Quasiperiodic Optical Lattices

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We study the kinetic temperature and the localization of cesium atoms in a three dimensional quasiperiodic optical potential created by the interference of five or six laser beams. Bragg scattering experiments show evidence of a quasiperiodic order for the atomic density. Temperature measurements are consistent with the topological invariance of the optical potential under phase variations of the laser beams. Numerical semiclassical Monte Carlo simulations show results in reasonable agreement with the experimental observations. [S0031-9007(97)04387-1]

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The discovery of quasicrystals by Shechtman *et al.* in 1984 [1] has opened a research field of considerable importance. Quasicrystals are long-range ordered materials that are not invariant under space translation [2,3]. For this reason, the electronic wave functions as well as the macroscopic properties of such materials display intriguing new features compared to crystals which possess translational and rotational symmetries [4]. The long-range order of quasicrystals appears, for example, in Bragg scattering experiments [1] where sharp diffraction peaks are observed. In optics, such kinds of long-range order can also be achieved by combining several monochromatic traveling waves to create an interference pattern. Indeed such a light field was used to trap polystyrene microspheres in a two dimensional (2D) quasiperiodic pattern by Burns *et al.* [5]. More recently, several groups used the interference pattern of several laser beams to cool and trap atoms in a lattice of micron-sized optical potential wells originating from the space-dependent light shift of the atomic levels. However, all the experiments on these structures, called optical lattices [6], used laser beam configurations that lead to *periodic* optical potentials. It is the aim of this paper to present an experimental and a numerical study of the behavior of cesium atoms in a quasiperiodic optical potential. We describe laser beam configurations suited to the generation of quasiperiodic potentials and show that atoms are efficiently trapped and cooled in such potentials. Using Bragg scattering [7,8], we show that the atomic density displays a quasiperiodic order. We also show that the atomic kinetic temperature does not depend on the relative phases of the laser beams in spite of the fact that phase variations are not equivalent to space translations for a quasiperiodic potential.

Consider an atom having a resonance frequency ω_0 interacting with n laser beams having the same frequency ω and wave vectors $\mathbf{k}_i$ ($i = 1, \ldots, n$). The light shift and the optical pumping at a given point are completely determined by the interference pattern of the laser beams. The space-dependent light shift acts as a potential for the atomic external degrees of freedom [9]. With four noncoplanar beams, one obtains a three dimensional (3D) periodic optical potential [10]. The resulting reciprocal space is spanned by three basis vectors $\mathbf{e}_i = \mathbf{k}_{i+1} - \mathbf{k}_1$ ($i = 1, 2, 3$) obtained from the differences between the wave vectors of the trapping beams [11]. If there are more than four beams, and if the expansions of $\mathbf{e}_4 = \mathbf{k}_5 - \mathbf{k}_1, \ldots, \mathbf{e}_n = \mathbf{k}_{n+1} - \mathbf{k}_1$ on $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ involve irrational numbers, the optical potential becomes quasiperiodic [12,13]. This potential can be described as the slice in $\mathbf{R}^3$ of a periodic potential in a higher dimensional space $\mathbf{R}^{n-1}$ exactly as in the case of solid state materials [14]. The role of the relative phases between the lattice beams is crucial here. In the case of a four-beam configuration, the topography is phase independent, and a phase variation just leads to a translation of the potential [10]. In the quasiperiodic case, a phase variation gives a translation of the potential in $\mathbf{R}^{n-1}$, but because the cut space $\mathbf{R}^3$ (i.e., the physical space) is fixed, one obtains a new cut that cannot be superposed on the initial one through a space translation. Nevertheless, in certain circumstances, the new potential is topologically similar to the initial one [15]. In the experiment, we start from the four-beam tetrahedron geometry shown in Fig. 1 [16]. The beams $\mathbf{k}_1$ and $\mathbf{k}_2$ (respectively, $\mathbf{k}_3$ and $\mathbf{k}_4$) propagating in the xOz (respectively, yOz) plane are polarized along $\mathbf{e}_y$ (respectively, $\mathbf{e}_x$), and the angle between the wave vectors is $2\Theta_x = 110° \pm 2°$ (respectively, $2\Theta_y = 110° \pm 2°$). We add to this configuration either a traveling or a standing wave along the z axis (see Fig. 1). All the beams have the same wavelength $\lambda = 852$ nm and the supplementary wave is σ^+ polarized with respect to Oz. The resulting optical potential is still periodic in the x and y directions but becomes quasiperiodic in the z direction. In the case of the traveling wave ($\mathbf{k}_5 = k\mathbf{e}_z$), we have a five-beam configuration, and for the standing wave ($\mathbf{k}_5 = k\mathbf{e}_z, \mathbf{k}_6 = -k\mathbf{e}_z$) we have a six-beam configuration [17].

