Universal Anharmonic Vibrator Description of Quasiband Structures in Collective Even-Even and Odd-*A* Nuclei

D. Bucurescu and N. Mărginean

National Institute of Physics and Nuclear Engineering, P.O. Box MG-6, Bucharest 76900, Romania

(Received 14 February 1997)

It is found that energy correlations within the bands of the good rotor odd-*A* nuclei are well described by an anharmonic vibrator formula, which indicates an unexpected constancy of the moment of inertia. Based on a large collection of quasibands in all collective even-even and odd-*A* nuclei, an empirical recurrence relation is proposed which allows one to calculate the excitation energies of any band starting from its lowest two. This simple two-parameter formula can be reduced approximately to that of a second order anharmonic vibrator. [S0031-9007(97)03520-5]

PACS numbers: 21.10.Re, 21.60.Ev

During the past few years, several unexpected, systematic features have been discovered by studying correlations between the excitation energies of low-lying collective bands. Most of these results concern the quasi-ground band of the even-even nuclei. The most remarkable is the finding that the yrast energies of the collective, *nonrotor* nuclei with $28 < Z < 82$ [i.e., those having $2.05 < R_{4/2} < 3.15$, with $R_{4/2} \equiv E(4_1^+)/E(2_1^+)$] are well described by a universal anharmonic vibrator (AHV) formula with *constant* anharmonicity ε_4,

$$E(n) = nE(1) + \frac{n(n-1)}{2}\,\varepsilon_4\,, \tag{1}$$

where $n = I/2$ can be interpreted as the number of phonons of the state of spin I (here $n = 1$ corresponds to the 2_1^+ state, $n = 2$ to 4_1^+, etc.) [1,2]. In a recent extension of this work it has been shown that a next order generalization of Eq. (1), including a second order anharmonicity term, $\frac{n(n-1)(n-2)}{6}\,\varepsilon_6$, gives an excellent description of the yrast bands of all types of even-even nuclei (the good rotor ones included) [3].

Progress along similar lines has also been made for the odd-mass nuclei. In Ref. [4] it was shown that the unique (or unnatural) parity structures in all nonrotational nuclei are well described by Eq. (1), with practically the same value of ε_4 as that of the even-even nuclei ($\simeq$150 keV). This study was extended in Ref. [5] to all the natural parity orbital (one-quasiparticle) quasibands in nonrotor nuclei, which were also shown to follow an AHV behavior [Eq. (1)].

In this Letter we present two other results which show that the AHV-type behavior of the quasiband structures in both the even-even and odd-*A* nuclei is truly universal. First, we show that the low-lying (Nilsson) bands in the good rotor odd-*A* nuclei are also excellently described by Eq. (1). This is very surprising, in view of the common present perception that the moment of inertia of all these nuclei (and bands) is not constant. Second, by studying a large collection of experimental quasiband structures in all types of collective nuclei (both even-even and odd-*A*), we observe a very simple and compact correlation between three consecutive excitation energies in the band, which is well satisfied up to the highest states (below the up (back) bending). This gives a recurrence relation which leads to a simple, universal two-parameter formula that can be used to describe the higher states of any band by starting from its lowest three states. With some reasonable approximation, this formula leads to a functional dependence on n identical to that of the second order AHV empirically proposed in [3].

We first study the band structures in the good rotor odd-*A* nuclei (i.e., those having cores with $R_{4/2} \geq 3.15$). In the rotor model their excitation energies are described by

$$E(I) = \frac{\hbar^2}{2\Im}\,[I(I+1) - K(K+1)]\,. \tag{2}$$

We consider a sequence of states of spins j, $j+2$, $j+4, \ldots$, from such a band; usually $j = K$ but for what follows this is not necessary. By analogy with the yrast band of the even-even nuclei, we define the in-band excitation energies,

$$\begin{aligned} E'(1) &= E(j+2) - E(j), \\ E'(2) &= E(j+4) - E(j), \ldots, \end{aligned} \tag{3}$$

and study the correlations between them.

