq. (2), in vector form, is

$$-i[\Omega - v_\parallel \mathcal{K}_\parallel]\vec{f} - \begin{pmatrix} \kappa\tilde{\phi}_+ \\ -\kappa\tilde{\phi}_- \end{pmatrix}\tilde{f}_0 = -i\Omega_*(v_\parallel)F_{M\parallel}\vec{\phi} , \tag{13}$$

$$-\omega_0\tilde{f}_0 - (\kappa\tilde{\phi}_- \quad -\kappa\tilde{\phi}_+)f = 0 , \tag{14}$$

where $\vec{f}$ and $\vec{\phi}$ are two-element matrices of $\tilde{f}_\pm$ and $\tilde{\phi}_\pm$, and $\kappa = 2k_x k_y$. Equation (14) for $\tilde{f}_0$ has no resonant contributions, and is kept separate because $\mathcal{W}$ is undefined when $k_\parallel = 0$. Substituting $\tilde{f}_0$ from Eq. (14) into Eq. (13) gives

$$[v_\parallel - \mathcal{W}]\vec{f} = -\mathcal{K}_\parallel^{-1}\Omega_*(v_\parallel)F_{M\parallel}\vec{\phi} , \tag{15}$$

where

$$\mathcal{W} \equiv \begin{pmatrix} v_p - is_0|u| & u \\ u^* & v_p^* + is_0|u| \end{pmatrix}$$

is the effective nonlinear phase velocity matrix, $v_p = \omega_+/k_\parallel$, $u \equiv -\kappa^2\phi_+^2/k_\parallel\omega_0$, $s_0 \equiv \mathrm{sgn}[\mathrm{Im}(\omega_0)]$. Equation (15) now has the form of Eq. (7), and so the closed fluid equation, Eq. (10) follows directly. Calculating eigenvalues and eigenvectors of $\mathcal{W}$ gives temperature equations

$$\partial_t\tilde{T}_{\parallel+} - i\omega_*^T\tilde{\phi}_+ + 2ik_\parallel\tilde{V}_{\parallel+} + \kappa\tilde{\phi}_+\tilde{T}_{\parallel0} + ik_\parallel\tilde{q}_+ = 0 , \tag{16}$$

$$\partial_t\tilde{T}_{\parallel0} - \kappa[\tilde{\phi}_+^*\tilde{T}_{\parallel+} - \tilde{\phi}_+\tilde{T}_{\parallel+}^*] = 0 , \tag{17}$$

where the kinetic heat flux is given by

$$\begin{aligned} ik_\parallel\tilde{q}_+ &= -\Delta[\gamma_+\tilde{T}_{\parallel+} + \kappa\tilde{\phi}_+\tilde{T}_{\parallel0}] \\ &\quad + ik_\parallel\tfrac{1}{2}[B_+ + B_-]\tilde{T}_{\parallel+} , \\ \omega_\pm &= v_{p,r} \pm s_0 s_1\sqrt{|u|^2 - (v_{p,i} + s_0|u|)^2} , \\ \Delta &= \frac{B_+ - B_-}{\omega_+ - \omega_-} , \end{aligned}$$

$B_\pm \equiv B_\pm(\omega_\pm)$ and $s_0 s_1$ in $\omega_\pm$ follow from continuity arguments. Equations (16) and (17), together with the equations for $\tilde{n}$ and $\tilde{V}_\parallel$ [easily following from Eqs. (3) and (4)], form a closed system of fluid equations with full nonlinear representation of resonant particles. Linearizing $\tilde{q}_+$ *and* taking $s_0 = 1$ gives the Chang-Callen closure [3] $\tilde{q}_+ = B_+(v_p)\tilde{T}_{\parallel+}$, and further taking $v_p \to 0$ gives the Hammet-Perkins closure [2], $\tilde{q}_+ = i\sqrt{8/\pi}\,\tilde{T}_{\parallel+}$.

We have solved this system of equations numerically, using an adjustable time-step predictor-corrector scheme. For eikonal frequencies we use $-i\omega_+ = \partial_t \ln\tilde{\phi}_+$ [from Eq. (12)] and $-i\omega_0 = \partial_t \ln\tilde{T}_{\parallel0}$ [from Eq. (14)]. Equation (15) has singularities at points where Δ diverges, which can be traced to degeneracy in $\mathcal{W}$. This occurs when $w_+ = w_-$, i.e., when $v_{p,i} = 0$ or $v_{p,i} = -2s_0|u|$. Fortunately, these singularities present no difficulty here because $v_{p,i}$ apparently goes through such points linearly in time, $v_{p,i} \sim t - t_0$, meaning that the singularity goes

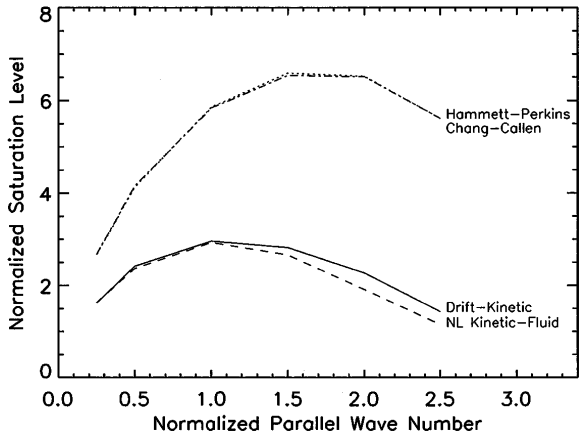

FIG. 1. Averaged potential, $|\tilde{\phi}_+|$, of the 3-mode ITG system for the four closures. Here, $k_x = k_y = 0.1$, and $\eta_i = 10$.

as $(t - t_0)^{-1/2}$, which has a convergent integral about $t = t_0$.

