Dissipation and Fluctuation at the Chiral Phase Transition

Tamás S. Biró[1,*] and Carsten Greiner[2,†]

[1]*Research Institute for Particle and Nuclear Physics, P. Box 49, H-1525 Budapest, Hungary*

[2]*Institut für Theoretische Physik, Universität Giessen, D-35392 Giessen, Germany*

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Utilizing the Langevin equation for the linear σ model we investigate the interplay of friction and white noise on the evolution and stability of collective pionic fields in energetic heavy ion collisions. We find that the smaller the volume, the more stable transverse (pionic) fluctuations become on a homogeneous disoriented chiral field background (the average transverse mass $\langle m_t^2 \rangle$ increases). On the other hand the variance of m_t^2 increases even more, so for a system thermalized in an initial volume of 10 fm^3 about 96% and even in 1000 fm^3 about 60% of the individual trajectories enter into unstable regions ($m_t^2 < 0$) for a while during a rapid one-dimensional expansion ($\tau_0 = 1$ fm$/c$). In contrast the ensemble averaged solution in this case remains stable. This result supports the idea of looking for disoriented chiral condensate (DCC) formation in individual events. [S0031-9007(97)04270-1]

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Ultrarelativistic heavy ion collisions offer the possibility to study hadronic matter at high initial temperatures and energy densities. Because of their small masses pions contribute most dominantly to the large multiplicity of produced particles. It was speculated that some of the observed pions are produced by the coherent decay of a semiclassical pion field [1,2]. The idea of so called "*disoriented chiral condensates*" (DCC) was then made widely known due to Bjorken, Kowalski, and Taylor [3]. They speculated that events can occur in which the classical pion field is oriented along a single direction in isospin space.

Rajagopal and Wilczek [4] have proposed that in a rapid quench through the second order chiral QCD phase transition such disordered chiral configurations may emerge. Unstable, exponentially increasing long wavelength pion fields can develop during the "roll down" period of the order parameter according to the pure classical equation of motion [4,5]. The spontaneous growth and subsequent decay of these configurations would give rise to large collective fluctuations in the number of produced neutral pions compared to charged pions, and thus could provide a mechanism explaining a family of peculiar cosmic ray events, the Centauros [6]. A deeper reason for these strong fluctuations lies in the fact that all pions are assumed to sit in the same momentum state and the overall wave function can carry no isospin [7].

However, the proposed quench scenario assumes a highly nonequilibrium initial configuration for the phase transition to occur, namely, that the potential governing the evolution of the order parameter and the long wavelength modes immediately turns to the classical one governing the vacuum structure at zero temperature. This represents a very drastic assumption as the soft and classical modes completely decouple in an *ad hoc* manner from the residual thermal fluctuations at or slightly below the critical temperature. It is not likely to happen in an ultrarelativistic heavy ion collision.

An alternative scenario was suggested by Gavin and Müller [8] who proposed to use instead of the bare potential the effective one-loop potential including the thermal fluctuations on the mean-field level. If the system cools rapidly enough [due to longitudinal ($D = 1$) or radial ($D = 3$) expansion] and, similar to the quench scenario, if the order parameter is small enough at the onset of the evolution below the critical temperature, DCC might emerge as well. The latter assumption, however, has also been criticized: if the order parameter stays in thermal contact with the fluctuations giving rise to the effective one-loop potential, then one also has to allow for the thermal fluctuations in the initial conditions [9,10]. Quenched initial conditions seem to be statistically unlikely.

In this Letter we address the question of the "likeliness" of an instability leading potentially to a DCC event in the light of different (dynamically preheated) initial conditions as raised by Blaizot and Krzywicki [11].

