The Aharonov-Bohm Effect Revisited by an Acoustic Time-Reversal Mirror

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We take advantage of the violation of time-reversal invariance for acoustic waves to achieve a new way of characterizing a vorticity field with a double time-reversal mirror (TRM). In particular, we show experimentally that the double TRM works as a vorticity amplifier. In the case of a vorticity filament, the sound-vorticity interaction is interpreted as the acoustical analog of the Aharonov-Bohm effect. Numerical experiments simulate the acoustical Aharonov-Bohm effect by propagating a plane wave which is scattered by a heterogeneous motionless medium. [S0031-9007(97)04366-4]

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In 1959, Aharonov and Bohm [1] described the way in which a curl-free magnetic vector potential modifies the wave front structure of an electronic wave function that obeys the Schrödinger equation. They concluded that for electrons traveling outside an infinitely long cylinder enclosing a magnetic field, the wave fronts outside the cylinder would be dislocated by an amount proportional to the magnetic flux within the cylinder. In 1980, Berry *et al.* [2], reasoning by analogy, also concluded that such dislocated wave fronts should occur for surface waves when they encounter a vortex. The aim of this paper is to describe experimentally and numerically how an acoustic wave is scattered by a vorticity filament. In particular, we show that an acoustic time-reversal mirror (TRM) amplifies the effect of vorticity on the acoustic wave. This leads to a new method of characterizing small vorticity fields. First of all, we present the acoustic double TRM and the first results obtained with a large vortex. Then an experiment is performed with a vorticity filament in water. Results are interpreted using an analogy with quantum mechanics. Finally, we compute numerically the sound-vorticity interaction which confirms that the double TRM acts as a vorticity amplifier.

Time reversal is based on the invariance of the wave equation when the time t is changed to $-t$. Experimentally the wave—sent by an acoustic source—is first received by an array of transducers, time reversed, and then refocused on the source [3]. We have shown recently that this invariance is broken if the propagation medium contains a vorticity distribution [4]: the time-reversed wave no longer focuses at the source. The defocalization is proportional to the Mach number of the flow $M = U/c$, where U is the characteristic velocity of the flow and c the velocity of the acoustic wave. If $M \ll 1$, as is the case for hydrodynamic flows, the defocalization is not strong enough to be detected. In order to amplify the effect of vorticity on the acoustic wave, we use a double TRM made up of two piezoelectric transducer arrays placed in front of each other on either side of the flow. First emission corresponds to a plane wave. Then, when one array emits, the other one receives, time reverses, and reemits the acoustic wave. In the absence of fluid motion, the double TRM ensures that the wave remains plane after several round trips. In the presence of a vortex, the initially plane wave is slightly distorted at each crossing, and this distortion increases linearly with the number of round trips. The analysis of this distortion allows us to solve the inverse problem, i.e., to define the vorticity field of the flow [5]. In fact, the double TRM appears as an artificial amplifier of vorticity: After N round trips through a vorticity field $\Omega(\vec{r})$, the wave front shape is equivalent, to first order, to the shape obtained after a single crossing through a vorticity field $2N\Omega(\vec{r})$. First, experiments were conducted with a large vortex (diameter $\sim$ 80 mm) rotating at a frequency $\Omega/2\pi \sim 1$ Hz. The two arrays of the double TRM are placed on a plane perpendicular to the axis of the solid rotation (see Fig. 2 below). Each array is made of 64 transducers. The element size along the x axis is 0.39 mm, and the spacing between two elements is 0.42 mm. Thus the total aperture of each array is about 25 mm. Each transducer element has its own amplifier, an 8-bit analog-to-digital converter, a storage memory, and an 8-bit digital-to-analog converter working at a 20 MHz sampling rate. The double TRM works with a central frequency of 3 MHz (wavelength is equal to 0.5 mm in water). Figure 1 shows the phase distortion at 3 MHz from -50 to 50 mm. A model based

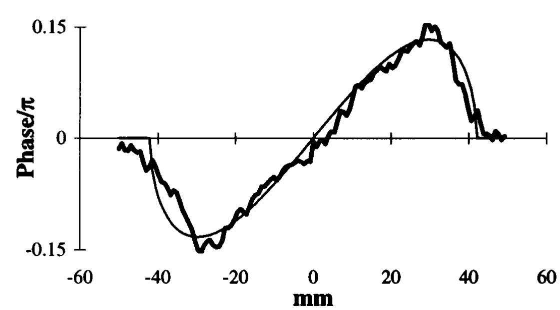

FIG. 1. Phase distortion after eight round trips through a large vortex (bold curve), and predicted distortion using geometrical acoustics model (thin curve).