We compare four theories: full drift kinetics [Eqs. (13) and (14) solved with a Vlasov code], the present model, and the two linear Landau-fluid closures.

The numerical results demonstrate that the nonlinear closure corrects at least two deficiencies of the linear closures. First, Fig. 1 shows the time-averaged saturation amplitude of $|\tilde{\phi}_+|$ for the four models. The nonlinear closure is much closer to the kinetic amplitude than the linear closures, repairing the discrepancy found in Ref. [5]. Second, Fig. 2 shows that the kinetic and nonlinear fluid closures both exhibit "bounces" in potential, while in the linear closures the potential reaches a steady level which remains fixed in time. The bounces arise from resonant ions orbiting in the potential well of $\tilde{\phi}_+$ [13]. Ions near the center of the well orbit with nearly the same frequency. After a half orbit the density perturbation, $\tilde{n}_+ = \tilde{\phi}_+$, is maximal. After a full orbit, the density is largely restored and $\tilde{\phi}_+$ returns to its original value, near 0. Thus, these bounces demonstrate that the nonlinear closure retains *particle trapping.*

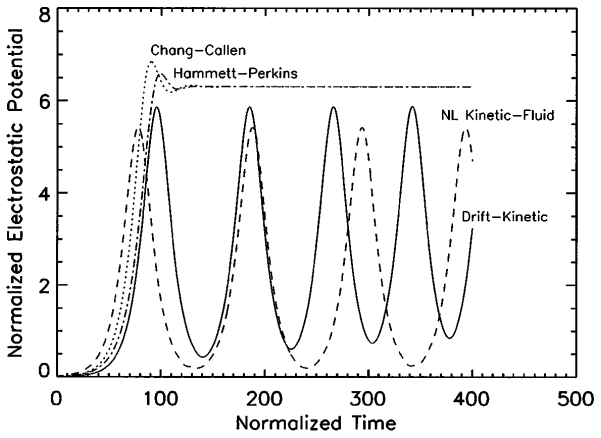

FIG. 2. Time evolution of $|\tilde{\phi}_+|$ for the four closures. Parameters as above, and $\Theta = 1.25$.

The trapping is a manifestation of a more fundamental property: *the nonlinear closure preserves the time reversal symmetry of the original kinetic equation.* This is verifiable by a time reversal transformation on Eq. (15), which reverses signs on ω, v_p, s_0, s_1, $\tilde{V}_\parallel$, $\tilde{v}_E$, $\mathbf{B}$, ω_*, ω_*^T, and κ, and takes $w_+ \leftrightarrow -w_-$ and $B_+(w_+) \leftrightarrow -B_-(w_-)$. In this case, $\tilde{q}_+$ reverses sign with the rest of Eq. (15). This is not the case for the linear closures, which in addition to linearization have no s_0 in $w_\pm$. This renders $B_\pm(w_\pm)$ irreversible, producing spurious damping.

In summary, we have given an exact fluid closure for the collisionless drift kinetic equation. This suggests that many resonance effects usually requiring kinetic equations can also be described by fluid equations. Numerical solution of the 3-mode ITG problem shows that the nonlinear closure is able to capture linear Landau damping and trapping, and maintains the reversibility of the original kinetic equation.

We are grateful to B. B. Afeyan, J. Cary, R. H. Cohen, A. M. Dimits, L. L. Lodestro, W. M. Nevins, and D. D. Ryutov for valuable discussions. We thank G. W. Hammet for providing us with the Vlasov code. This work was supported by U.S. DOE Contract No. W7405-ENG-48 (LLNL), and Grant No. DE-FG03-97ER54424 (University of Colorado), and contributes to the Numerical Tokamak Turbulence Project, a DOE Grand Challenge Application.

[1] R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory*, edited by T. M. O'Neil and D. L. Book (Benjamin, New York, 1969), pp. 89–117.
[2] G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
[3] Z. Chang and J. D. Callen, Phys. Fluids B **4**, 1167 (1992).
[4] N. Mattor, Phys. Fluids B **4**, 3952 (1992).
[5] S. E. Parker *et al.*, Phys. Plasmas **1**, 1461 (1994).
[6] A. M. Dimits *et al.*, Phys. Rev. Lett. **77**, 73 (1996).
[7] S. E. Parker and D. Carati, Phys. Rev. Lett. **75**, 441 (1995).
[8] E. A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).
[9] The physical origin of this jump is a kinetic transient, which vanishes more slowly than the exponentially damped mode, even in the "fluid limit," $|w| \gg 1$. Fully developed turbulence does not generally encounter this region, because both the transients and such modes generally damp away in the initial stages of evolution.
[10] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), pp. 1018–1026.
[11] T. H. Stix, *Waves in Plasmas* (American Institute of Physics, New York, 1992), pp. 199–216.
[12] W. W. Lee and W. M. Tang, Phys. Fluids **31**, 612 (1988).
[13] A. M. Dimits, Phys. Fluids B **2**, 1768 (1990).