In a simplified model we study the influence of the thermal modes ("fluctuations") on the off-equilibrium evolution of the order parameters. These chiral fields are treated semiclassically within the chiral 0(4) σ model. The long wavelength modes, especially the zero modes used as order parameters, represent an open system which constantly interacts with the thermal fluctuations (the "reservoir" or "heat bath"). On the one-loop level the interaction with the hard modes will generate an effective mass term $\frac{1}{2}\lambda T^2$ [8–10], leading to the effective Hartree-Fock potential. It was recently shown in detail that in the ϕ^4 theory hard modes can be integrated out on the two-loop level leading to *dissipation* and *noise* in the quasiclassical limit for the propagation of the long wavelength fields [12]. The resulting equations of motion are of Langevin-type. In the weak coupling limit, the friction coefficient η is directly related to the on-shell plasmon damping rate, $\eta \equiv 2\gamma_{pl}$. The noise term shows up as an imaginary part in the effective action and is related to the friction via the fluctuation-dissipation theorem. The interplay of noise and dissipation

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guarantees that the soft modes eventually become thermally populated on the average.

We propose to study the following Langevin equations of motion for the order parameters $\Phi_a = \frac{1}{V}\int d^3x\,\phi_a(\mathbf{x},t)$ in a volume V:

$$\begin{aligned}\ddot{\Phi}_0 + \left(\frac{D}{\tau} + \eta\right)\dot{\Phi}_0 + m_0^2\Phi_0 &= f_\pi m_\pi^2 + \xi_0\,,\\ \ddot{\Phi}_i + \left(\frac{D}{\tau} + \eta\right)\dot{\Phi}_i + m_0^2\Phi_i &= \xi_i\,,\end{aligned} \tag{1}$$

with $\Phi^a = (\sigma, \pi^1, \pi^2, \pi^3)$ being the chiral meson fields and

$$m_0^2 = \lambda\left(\Phi_0^2 + \sum_i \Phi_i^2 + \frac{1}{2}T^2 - f_\pi^2\right) + m_\pi^2\,. \tag{2}$$

Here τ is the proper time of the expanding system and the "dot" denotes the derivative with respect to τ. The above equations assume a D-dimensional scaling expansion, resulting in an additional effective damping term D/τ [8–10].

Before presenting our results some comments are in order: We use the standard parameters $f_\pi = 93$ MeV for the pion decay constant, $m_\pi = 140$ MeV for the pion mass, and $\lambda = 20$ for the coupling constant. With this choice we are obviously in the strong coupling regime, so our conclusions drawn from the investigation of the weak coupling regime [12] are rather a motivation for our present usage of simple effective terms for noise and dissipation.

We treat the dissipation term as Markovian, which assumes a clear separation among the time scales of the hard and soft modes. In the semiclassical Markovian approximation the noise is effectively white and at the two-loop level it is Gaussian [12],

$$\begin{aligned}\langle \xi_a(t)\rangle &= 0\,,\\ \langle \xi_a(t_1)\xi_b(t_2)\rangle &= \frac{2T}{V}\,\eta\,\delta_{ab}\,\delta(t_1 - t_2)\,,\end{aligned} \tag{3}$$

where T is the temperature, V the volume, and η the friction coefficient.

Finally we have to specify the friction coefficient η for the σ and pion field. The on-shell ($\omega = m$) plasmon damping rate for standard ϕ^4 theory arising from the "sunset" diagram [12,13] can be easily calculated in the 0(4) model, assuming that all four masses m for the fluctuations (i.e., hard quanta) are equal:

$$\eta = 2\gamma_{pl} = \frac{9}{16\pi^3}\,\lambda^2\,\frac{T^2}{m}\,f_{Sp}(1 - e^{-\frac{m}{T}})\,, \tag{4}$$

where $f_{Sp}(x) = -\int_1^x dt\,\frac{\ln t}{t-1}$ defines the Spence function. Admittingly this "choice" is only a crude estimate as the zero modes do not evolve on shell during the (possibly unstable) evolution. Thus the dissipation and noise correlation should better be described by non-Markovian terms including memory effects. In addition, the O(4) transverse and longitudinal mass for the fluctuations [8–10],

$$\begin{aligned}m_t^2 &= \lambda\left(\Phi_0^2 + \sum_i \Phi_i^2 + \frac{1}{2}T^2 - f_\pi^2\right) + m_\pi^2\,,\\ m_l^2 &= m_t^2 + 2\lambda\left(\Phi_0^2 + \sum_i \Phi_i^2\right),\end{aligned} \tag{5}$$

are not really equal.

