Vortex-Nucleus Interaction and Pinning Forces in Neutron Stars

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We calculate the force that pins vortices in the neutron superfluid to nuclei in the inner crust of rotating neutron stars, relying on a detailed microscopic description of both the vortex radial profile and the inner crust nuclear structure. The contribution to the pinning energy from pair condensation is estimated in the local density approximation with realistic nucleon-nucleon interactions. The kinetic contribution, included inconsistently in previous approaches, is evaluated in the same approximation and found to be relevant. The vortex-nucleus interaction turns out to be attractive for stellar densities greater than $\sim 10^{13}$ g/cm^3. In this region, we find values for the pinning force which are almost 1 order of magnitude lower than the ones obtained so far. This has direct consequences on the critical velocity differences for vortex depinning.   [S0031-9007(97)04389-5]

The calculation of the interaction energy between a vortex and a nucleus has been of high concern since the vortex pinning model was proposed by Anderson and Itoh [1] to explain pulsar glitches, that is, sudden spin-ups in the neutron star rotation. The idea is to calculate the difference in energy between a configuration with the nucleus outside the vortex core and one with the nucleus at the center of the core. This is done by taking the configuration of a vortex alone as the one of zero energy, and then calculating the energies of the two configurations with the nucleus present. The first estimates [2] considered only the difference in pairing condensation energy, calculated in a crude model with uniform densities for both nuclear and vortex matter. As made clear by Epstein and Baym [3], however, the difference in energy between a vortex alone and one with a nucleus comes from two contributions, one of which is kinetic and the other condensational. These authors also introduced a realistic density profile for the nuclei present in the neutron star crust, which has a relevant effect on the results for the pairing energies. To date, their treatment is the most refined available in the literature, although, as discussed later, they use only the condensational contribution to evaluate the pinning energy.

The point of view of Epstein and Baym [3] was to use the Ginzburg-Landau approximation to evaluate the pairing properties of the superfluid crust. In this scenario, the core radius was taken to be ξ_{GL}, the Ginzburg-Landau order parameter. The conditions of applicability of the Ginzburg-Landau theory, however, are far from satisfied in the case under discussion. Indeed, the neutron star crust is practically a zero-temperature case ($T \sim 0.01$ MeV), while for the Ginzburg-Landau approach to be valid, the temperature of the system should be close to the transition one ($T_c \sim 0.5$ MeV). Moreover, the density variations due to the presence of the nucleus are quite steep, which is also in contrast with the requirements of the Ginzburg-Landau theory. As a

matter of fact, Epstein and Baym must rescale their results for the Ginzburg-Landau coherence lengths by factors in the range 2–12, in order to reproduce experimental condensation energies for ordinary nuclei.

For these reasons we felt the need to change the theoretical framework and use a more realistic approach to treat the radial dependence of the pairing gap in the inner crust of neutron stars. Our model is based on the local density approximation to evaluate the pairing properties of the system [4]. This approach, when applied to ordinary finite nuclei, gives realistic values for their condensation energies [5]. Application of this model to the case of the inner crust of neutron stars, where a lattice of neutron rich nuclei (described in terms of Wigner-Seitz cells) is permeated by a gas of unbound superfluid neutrons, can be found in Ref. [6]. A full BCS treatment of the problem, although more satisfactory, would bring about many difficulties, due to the different symmetries and yet comparable dimensions of the nucleus and the vortex core.

Superfluid matter in a straight vortex moves with a velocity field

$$\mathbf{v}(\mathbf{x}) = \frac{\hbar}{2m_N r} \, \mathbf{e}_\vartheta \,, \tag{1}$$

where r is the radial distance of the point $\mathbf{x}$ to the vortex axis, m_N is the nucleon mass, and $\mathbf{e}_\vartheta$ is the tangent unit vector. From this equation we can readily see the need for a layer of normal matter, called vortex core, surrounding the axis and corotating with the solid crust. This is so since the curl of the velocity field of a superfluid has to be zero everywhere. The field we are considering satisfies this condition at every point but on the axis. This singular behavior can be avoided by assuming that, along the axis, matter is not superfluid. This point can be understood also in another way. Equation (1) states that the velocity and the kinetic energy density of the superfluid tend to infinity as the axis is approached. This is clearly impossible, thus

indicating that at some point close to the axis neutron matter has to undergo a transition to a normal state, where it can be assumed to be static in a frame where the nuclear lattice is at rest.