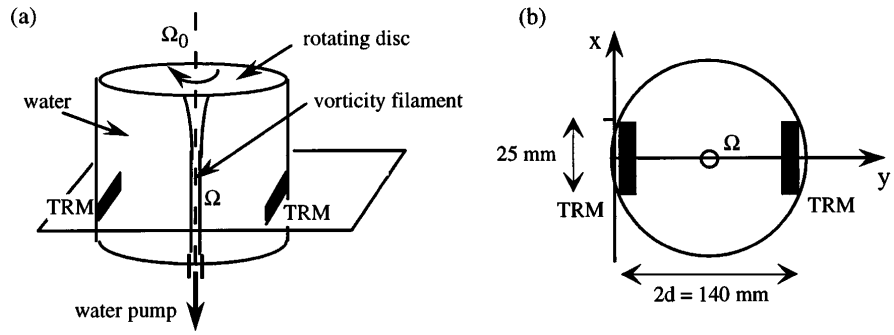

FIG. 2. Experimental setup with a vorticity filament (a) overall view, (b) double TRM in the $z = 0$ plane.

on geometrical acoustics describes quite simply the experimental results. In this approach, the wave velocity is locally modified by the flow velocity. At a given point in the vortex, we add to the wave velocity, c, the component of the flow velocity projected along the acoustic ray. If $\vec{U}(\vec{r})$ is the flow velocity and $\vec{n}(\vec{r})$ the unit vector tangential to the acoustic ray at position $\vec{r}$, we can write the new local wave velocity $c'(\vec{r}) = c + \vec{U}(\vec{r}) \cdot \vec{n}(\vec{r})$. Moreover, in first order approximation, we can neglect the deflection of the wave by the vortex, leading to $\vec{n}(\vec{r}) = \vec{e}_y$, where $\vec{e}_y$ is a y axis unit vector. The phase shift measured in the x direction is after N round trips

$$\frac{\Delta \varphi_N(x)}{2N} = -\frac{\omega}{c} \int_{-d}^{d} \frac{\vec{U}(\vec{r}) \cdot \vec{n}/c}{1 + \vec{U}(\vec{r}) \cdot \vec{n}/c} \, dy \,, \qquad (1)$$

where $2d$ is the distance between the two arrays and ω the angular frequency of the acoustic wave. Here, the large vortex is a flow in solid rotation such that $\vec{U}(\vec{r}) = \Omega r \vec{u}_\theta$ (where $\vec{u}_\theta$ is the angular unit vector). For small Mach number ($M \ll 1$), Eq. (1) can be simplified,

$$\frac{\Delta \varphi_N(x)}{2N} = -2\Omega \frac{\omega x}{c} \sqrt{R^2 - x^2} \,. \qquad (2)$$

Equation (2) shows that the phase shift is proportional to the product $N\Omega$, which confirms that the TRM acts as a vorticity amplifier. In addition, the fit between the theoretical and the experimental phase shifts (Fig. 1) provides the values for Ω and R ($\Omega/2\pi = 0.3$ Hz and $R = 42$ mm).

What happens now if a flow is induced by a vorticity filament? The experimental setup is as follows: A vorticity filament is generated in a cylinder ($R = 70$ mm) filled with water between a rotating disk placed at the top of the cylinder and a narrow tube at the bottom of the cylinder through which water is pumped out [Fig. 2(a)]. As before, the double TRM is placed on either side of the cylinder on a plane perpendicular to the axis of the vorticity field [Fig. 2(b)]. Acoustically speaking, the vorticity filament is generally penetrable but, for strong suction, the filament becomes a little air tornado which is impenetrable. It appears visually that the radius of the vortex is a bit smaller than the radius of the tube (2 mm). After a single crossing through the vorticity field, the incident plane wave front appears to be dislocated (Fig. 3). More precisely, Figs. 4(a) and 4(b) represent the phase and amplitude distortions at 3 MHz which show that the dislocation is modulated by a scattering effect due to the vortex core. From the theoretical point of view, this dislocation is predicted by the geometrical acoustics model used above. Indeed, the vorticity filament creates a flow such that $\vec{U}(\vec{r}) = (\Gamma/2\pi r)\vec{u}_\theta$ outside the vortex core, where Γ is the circulation of the flow. Using Eq. (1), the dislocation depends on a parameter α such as

$$\frac{\Delta \varphi_N(x)}{2N} = 2\alpha \tan^{-1}\left(\frac{d}{x}\right), \qquad \text{with } \alpha = -\frac{\omega \Gamma}{2\pi c^2} \,. \qquad (3)$$

However, geometrical acoustics is no longer valid in the core of the vortex because the filament size is of the same order as the acoustic wavelength. In this case, we study the sound-vorticity interaction using a wave approach built on the acoustical analog of the Aharonov-Bohm effect. Indeed, if we replace vorticity by a magnetic field, it appears that the Schrödinger equation, which describes the quantum interaction between a beam of particles and a magnetic field confined in a filament, is the analog, for small Mach number, of the sound propagation equation in