The phenomenon of long wavelength DCC amplification occurs in periods when the transverse mass squared m_t^2 becomes negative. This happens during a quench scenario or rapid cooling. For the sake of simplicity we treat η as a constant, obtained using (4) and $m/T \approx 1$ throughout the evolution. At $T = T_c \equiv \sqrt{2f_\pi^2 - 2m_\pi^2/\lambda} = 123$ MeV the friction $\eta = 2\gamma_{pl} = 2.2\ (\mathrm{fm}/c)^{-1}$ is rather strong. Therefore we also investigate scenarios with 1/2 and 1/4 of this value.

Aside from a theoretical justification one can regard the Langevin equation as a practical tool to study the effect of thermalization on a subsystem, to sample a large set of possible trajectories in the evolution, and to address also the question of all thermodynamically possible initial configurations in a systematic manner. Applying this we are able to study the up to now unknown influence of thermal fluctuations on the growth of disoriented chiral domains. We expect that the noise term leads to a subsequent thermalization making DCC formation less likely, but allowing also for large fluctuations on an event by event basis. Thus we need to study how fast the system cools and destabilizes and how fast it can thermalize.

The scaling expansion and cooling of the system is described by the equations

$$\frac{\dot{T}}{T} + \frac{D}{3\tau} = 0, \qquad \frac{\dot{V}}{V} - \frac{D}{\tau} = 0\,. \tag{6}$$

In the framework of the model described so far we investigate different evolution scenarios for the order parameters Φ_a. For comparison we calculate the pure classical ($\eta = 0, T = 0$) and the one-loop annealing scenario ($\eta = 0$) for the $k = 0$ modes with quenched initial condition ($\Phi_a = 0, \dot{\Phi}_a = 0, a = 0\ldots3$). These scenarios contain long and highly unstable periods of the evolution during a rapid ($\tau_0 = 1$ fm$/c$) one-dimensional ($D = 1$) scaling expansion. This can be inspected in the lower part of Fig. 1 where the characteristic quantity $\mu_t = \mathrm{sgn}(m_t^2)\sqrt{|m_t^2|}$ is plotted as a function of time.

In the two-loop motivated Langevin scenario we let the system thermalize at temperature $T = T_c$ for 10 fm$/c$ from the quenched initial condition and then switch on the one-dimensional expansion according to Eq. (6) with $\tau_0 = 1$ fm$/c$. The outcome of thermalization depends on the volume occupied by the $k = 0$ modes—in agreement with the equipartition theorem. The middle part of Fig. 1 shows the (ensemble) average evolution of 1000 trajectories, each propagated according to the Langevin equation (1), for different initial volumes of $V_0 = 1, 10, 100,$ and 1000 fm^3.