This is the point of view we took to define the radius core. In this case, the distance where the transition occurs can be obtained equating the kinetic energy density, due to the rotation around the axis, to the condensation energy per unit volume. Closer to the axis the kinetic term increases rapidly, making it energetically unfavorable for matter to remain superfluid. The kinetic energy per unit volume is

$$\mathcal{E}_{kin} = \frac{\hbar^2 n}{8 m_N r^2}, \tag{2}$$

where $n = n(r)$ is the superfluid particle (neutron) density. Because of the superfluid state, a unit volume of matter has an energy lower by

$$\mathcal{E}_{cond} = -\frac{3\Delta^2 n}{8\varepsilon_F}, \tag{3}$$

compared to a unit volume of normal matter. Here $\Delta = \Delta(r)$ is the energy gap calculated in the semiclassical approximation, and $\varepsilon_F = \varepsilon_F(r)$ is the local Fermi energy. In the local density approximation, the different quantities depend parametrically on r via the local Fermi momentum (see Ref. [6]). Equating Eqs. (2) and (3) to zero, one gets an equation in r, whose solution is the transition radius R_t. This argument can be readily generalized to the case in which a nucleus is set at the center of the vortex core, thus modifying its structure. Numerical calculations, using a realistic neutron density profile as given by Negele and Vautherin [7], were performed to obtain the shape of the core. In this case, the transition radius $R_t = R_t(z)$ will depend also on the coordinate z, due to the spherical symmetry of the nucleus. In the actual calculations, we also took into account the density variation of the neutron superfluid due to the centrifugal potential induced by the rotation, as follows from the local density approximation.

As said before, the vortex alone was considered as the zero energy state. Setting a nucleus within the flow changes the density profile and the velocity field, thus causing a variation in the kinetic and condensation energy. Depending on where the nucleus is placed, the energies will be modified by a different amount. We considered the two cases of a nucleus right at the center of the core (case I) and just barely out of it (case II).

In case I, the kinetic term was obtained by a numerical integration of the kinetic energy density, given by Eq. (2). The condensation energy was obtained via a numerical integration of Eq. (3) over the volume occupied by the superfluid. In both cases, the realistic density profiles were used. We point out that Epstein and Baym [3] neglect the kinetic contribution in this case, while our results show that it is relevant. Incidentally, a simple calculation based on their approach (and in the simplified scenario of purely axial symmetry, i.e., with a "cylindrical" nucleus) gives a kinetic effect of magnitude comparable to ours.

Epstein and Baym [3] gave a good estimate of the kinetic energy when the nucleus is out of the vortex core, and we took that as the appropriate value. To find the condensation energy term in case II, we proceeded as before by numerical integration. We point out that Epstein and Baym, after calculating the kinetic contribution, do not include it in the evaluation of the pinning energies. In this sense their results effectively include only the pairing contribution calculated in the Ginzburg-Landau approximation.

Subtraction of the energy of a nucleus outside the core and that of one inside yields the pinning energy E_{pin}. The pinning force F_{pin} is defined as E_{pin} divided by the minimum distance between the nucleus and the vortex axis. This was taken to be $R_t + R_N$, where R_N is the nuclear radius.

We performed our calculations for different zones in the inner crust of the neutron star. The physical properties of these zones were obtained by Negele and Vautherin [7] and we report them in Table I. The calculations were done using different nucleon-nucleon residual interactions, namely, Argonne's potential [8] and Gogny's effective interaction [6], and with the nucleon effective mass varying with density. Incidentally, it turns out that setting the effective mass equal to that of a free nucleon does not change the results significantly.

In Tables II and III we report the results obtained. The transition radius R_t is the core radius of the vortex alone. In order to compare kinetic and pairing contributions, we give the values for $\Delta E_{kin} = E_{kin,out} - E_{kin,in}$ and $\Delta E_{cond} = E_{cond,out} - E_{cond,in}$, so that the pinning energy is $E_{pin} = \Delta E_{kin} + \Delta E_{cond}$. The subscript "in" refers to

TABLE I. Physical parameters of the four regions in the inner crust. The values are taken from Negele and Vautherin [7]. The baryon densities ρ_b of the four zones are given in g/cm^3, the densities of the free neutron gas n_{nG} in fm^{-3} and the radii of the nuclei R_N and those of the Wigner-Seitz cells R_{WS} in fm.

Zone	1	2	3	4
ρ_b	1.51×10^{12}	9.55×10^{12}	3.39×10^{13}	7.76×10^{13}
n_{nG}	4.79×10^{-4}	4.68×10^{-3}	1.82×10^{-2}	4.37×10^{-2}
R_N	6.0	6.73	7.32	6.72
R_{WS}	44.0	35.5	27.0	19.4

TABLE II. Results of the calculation with the Argonne interaction. The radii of the vortex core R_t are given in fm, the energies in MeV, while the pinning forces are in MeV/fm. As explained in the text, the pinning forces are given only for positive pinning energies, since in the threading regime they do not derive from the values of E_{pin} [9].

Zone	1	2	3	4
R_t	3.87	2.93	3.62	7.02
ΔE_{kin}	−2.59	0.52	5.36	1.25
ΔE_{cond}	−0.31	−0.42	0.63	2.69
E_{pin}	−2.90	0.10	5.99	3.94
F_{pin}		0.01	0.55	0.29

the state in which the nucleus is at the center of a vortex core, and "out" to the case of a nucleus whose center is at a distance $R_t + R_N$ from the core axis. When the pinning energy is positive, the vortex pins to nuclei. When the pinning energy is negative, the vortex tends to avoid nuclei in its path through the lattice. We refer to this scenario as threading (or interstitial pinning). Only in the pinning case can we calculate the pinning force as just described. In the threading case, instead, it is much easier for the vortex to move through the nuclear array, and the pinning force is orders of magnitude smaller than the values one would obtain from E_{pin} (cf. Ref. [9]).