To analyze the distribution of atoms in the quasiperiodic lattice as well as their temperature, we first report the results of a semiclassical Monte Carlo simulation of the atomic motion for the five-beam configuration of Fig. 1.

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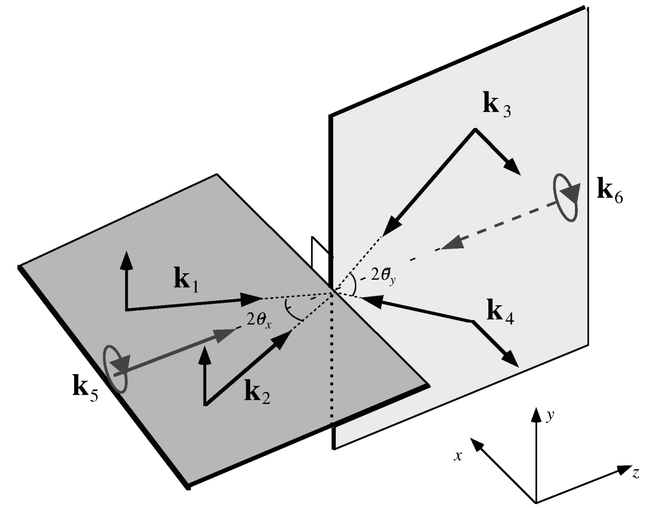

FIG. 1. Beam configuration used to create a quasiperiodic optical lattice. In the experiment, the angles are $\Theta_x = 55° \pm 1°$ and $\Theta_y = 55° \pm 1°$. The supplementary beams $\mathbf{k}_5$ and $\mathbf{k}_6$ are aligned along the z direction and are σ^+ polarized.

For the sake of simplicity, we consider the model situation of a $J = 1/2 \rightarrow J' = 3/2$ transition and constrain the atomic motion in the xOz plane [18]. These approximations were already used in the case of 3D periodic lattices and gave results in good agreement with experimental studies [19]. Moreover, as is usually done for quasicrystals [3,20], we use a rational approximant for the quasiperiodic lattice [the modulus of the projection of $\mathbf{k}_i$ ($i = 1, \ldots, 4$) along Oz is assumed to be $|\mathbf{k}_i \cdot \mathbf{e}_z| = 4k/7$]. With such an assumption, the potential becomes periodic but the spatial period along Oz (7λ) is much larger than the typical distance between two potential wells ($<\lambda/2$). Figure 2(a) shows a map of the atomic density in the xOz plane. Contrary to the case of periodic optical lattices where atoms are randomly distributed among the wells, the atoms are here located in a few wells which correspond to the deepest potentials (these wells have an elliptical polarization with a dominant σ^+ component because of the polarization of the fifth beam). For these deepest wells, the occupation probability is much larger than the average occupation probability in a periodic lattice. The Fourier transform of the atomic density that contains all the information for Bragg scattering experiments exhibits peaks of various intensity shown in Fig. 2(b). The comparison with the pattern [Fig. 2(c)] obtained with a four-beam periodic lattice ($I_5 = 0$) shows the occurrence of extra peaks in Fig. 2(b) characteristic of the quasiperiodic structure.