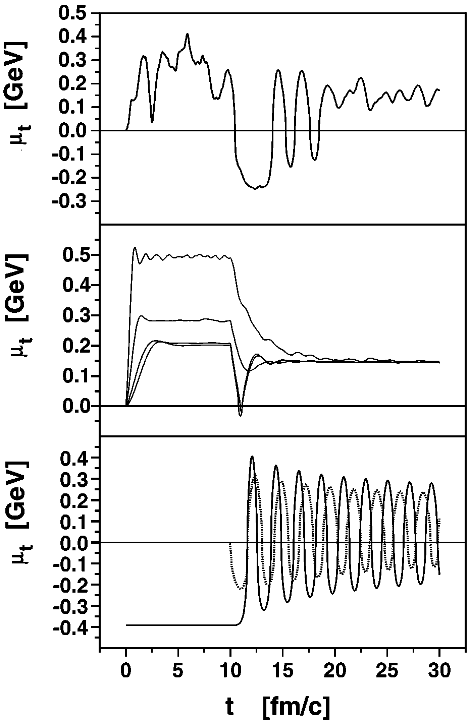

FIG. 1. (upper) $\mu_t(t) = \mathrm{sgn}(m_t^2)\sqrt{|m_t^2|}$ is shown for the most unstable event of 1000 propagated according to (1) with an initial volume $V_0 = 10$ fm^3 (the one-dimensional expansion starts at $t = 10$ fm$/c$—compare text). (middle) The ensemble averaged value $\langle \mu_t(t) \rangle \equiv 1/N \sum_i \mu_t^{(i)}(t)$ (for $N = 1000$ events) is given for initial volumes $V_0 = 1$, 10, 100, and 1000 fm^3 (from top to bottom). (lower) For comparison also the deterministic solution for $\mu_t(t)$ [$\eta, \xi = 0$ in (1)] for the pure [$T = 0$ in (2)] (solid line) and annealing scenario (dotted line) is shown for quenched initial conditions. The one-dimensional expansion is started also at $t = 10$ fm$/c$.

[For $D = 1$ the volume $V(\tau)$ increases linear with time by τ/τ_0. For a typical duration of the expansion of $\tau \sim 10$ fm$/c$ the final volume is then a factor of 10 larger. Initial volumes of about 5–25 fm^3 seems reasonable for a resulting DCC domain. We will thus focus our discussions mainly on $V_0 = 10$ fm^3.] Because of the high friction coefficient η the equipartition is set after the first few fm$/c$. We find that the smaller the volume, the larger m_t^2.

The volume dependence of the thermalized chiral order parameter fields can be analyzed using the virial theorem

$$\Phi_a \frac{\partial H}{\partial \Phi_a} = \dot{\Phi}_a \frac{\partial H}{\partial \dot{\Phi}_a} = T\,. \qquad (7)$$

In the $m_\pi = 0$ limit the large volume limit is calculated as [with $f = \frac{1}{2}(T_c^2 - T^2)$]

$$\sum_{a=0}^{3} \Phi_a^2 = \begin{cases} f + \frac{4T}{\lambda V f} + \mathcal{O}(1/V^2), & T < T_c\,, \\ 2\sqrt{\frac{T_c}{\lambda V}}\,, & T = T_c\,, \\ \frac{4T}{\lambda V f} + \mathcal{O}(1/V^2), & T > T_c\,. \end{cases} \qquad (8)$$

From this $m_t^2 = \lambda(\sum \Phi_a^2 - f) = \mathcal{O}(1/V)$ follows for $T < T_c$ in accordance with the Goldstone theorem.

At the end of the expansion μ_t relaxes to the value of $m_\pi = 140$ MeV, as can be seen from the curves in the middle part of Fig. 1. These ensemble averaged curves do not show any significant period of instability. This result is in agreement with those of Randrup [14] for a one-dimensional expansion from a thermalized initial condition.

This lack of instability in the averaged evolution does not mean, however, that DCC formation cannot be expected in heavy ion collisions: Using the Langevin equation we are able to explicitly single out particular evolutions which are the most unstable. The upper part of Fig. 1 presents such an evolution preheated in a $V_0 = 10$ fm^3 initial volume at $T = T_c$. Here, in spite of the initial thermalization and ongoing noise during the expansion, quite significant unstable periods develop. For quantifying the strength of instability we define the quantity [14]

$$G = \int |m_t| \Theta(-m_t^2)\, dt\,. \qquad (9)$$

The amplification of small amplitude instabilities with $k = 0$ is then $\exp G$. For the particular event shown in Fig. 1 $G = 4.7371$. The distribution of this quantity, $P(G)$, is shown in Fig. 2 for an initial volume of $V_0 = 10$ fm^3, the 1000 individual trajectories are all preheated to $T = T_c$.

We observe that only in 4.3% of all cases an unstable period is missing. The average trajectory shows, however, no signal, because the unstable periods occur at different times. Therefore the middle part of Fig. 1 does *not* allow for drawing a conclusion.