We now consider the whole collection of bands based on intrinsic (Nilsson) states. For this study we have extracted from the nuclear data sheets [6] a collection of 225 bands for which at least two transitions are known. Together with the bands, based on the unique parity orbitals [4], we have a total of 303 bands. These are the different Nilsson bands in the deformed ("good rotor") nuclei, between ^{155}Sm and ^{187}Os. The correlations between $E'(2)$, $E'(3)$, and $E'(1)$, respectively, are shown in Fig. 1. The result is very striking: The experimental data define very compact straight lines. In particular, the correlation of $E'(2)$ with $E'(1)$ is of the AHV type (1), i.e., of slope exactly 2 and with very little scatter of the points around the fitted straight line (one standard deviation of

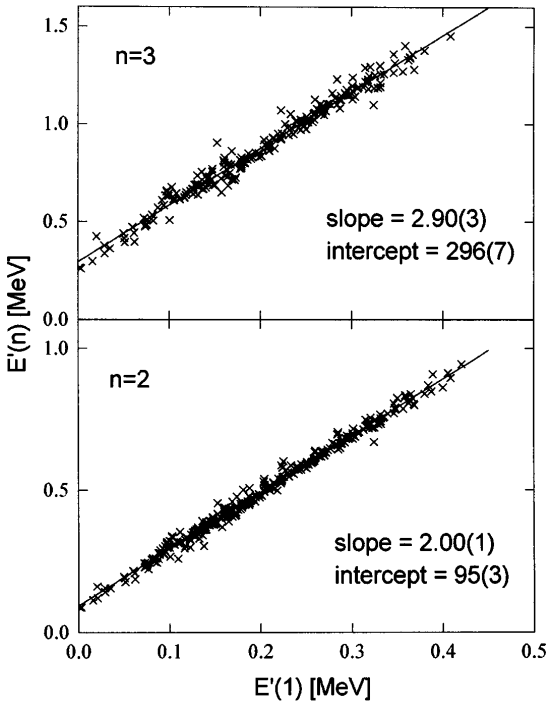

FIG. 1. Correlations of in-band excitation energies for 303 Nilsson bands in the deformed odd-A nuclei (between ^{155}Sm and ^{187}Os). Energies are relative to the lowest state of spin j [see Eq. (3)]. The crosses are the experimental points and the curves are fitted straight lines whose slope and intercept (in keV) are indicated.

only 17 keV). The intercept ε_4 of Eq. (1) is 95 $\pm$ 3 keV, notably smaller than that of $\sim$150 keV for the nonrotor even-even nuclei [1] and unique parity structures in odd-A nuclei [4], or of 115 keV deduced for a collection of different natural parity bands in nonrotor odd-A nuclei [5]. The excellent AHV description of all these rotational bands, although surprising, has a simple meaning. It is clear that such phononlike correlations need not imply anharmonic vibrations. Formula (1) describes exactly a set of rotational bands (2) *if* all of them have the same moment of inertia [$\Im$ in Eq. (2) is a *constant*]. In this case, ε_4 in Eq. (1) is related to the constant moment of inertia (MI) $\Im$,

$$\varepsilon_4 = 4\,\frac{\hbar^2}{\Im}\,. \tag{4}$$

Thus, in terms of the rotor model, the empirical result of Fig. 1 implies an unexpected fact: For all of the rotor nuclei between $A \sim 150$ and 190, all Nilsson bands have almost the same, constant value for the MI at the lowest rotation frequencies. This value, corresponding to $\varepsilon_4 = 95 \pm 3$ keV, is $\Im = (42.1 \pm 1.3)\hbar^2\ \mathrm{MeV}^{-1}$. This constancy, or rather the narrow distribution around a certain value, un-noticed until now (note that, on the basis of the $A^{5/3}$ dependence of the MI, one expects a variation of about 36% between the masses 155 and 187), is characteristic only to the odd-A nuclei. In the corresponding even-even core nuclei (from ^{154}Sm to ^{186}W) the behavior is completely different: ε_4 deduced from Eq. (1) varies strongly from nucleus to nucleus and, as a result, the $E(4_1^+)$ versus $E(2_1^+)$ plot bends down systematically from the AHV line (1) towards a line of slope 3.33 [1]. Alternatively, the MI for the 2_1^+ state [which is essentially the reverse of $E(2_1^+)$] varies continuously from about 20 to $40\hbar^2\ \mathrm{MeV}^{-1}$ [7]. It is interesting to note that the "constant" MI value observed for the odd-A nuclei is practically equal to the maximum value reached by the even-even nuclei.

The constancy of the MI's has also been checked by determining them directly from the transition energies in the usual way. Since, in determining $\Im$ from the correlation of Fig. 1 via Eq. (4), one uses three states of the band (or two transition energies), it is likely that what is determined is the dynamic MI, $\Im^{(2)}$. We have therefore determined $\Im^{(2)} = 4/[E'(2) - 2E'(1)]$ as well and found that its distribution is very close to a *normal* one, with a mean of $(43.2 \pm 5.3)\hbar^2\ \mathrm{MeV}^{-1}$, which is the same as the value deduced from Fig. 1, except for the larger standard deviation.