To check experimentally that the atomic density is quasiperiodic, we performed Bragg scattering on cesium atoms in the five-beam quasiperiodic lattice. The atoms are first cooled and trapped in a magneto-optical trap (MOT). The laser beams and the inhomogeneous magnetic field of the MOT are then switched off, and the five beams of the lattice (originating from the same laser

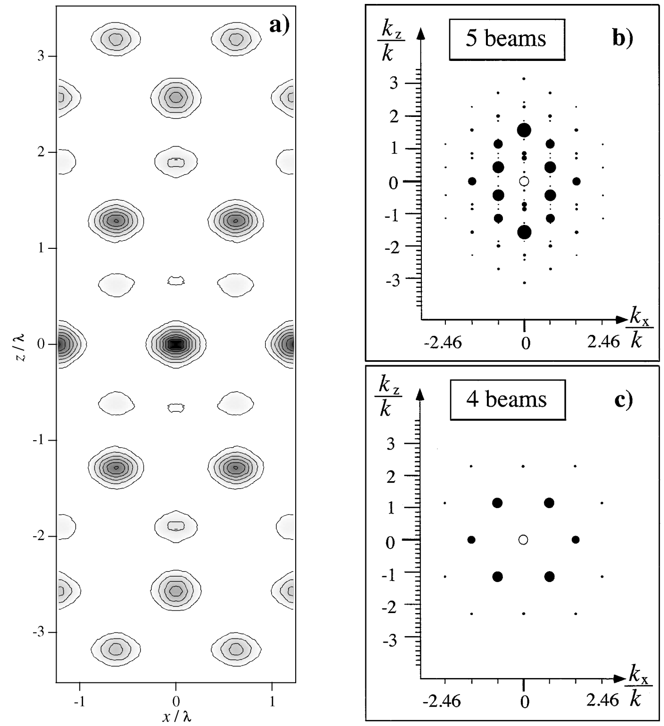

FIG. 2. (a) Atomic density in the case of a five-beam quasiperiodic optical lattice obtained from a 2D semiclassical Monte Carlo simulation for a $J = 1/2 \rightarrow J' = 3/2$ atomic transition (and $\Delta_1' = 100E_r$, $\Delta = -30\Gamma$). The five beams have the same intensity and detuning. (b) Atomic density in the reciprocal space (i.e., Fourier transform of (a) for the five-beam quasiperiodic lattice). (c) Atomic density in the reciprocal space for a periodic optical lattice ($I_5 = 0$). Natural units are $k/7$ for the z axis and $k\sin(55°) \simeq 0.82k$ for the x axis. The area of the dots is proportional to the peak intensity. The weakest peaks are not shown.

diode) are applied to the atoms. As shown in solid state textbooks, the Bragg condition states that the difference between the wave vectors of the incident and diffracted beams belongs to the reciprocal lattice. Because the reciprocal lattice is spanned by vectors $\mathbf{e}_i = \mathbf{k}_{i+1} - \mathbf{k}_1$, a probe beam P having almost the same wave vector as a lattice beam $\mathbf{k}_p \simeq \mathbf{k}_5$ is diffracted in directions close to the other lattice beams $\mathbf{k}_i$ [7,10]. In the experiment, the five beams of the quasiperiodic lattice have intensities I up to 15 mW/cm^2 and a detuning $10\Gamma \leq |\Delta| \leq 30\Gamma$ from the $6S_{1/2}(F = 4) \rightarrow 6P_{3/2}(F' = 5)$ transition ($\Gamma/2\pi = 5.3$ MHz is the natural width of the upper level). The probe beam is nearly parallel to $\mathbf{k}_5$, its intensity and detuning being $I_p = 50\ \mu$W/cm^2, $\Delta_p = -\Gamma$. The large frequency difference [typically $(\Delta_p - \Delta)/2\pi > 100$ MHz] between the probe and lattice beams ensures that no resonant four-wave mixing process [10] disturbs the measurements [8]. Note that in these experimental conditions we are probing the peak at $k_x = 0.82k$, $k_z = 3k/7$ of Fig. 2(b), i.e., a Fourier component of the density which is characteristic of the quasiperiodic order. The alignment of the probe beam along the fifth beam direction has an accuracy of about 0.5 mrad, and in order to maximize the scattered signal, the probe beam diameter is matched to the cloud dimension ($\simeq 1.5$ mm). We record the beat note at the frequency $\Delta_p - \Delta$ between the diffracted probe and a lattice beam using a fast photodiode coupled to a spectrum analyzer. The heterodyne detection sensitivity allows potential exploration of the domain of very low Bragg reflectivity. Moreover, by using the zero span feature of the spectrum analyzer it is possible to extract a time resolved Bragg signal. Bragg reflectivities on the order of 10^{-4}–10^{-3} are observed. Furthermore, it is possible to follow the buildup of the quasiperiodic order by varying the relative intensity I_5/I of the fifth beam. The experimental dependence of the Bragg reflectivity is in satisfactory agreement with the dependence found in the numerical simulation (Fig. 3) [21].