In order to review the tendencies Table I summarizes the most important properties of the distribution of the amplification factor in the Langevin scenario. The fraction of events with no instability at all are written in the second column for the various initial volumes. In these cases no

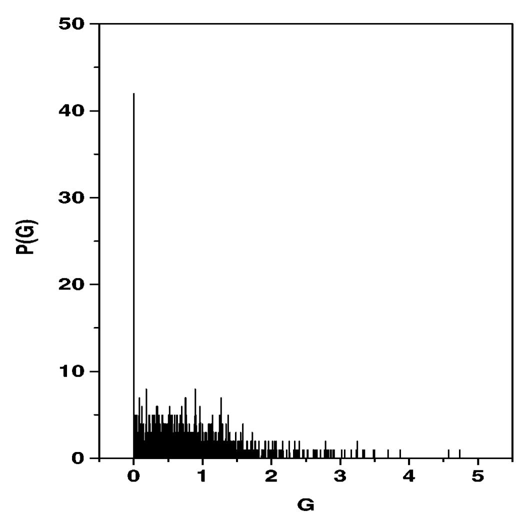

FIG. 2. Distribution of the enhancement factor G for $V_0 = 10$ fm^3. The most unstable event has $G = 4.7371$.

TABLE I. Statistical properties of individual evolutions according to the Langevin scenario. The upper table belongs to the friction $\eta = 2\gamma_{pl}$, the middle to $\eta = \gamma_{pl}$, and the lower one to $\eta = \gamma_{pl}/2$.

V	$P(G = 0)$	$\langle G \rangle$	$G_{\max}$
1	0%	3.0983	7.1163
10	4.2%	0.8540	4.7371
100	40.3%	0.2728	2.0309
1000	40.4%	0.1082	0.6780
1	0%	3.0883	8.5129
10	7.0%	0.7684	4.5744
100	48.4%	0.2011	1.8699
1000	56.5%	0.0557	0.5327
1	0%	3.1088	9.0965
10	12.3%	0.7775	5.6462
100	54.4%	0.1620	1.8172
1000	59.7%	0.0483	0.5063

DCC signal can develop at all. The average growth factor $\langle G \rangle$ and its maximum within 1000 events, $G_{\max}$, are shown in the third and fourth columns, respectively.

In the late phase of the expansion the friction (4) is overestimated by employing $m/T = 1$, as m/T significantly increases due to the cooling and the thermal fluctuations decouple. In fact by applying a smaller friction the unstable oscillating periods appear for a longer time similar to the annealing scenario. On the other hand, a variation of the friction parameter, η, to its $1/2$ or $1/4$ value does not change significantly the characteristics of the distribution $P(G)$ (cf. Table I).

In conclusion we have investigated the chiral O(4) model in a Langevin scenario, motivated by the two-loop results of the nonequilibrium field theory. In this approach the $k = 0$ order parameter fields are coupled to a thermal bath via mass, friction, and noise terms. We preheated the system at $T = T_c$ using its own dynamics and then switched over to a one-dimensional scaling expansion. Average and statistical properties of individual solutions of the Langevin equations were studied with the emphasis on such periods of the time evolution when the transverse mass becomes imaginary and therefore an exponential growth of unstable fluctuations in the collective fields can be expected.

We have found that in different realistic initial volumes ranging from 1 to 1000 fm^3, where the average evolution does not show any sensible instability, individual events lead to sometimes significant growth of fluctuations. Both the strength ($G \approx 2$–5) and the probability (50%–90%) of such events are remarkably high. The most extreme events are quite similar to the predictions of the quenched models. While in general the damping stabilizes the evolution of long wavelength fields, the fluctuations increase the chances for prolonged unstable periods in about 60%–90% of all events. All these estimates are optimistic, because the back reaction of the soft but $k \neq 0$ modes will lead to a shortening of the unstable periods [15].

Our findings support the idea of looking for DCC formation experimentally in individual events.

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*Electronic address: tsbiro@sunserv.kfki.hu

†Electronic address: greiner@theorie.physik.uni-giessen.de

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