For the correlation $E'(3)$ versus $E'(1)$ (Fig. 1) and for the higher states, the slopes of the straight line fits deviate more and more from the AHV prescriptions. These deviations reflect the known fact that the MI does not remain constant along the band, but generally increases as one goes upwards in the band.

With this result added to the previous ones [1,2,4,5] there is now rich evidence that both the yrast bands of all nonrotor even-even nuclei with $28 < Z < 82$ and the one-quasiparticle bands in *all* odd-A nuclei from the same region are rather well described by the AHV relation (1), with an almost *constant* ε_4 (which may depend somewhat on the mass region or the character of the nuclei considered—deformed or nondeformed). Generally, the experimental data show an excellent AHV correlation for the first two energies in the band, which persists reasonably well for the third excitation energy, but the scattering of the points around the average AHV behavior increases continuously for the higher states in the band, until this correlation is practically washed out. A closer inspection of the experimental plots reveals, however, that the deviations of the points corresponding to different states from the mean AHV behavior are rather well correlated. To unravel these correlations we have studied different relationships between the experimental excitation energies of more than three successive states from the band. This was made for our entire collection of bands, now numbering 64 bands in the deformed even-even nuclei, 158 in the nondeformed ones, 303 in the deformed odd-A nuclei, and 582 in the nondeformed odd-A ones, for nuclei between As and Hg. Let us denote by $E_\gamma(n)$ the "transition energy" from the state n to the state below it, $(n-1)$: $E_\gamma(n) = E(n) - E(n-1)$ [or $E'(n) - E'(n-1)$ in the odd-A nuclei]—see Eqs. (1) and (3)—are actually the $E2$ γ-ray transition energies in the band. We have observed that for any

three consecutive transitions (from the states labeled by $n-2, n-1, n$) the energy of the "middle" one is well approximated by the arithmetic mean of the two adjacent ones, below and above it. This is actually the prediction of both the AHV, Eq. (1), and the rotational formula (2):

$$E_\gamma(n) = 2E_\gamma(n-1) - E_\gamma(n-2)\,. \qquad (5)$$

Figure 2 shows the correlation between the experimental quantities $E_\gamma(n)$ and $[2E_\gamma(n-1) - E_\gamma(n-2)]$ for our entire collection of bands in both the even-even and odd-A collective nuclei. Each band, for which at least three transitions (or three "excited" states) are known, contributes with points in Fig. 2 up to the highest transition (n value) below the (up) back bending. We have a total of 754 such bands, and we considered up to 9 transitions in each. We note that the correlation in Fig. 2 is an extremely tight envelope very close to the straight line (5) (of slope 1 and intercept 0). Remarkable in this plot is that, although we go as high as possible in n for each band, the scattering of the points remains very small in comparison with that of the correlation plots between the excitation energies, such as those in Fig. 1. Actually, since the overwhelming majority of the experimental points lie within a very narrow envelope, such a plot is useful to evidence "irregularities" in the bands, which are immediately highlighted by points with a large deviation from the average behavior. Thus, an examination of such points in our case (Fig. 2) revealed either possible up bendings not eliminated by our procedure or other phenomena such as band perturbation due to strong Coriolis mixing. Note that in Fig. 2 we take both the "AHV" class of nuclei (vibrational, transitional, and deformed odd-A), which generally shows "random" deviations from formula (1) (see, however, the considerations of Ref. [8]), and the rotational even-even nuclei for which ε_4 varies strongly and in a systematic manner [1,8]. Since we see practically the same type of correlation in each case, this gave us the incentive to treat all bands, in all nuclei, on the same footing, as we discuss next.

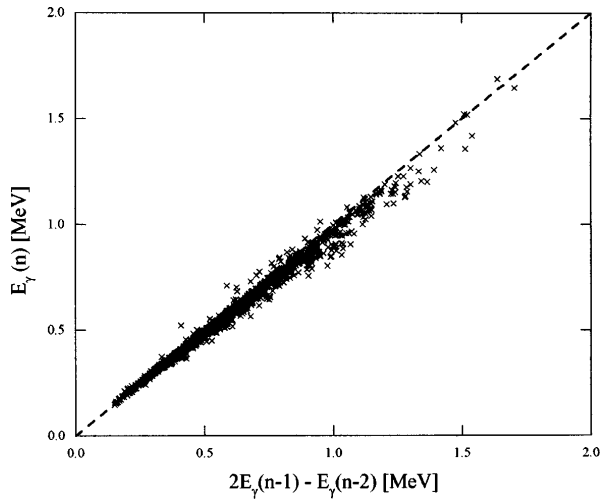

FIG. 2. Correlation between experimental energies of three consecutive in-band transitions [see discussion of Eq. (6)]. A number of 754 bands in the even-even and odd-A nuclei (both deformed and nondeformed) with at least three known transitions contribute to this plot (a band with n transitions gives $n-2$ points in the plot), and up to 9 transitions ($n = 9$) were considered for each band.