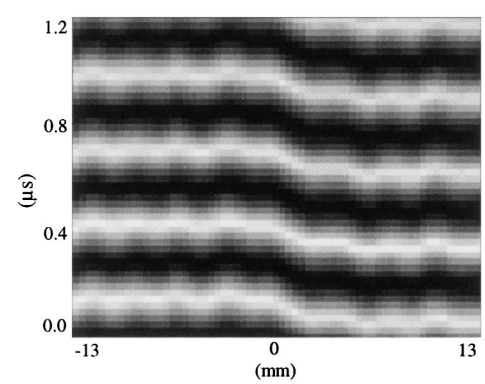

FIG. 3. Distortion of an incident plane wave after a single crossing through a vorticity filament.

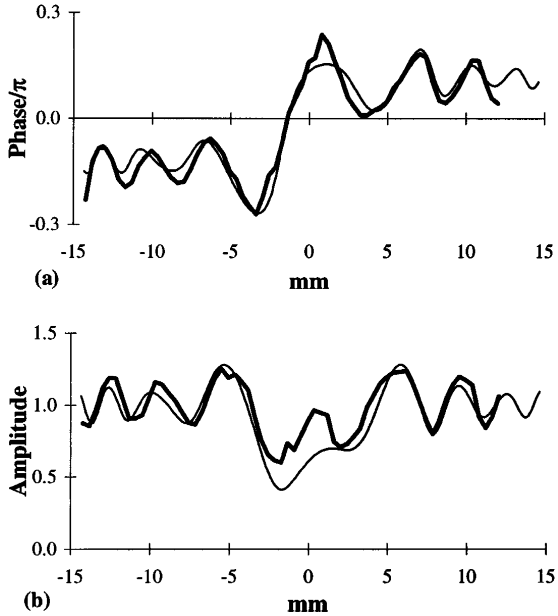

FIG. 4. (a) Phase distortion and (b) amplitude distortion of a plane wave through an impenetrable vorticity filament: experimental measurements (bold curve) and quantum approach (thin curve).

a filamentary vorticity field [2,5,6]. In the most general case, the quantum solution to the Schrödinger equation gives a complicated result [2,7]. Nevertheless, in the case of an infinite magnetic string (which corresponds to a vorticity filament whose core is smaller than the acoustic wavelength), the far field solution is quite simple:

$$\Psi_\alpha(r,\theta) = \exp[-ikr\cos(\theta) + i\alpha\theta] - \sin(\pi\alpha)\frac{\exp(i\theta/2)}{\cos(\theta/2)}\frac{\exp(ikr + i\pi/4)}{(2\pi kr)^{1/2}}. \quad (4)$$

The first term of the wave function Ψ_α describes the dislocation of the incident plane wave, whereas the second term puts in evidence the wave scattered by the vortex core. As expected, the acoustical analog of the quantum parameter α [2] is the parameter α which describes the phase dislocation in the geometrical acoustics model [see Eq. (3)]. For penetrable or impenetrable vortices of different core size, the fit between the quantum solution and the experimental measurements enables the characterization of the vorticity field. For example, we deduce from Fig. 4 the parameter $\alpha = 0.095$ and the radius of the vortex core $r_0 = 1.3$ mm with an accuracy of 10%. The great interest of this measurement lies in the fact that independent measurements of the vortex core with classical Doppler techniques (laser Doppler velocimetry or ultrasound Doppler velocimetry) are impossible [5].

The final point to verify is whether the TRM still acts as a vorticity amplifier in the case of a vorticity filament. The quantum approach, which only considers the interaction between a plane wave and a confined vorticity field, is not able to solve the problem of the interaction of the time-reversed wave with the same vorticity field. In order to do that, we built a numerical experiment with a finite differential code which simulates as closely as possible our experimental approach. The basic principle of this numerical experiment is, on the one hand, to solve numerically the propagation equation,

$$\Delta p(\vec{r},t) - \frac{1}{c'^2(\vec{r})}\frac{\partial^2 p(\vec{r},t)}{\partial t^2} = 0, \quad (5)$$

and, on the other hand, to take into account the vorticity field by modifying the wave velocity as we did previously with the geometrical acoustics model,

$$c'(\vec{r}) = c + \vec{U}(\vec{r})\vec{e}_y, \qquad \text{where } \vec{U}(\vec{r}) = \frac{\Gamma}{2\pi r}\vec{u}_\theta. \quad (6)$$

This amounts to considering the moving homogeneous medium as a motionless heterogeneous medium. Three aspects are then studied: First, we compare the phase and amplitude distortion measured numerically after a single crossing through the vorticity field to the quantum wave function calculated in the same configuration. The excellent agreement observed in Fig. 5 justifies the use of this numerical model to treat sound-vorticity interaction. Second, we observe the distortion of the incident plane wave front during its propagation through the flow (Fig. 6). This confirms that the scattered wave front can be interpreted as the interference between a wave scattered by the vortex core and a wave dislocated by the flow induced. Third, we compare the numerical acoustic field obtained in the two