A general look at the results shows that there is pinning on nuclei for densities greater than $\sim 10^{13}$ g/cm^3. This general trend is in agreement with what has been obtained so far in the literature. As already mentioned, the kinetic energy contributions are relevant, as can be seen from the relative values of ΔE_{kin} and ΔE_{cond}. In particular, due to the interplay between the spherical geometry of the nucleus and the cylindrical geometry of the vortex, the kinetic energy difference can be also negative. The Argonne and Gogny cases are quite similar, although Gogny gives pinning only at slightly larger densities. The fact that these very different interactions (Argonne is a bare nuclear potential; Gogny is an effective interaction) give results for the pinning that agree within a factor of 2 is gratifying, since the choice of the nucleon-nucleon interaction to be used in the calculations discussed here is an open and controversial issue.

We now compare our results with those obtained by other authors. In Table IV we report the values for

TABLE III. Results of the calculation with the Gogny interaction. The radii of the vortex core R_t are given in fm, the energies are in MeV, and the pinning forces in MeV/fm.

Zone	1	2	3	4
R_t	2.88	2.44	2.82	5.13
ΔE_{kin}	−3.90	−0.36	6.32	5.89
ΔE_{cond}	−0.28	−1.24	−0.37	1.60
E_{pin}	−4.18	−1.60	5.95	7.49
F_{pin}			0.59	0.63

TABLE IV. Results from the Ginzburg-Landau approximation. The pinning energies are taken from Epstein and Baym [3], the pinning forces from Link and Epstein [9]. The energies are given in MeV and the forces in MeV/fm.

Zone	1	2	3	4
E_{pin}	−4.4	0.4	15.0	9.0
F_{pin}		0.11	3.6	1.9

the pinning energies obtained by Epstein and Baym [3], as well as the results for the pinning force calculated by Link and Epstein [9], from those energies. We are reminded that the pinning energies of Epstein and Baym are only condensational (i.e., they correspond to the term ΔE_{cond}). We notice how their pairing energy differences are much larger than ours. This is due to the fact that, in order to reproduce experimental condensation energies for ordinary nuclei in the Ginzburg-Landau approach, they must divide their coherence lengths by factors in the range 2–12 (depending on the pairing gaps they use). In turn, this amounts to multiplying the condensation energies by factors in the range 4–144. Finally, after averaging the results obtained from two sets of pairing gaps ("Takatsuka" and "Chen et al." gaps), they obtain the "best estimates" for the pairing energy difference reported in Table IV. Numerically, however, the kinetic contribution included by us partially makes up for the difference, since it presents relevant positive values at larger densities.

To complete the comparison between our results and those obtained by Epstein and Baym in the Ginzburg-Landau approximation, we first observe that the pairing gaps calculated in neutron matter with the Argonne interaction [8] and those calculated by Takatsuka [10] are practically the same in the density range corresponding to the inner crust. Therefore, it is instructive to compare the difference in pairing energy ΔE_{cond} obtained in the present paper with the Argonne potential, and that obtained by Epstein and Baym with the Takatsuka gaps (which can be deduced from Table IV of Ref. [3]). These results are reported in Table V. The two sets of values differ by 1 order of magnitude, thus confirming the striking difference between the two approaches. We have already discussed how the local density approximation is expected to be a better approach than the Ginzburg-Landau one for the situation under study.

TABLE V. Difference in pairing energy ΔE_{cond}, obtained in this paper in the local density approximation with the Argonne interaction, and obtained by Epstein and Baym [3] in the Ginzburg-Landau approximation with the Takatsuka gaps [10]. The energies are given in MeV.

Zone	1	2	3	4
Argonne	−0.31	−0.42	0.63	2.69
Takatsuka	−4.2	−4.8	11.9	17.1

From a general look to the previous results, we see that our treatment gives pinning forces that are smaller than those obtained in the previous approaches by almost 1 order of magnitude. We point out that having too large values for the pinning force has been one of the problems of the vortex pinning model. In this sense, the results of our approach seem to go in the right direction.

In conclusion, we have proposed a microscopic model to calculate the vortex-nucleus interaction in the inner crust of rotating neutron stars. We have treated the pairing energies in a semiclassical approximation, which is better suited to deal with the system under discussion than the Ginzburg-Landau approach followed so far. We have also included the kinetic contribution to the pinning energy, which turns out to be relevant. We have used realistic density profiles for the Wigner-Seitz cells and different realistic nucleon-nucleon interactions to test their influence. We have defined the radius of the vortex core and the density profile of the rotating superfluid in a way which is consistent with the semiclassical approach followed. In particular, we have not introduced any arbitrary scaling factor in our model. We have obtained results that differ by almost 1 order of magnitude from those obtained in previous less refined approaches. These results are likely to have important effects in relation to pulsar glitches. For example, critical velocity differences for depinning are directly related to the pinning forces. These applications, however, are beyond the scope of the present work.

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