The experimental data in Fig. 2 show a correlation which can be well described by a straight line but with slope and intercept slightly different from 1 and 0, respectively. So, a more correct formula which reproduces the average behavior of our rich experimental collection is

$$E_\gamma(n) = c_1[2E_\gamma(n-1) - E_\gamma(n-2)] + c_2\,, \qquad (6)$$

where c_1 and c_2 are parameters to be determined from the data. We can now use this formula to calculate excitation energies $E(n)$ recurrently, starting from the lowest transitions in the band,

$$E(n) = E(n-1) + c_1[2E(n-1) - 3E(n-2) + E(n-3)] + c_2\,. \qquad (7)$$

We have performed least squares fits to the experimental band energies, not for individual nuclei but for different sets of nuclei (such as even-even and odd-A, deformed or nondeformed, respectively), as well as for the entire set of bands, and determined the values of c_1 and c_2. The "theoretical" energies were calculated with Eq. (7), starting from $n = 3$; the first two energies in the band, $E(1)$ and $E(2)$, were taken equal to the experimental values. We have found that generally c_1 takes values between 0.92 and 0.98, and c_2 between 7 and 22 keV. It is interesting to see how precisely this formula works. Figure 3 gives statistical results of three such global fits to band energies with Eq. (7) by showing the distribution of the mean relative deviation of the excitation energies calculated for each band as $\frac{\delta E_x}{E_x} = \frac{1}{n-2}\,\Sigma\,\frac{|E_{\rm exp}(i)-E_{\rm th}(i)|}{E_{\rm exp}(i)}$, the sum running from 3 up to the highest state n in the band. The formula works best in the case of the well-deformed (good rotor) nuclei: 74.7% of the 253 bands have $\frac{\delta E_x}{E_x}$ less than 2%, and 58% of them have less than 1%. In the case of the nondeformed nuclei (which have the largest scattering in the plot of Fig. 2), the results are weaker: 61.7% of the 501 bands have $\frac{\delta E_x}{E_x} < 2\%$, and the tail at higher deviations is larger. For *all* the bands taken together, 65.7% of the 754 bands have $\frac{\delta E_x}{E_x} < 2\%$.

We do not insist on the description that this approach can give in particular cases, since it is intended to reproduce only the mean behavior of the experimental data (as shown in Fig. 2). But we shall deduce what type of n dependence (ultimately, a dependence on the level spin) is provided by the recurrence relation (7) for the level energies $E(n)$. An explicit n dependence can be obtained in the *approximation* $c_1 = 1.0$; in this case, Eq. (7) leads to

$$E(n) = nE(1) + \frac{n(n-1)}{2}\,\epsilon_4 + \frac{n(n-1)(n-2)}{6}\,\epsilon_6\,, \qquad (8)$$

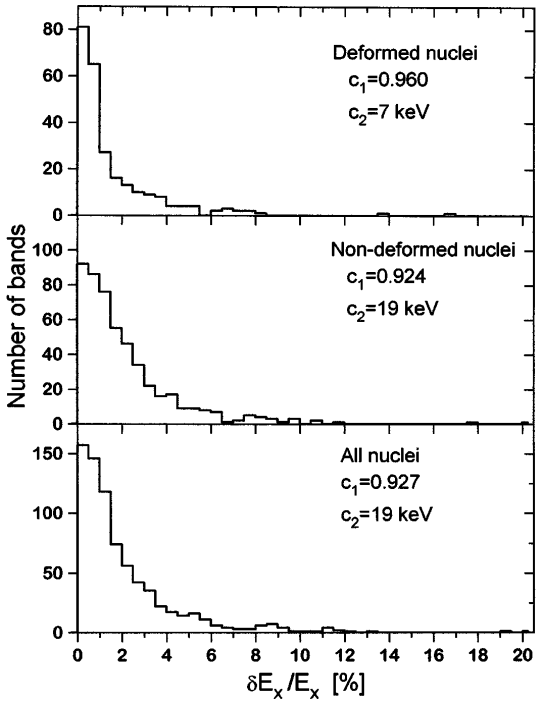

FIG. 3. Statistical distributions of the mean relative deviation of the band excitation energies (see text for the definition) calculated with Eq. (7) for the three indicated classes of nuclei. In each case, the values obtained for the parameters c_1 and c_2 from least squares fit to the data are given.