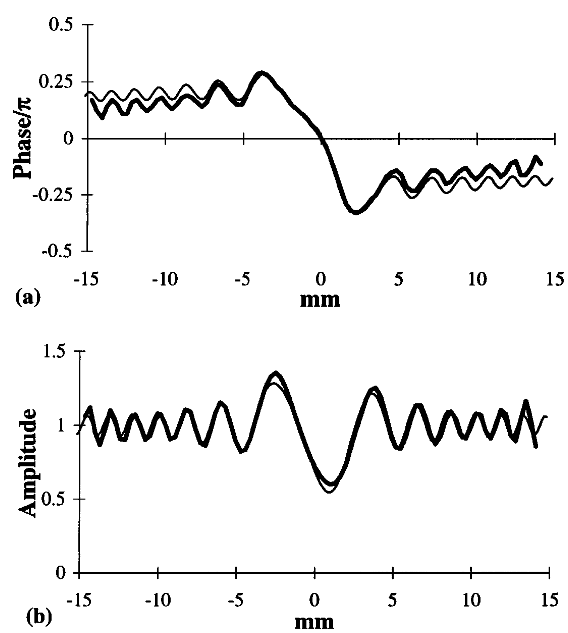

FIG. 5. (a) Phase distortion and (b) amplitude distortion of a plane wave through a penetrable vorticity filament: numerical experiments (bold curve) and quantum approach (thin curve); $\alpha = 0.25$ and $r_0 \ll \lambda$.

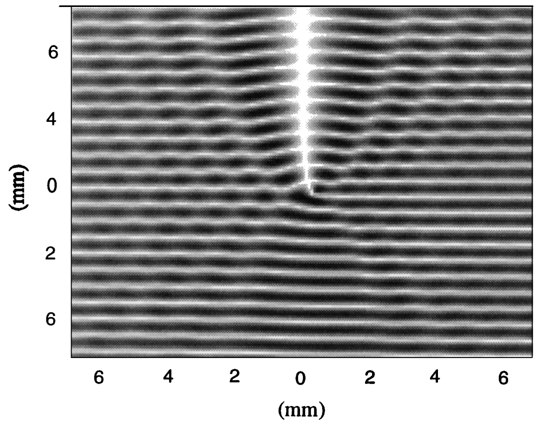

FIG. 6. Propagation of a plane wave through a filament of vorticity using a finite differential code.

following configurations: (1) a single crossing through a vorticity string with $\alpha = 0.5$, and (2) a round trip through a vorticity string with $\alpha = 0.25$. The good agreement (Fig. 7) confirms that the TRM acts as a vorticity amplifier, even for a vorticity field whose size is comparable to the acoustic wavelength.

To conclude, we show experimentally that a double TRM works as a vorticity amplifier and enables us to characterize different kinds of vortices. The acoustical Aharonov-Bohm effect has been investigated through the measurement of the phase and amplitude distortions of a plane wave through the flow induced by a vorticity filament. Finally, the acoustical Aharonov-Bohm effect has been numerically interpreted as the scattering of a plane wave through a heterogeneous motionless medium.

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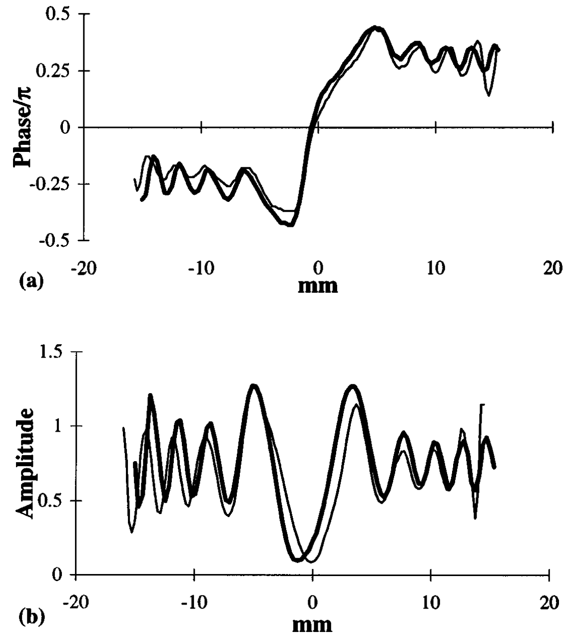

FIG. 7. (a) Phase distortion and (b) amplitude distortion of a plane wave through a penetrable vorticity filament: a simple path with $\alpha = 0.5$ (bold curve), a forward and backward propagation with $\alpha = 0.25$ (thin curve).

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