In another series of experiment, we measured the kinetic temperature of the cesium atoms. We used a ballistic method to measure the velocity distribution, the beam used for the measurement being located 15 cm below the lattice. For these experiments the lattice beams were applied for 150 ms, a time much longer than the time needed to reach steady state (less than 1 ms). The variation of the temperature versus the light shift $\hbar\Delta_1'$ per beam [22] is shown in Fig. 4. The temperatures obtained for the five-beam and six-beam quasiperiodic lattices are compared to the values found for the four-beam lattice ($I_5 = I_6 = 0$). The three curves of Fig. 4 have similar shapes. In particular, a linear dependence with a slope a_n ($n = 4, 5, 6$) is found for large light shifts. The ratios a_5/a_4 and a_6/a_4 are, respectively, equal to 1.55 ± 0.12 and 2.12 ± 0.11. The numerical simulation predicts a value 1.49 ± 0.06 for a_5/a_4 in good agreement with the experiment [23].

Another interesting measurement is the statistical distribution of temperatures for a time sufficiently long that the beam phases have drifted in a random way. Although the experimental setup is sufficiently stable to maintain constant relative phases over 1 sec, the relative phases have kept no memory of their initial values after a few minutes. We recorded the value of the temperature in the six-beam quasiperiodic lattice during two hours with a measurement made every 20 sec. For a given value of I and Δ, the temperature does not exhibit any significant variation and appears as a flat curve with a constant noise (which corresponds to a spread of temperature). In the range $15 \leq T \leq 25$ μK (and $\Delta = -10\Gamma$) that we studied, this spread is 0.3 μK (i.e., $1.5E_r$ where E_r is the recoil energy), a value identical to the spread measured in the case of a periodic lattice. This spread is probably due to intensity and frequency fluctuations in the laser diodes. This result shows that the modification of the optical potential due to phase variation has no measurable effect on the temperature. This is expected because the same statistical distribution of potential depths is found whatever the phases.

This experiment gave additional information on the capture efficiency in a quasiperiodic lattice. From the area of the time-of-flight signal, it was possible to deduce that the number of atoms captured in the quasiperiodic lattices is about 40% larger than in the four-beam lattice. By measuring the number of atoms as a function of the duration of the lattice phase, we found that the decay time of the six-beam quasiperiodic lattice is 0.6 sec, a value 20% larger than in the four-beam lattice in the same experimental conditions. The six-beam quasiperiodic lattice thus appears to be more efficient and more robust than the four-beam lattice [24]. These results may be associated with two observations in the numerical simulation: first an increase in the velocity capture range and second