where $\epsilon_4 = E(2) - 2E(1)$ and $\epsilon_6 = 2c_2$, with $E(1)$ and $E(2)$ being the experimental energies of the first two (excited) states. This is exactly the second order AHV formula proposed in Ref. [3] for the description of any type of band. In some particular cases, the best results will be obtained by leaving ϵ_4, ϵ_6, and even $E(1)$ as free parameters. Thus we recognize the recurrence formula (7) as representing a *generalized* anharmonic vibrator (GAHV).

The discussion above has been restricted to correlations between quasiband states that differ in spin by steps of 2 units. In the odd-A nuclei, one can consider correlations for consecutive states that differ in spin by 1 unit. In the good rotor nuclei, we have observed nice correlations of the type (1), with an ε_4 value of about 23 keV [as expected from Eq. (2) with the $\Im$ deduced from the $\Delta I = 2$ step correlations], and Eq. (7) also works well. However, the $K = \frac{1}{2}$ bands and those with strong Coriolis distortion do not enter into this systematic, although they work well in the step 2 case. On the other hand, the step 1 correlations do not work well for many nonrotational cases. The problem of the systematic features of the shift between the "favored" and "unfavored" states (briefly considered in [9] in a restricted case) needs further consideration.

In summary, in this Letter we presented the following two results concerning the quasiband structures in atomic nuclei.

First, we have shown that all one-quasiparticle (or Nilsson) bands in the well-deformed odd-A nuclei (between Sm and Os) show, for the lowest states, global energy correlations which are very well described in terms of the universal anharmonic vibrator equation (1). Within the rotor model, this signifies that the moments of inertia of all these bands at their lowest rotation frequencies have a rather narrow distribution around a constant value. This finding is rather unexpected. One anticipates that the MI of the odd-A nuclei should be larger than that of the adjacent even-even nuclei due to the reduction of the pairing through blocking, but not that it is almost independent of mass and the orbital occupied by the odd particle. Furthermore, this mean, constant value of the MI corresponds to the highest value of the MI of the even-even cores in the 2_1^+ state. On the whole, this surprising experimental finding represents a challenge for microscopic models.

Second, by examining a very large collection of data, we have found that the experimental excitation energies within the quasiband structures of all the collective even-even and odd-A nuclei very accurately obey a simple relationship, Eq. (7), which can be used to calculate recurrently all energies in the band starting from the lowest two. This recurrence formula corresponds to a *generalized anharmonic vibrator*, and, within a natural approximation, it leads to the second order AHV formula proposed in Ref. [3] for the description of both vibrational and rotational bands in the even-even nuclei.

We conclude that the yrast structures of all the collective even-even nuclei, as well as the one-quasiparticle band structures in the odd-A nuclei from the medium-heavy mass range (between $Z = 30$ and 80), are well described by a GAHV formalism [Eq. (7)]. This reveals an unexpected simplicity and unity of the low-energy collective phenomena, which represents a serious challenge for nuclear structure theories.

We are grateful to Professor A. W. Thomas for a critical reading of the manuscript.

[1] R. F. Casten, N. V. Zamfir, and D. S. Brenner, Phys. Rev. Lett. **71**, 227 (1993).
[2] N. V. Zamfir and R. F. Casten, Phys. Lett. B **341**, 1 (1994).
[3] N. V. Zamfir and R. F. Casten, Phys. Rev. Lett. **75**, 1280 (1995).
[4] D. Bucurescu, G. Căta-Danil, M. Ivaşcu, L. Stroe, and C. A. Ur, Phys. Rev. C **49**, R1759 (1994).
[5] D. Bucurescu, N. Mărginean, I. Căta-Danil, M. Ivaşcu, L. Stroe, and C. A. Ur, Phys. Lett. B **386**, 12 (1996).
[6] Evaluated nuclear structure data file (ENSDF), maintained by the National Nuclear Data Center, Brookhaven National Laboratory.
[7] J. M. Espino and J. D. Garrett, Nucl. Phys. **A492**, 205 (1989).
[8] R. V. Jolos, P. von Brentano, N. V. Zamfir, and R. F. Casten, Phys. Rev. C **54**, R2146 (1996).
[9] D. Bucurescu, N. V. Zamfir, G. Căta-Danil, M. Ivaşcu, L. Stroe, C. A. Ur, and R. F. Casten, Phys. Lett. B **376**, 1 (1996).