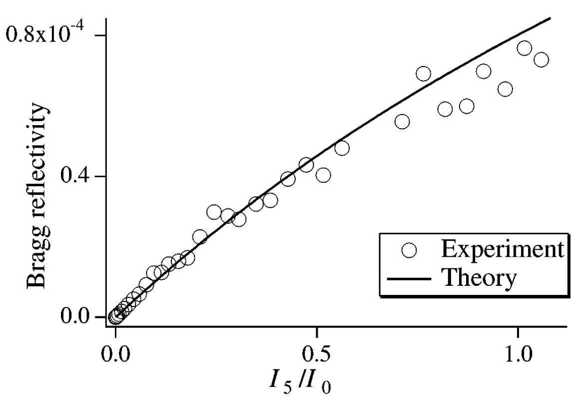

FIG. 3. Variation of the Bragg scattering reflectivity versus I_5/I. The experimental results for $\Delta = -30\Gamma$ and $I = 7.5$ mW/cm^2 are compared with a theoretical curve obtained from the 2D semiclassical Monte Carlo simulation. The Bragg reflectivity grows linearly with I_5/I for $I_5 \ll I$.

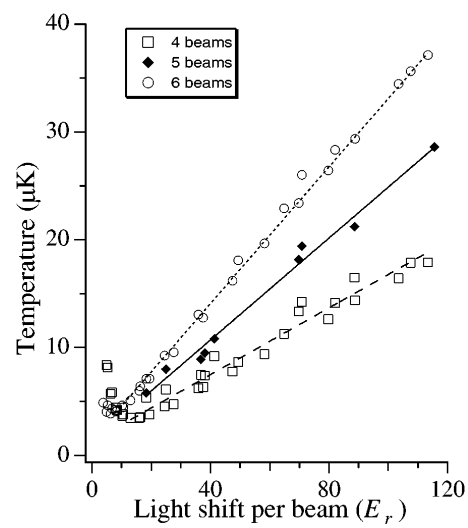

FIG. 4. Variation of the kinetic temperature of cesium atoms as a function of the light shift per beam $\hbar\Delta_1'$ for periodic and quasiperiodic optical lattices (the data were taken for $\Delta = -10\Gamma$, -20Γ, -30Γ and for several lattice beam intensities). For comparison with other experiments note that $\Delta' = 8\Delta_1'$ in a four-beam lattice.

a significant reduction in the spatial diffusion along the quasiperiodic direction.

Additional information is obtained using probe transmission spectroscopy [6,25]. The probe P' is aligned along Oz. As expected from the numerical study of the atomic density, the signal obtained with a σ^+ probe is larger than the signal obtained with a σ^- probe by about 1 order of magnitude. Because the depth and the curvature of the σ^+ wells are, on the average, larger than those of the σ^- wells, the Raman transition for a σ^+ probe is centered on a frequency Ω^+ larger than the one Ω^- found with a σ^- probe. Furthermore, following the idea of Weidemüller *et al.* [8], we checked that the Bragg reflectivity measured with the Bragg probe P is strongly reduced when the second probe beam P' is tuned to the Raman transition with an intensity sufficient to reach saturation.

Future experiments should include more complex structures such as a Penrose-like tiling [26]. One interesting feature of optical lattices is that we have a wide range of possibilities for the laser beam directions (and polarizations). In particular, contrary to the presently observed quasicrystals, it might be possible to achieve quasiperiodic potentials where different "local isomorphism classes" could be visited [27]. The study of atomic transport inside a quasiperiodic lattice is also very promising, particularly in the case of nondissipative optical lattices where quantum effects associated with the atomic wave function are crucial [28].

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[22] More precisely $\hbar\Delta'_1$ is the level shift of the Zeeman sublevel $m = J$ in a σ^+ circularly polarized traveling wave having intensity I. Because the beam polarizations are linear, the optical potential depth Δ' is $\Delta' = 2\Delta'_1$ in a 1D lin⊥lin lattice (see [9] and references therein) and $\Delta' = 8\Delta'_1$ in a four-beam lin⊥lin lattice (the field *intensity* at the bottom of the wells is 4 times larger than in the 1D case [11]).
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