Postmodern Technicolor

Thomas Appelquist
Department of Physics, Yale University, New Haven, Connecticut 06511

John Terning
Department of Physics, University of California, Berkeley, California 94720

L. C. R. Wijewardhana
Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221
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Using new insights into strongly coupled gauge theories arising from analytic calculations and lattice simulations, we explore a framework for technicolor model building that relies on a nontrivial infrared fixed point, and an essential role for QCD. Interestingly, the models lead to a simple relation between the electroweak scale and the QCD confinement scale, and to the possible existence of exotic leptoquarks with masses of several hundred GeV. [S0031-9007(97)04192-6]

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Electroweak symmetry breaking and fermion mass generation are still open problems in particle physics. The minimal standard model can account for all current experiments, but, with its light Higgs boson, is technically unnatural.

An early proposal for avoiding this problem was given in the form of a new scaled-up, QCD-like interaction: technicolor (TC). Problems with accounting for the charm and strange quark masses without flavor changing neutral currents ruled out such a simple possibility and led to the development of modern TC theories [1], known as walking TC: theories with vanishing or small ultraviolet β functions.

Precision electroweak measurements have shown, however, that even walking theories may be inadequate since they seem to predict [2] too large a value for the S parameter. The measurement of the top quark mass provides an additional problem, since it seems difficult to produce a large enough top quark mass without a very small scale for the additional [e.g., extended technicolor (ETC)] interactions necessary to couple the technifermion condensate to the top quark. Such interactions violate weak isospin symmetry, since they must also produce a small bottom quark mass. But without suppression by a relatively large scale, they lead to problems with the T parameter (also known as $\Delta\rho_* = \alpha T$).

In this Letter, we make use of some recent observations about SU(N) gauge theories to explore a technicolor framework that provides a potential solution to these problems. The most important of the observations is that SU(N) theories can exhibit an infrared (IR) fixed point [3] for a certain range of flavors (such fixed points have also been found in supersymmetric [4] gauge theories). This behavior has been examined further through analytic studies [5,6] and lattice simulations [7]. An IR fixed point, which naturally incorporates walking, is an essential ingredient in this framework for postmodern technicolor theories (PTC). The use of an IR fixed point to implement walking was considered by Lane and Ramana as part of their study of multiscale technicolor models [8]. An important difference, however, is that in the framework discussed here, the technicolor fixed point coupling is not strong enough by itself to break the electroweak symmetry. The addition of QCD is necessary, and this has interesting consequences.

Consider an SU(N) gauge theory with N_f flavors. At two loops it has a nontrivial IR fixed point for a range of N_f; the coupling at the fixed point is given by the ratio of the first two coefficients in the β function,

$$\alpha_* = -\frac{b}{c}\,, \tag{1}$$

where

$$b = \frac{1}{6\pi}\,(11N - 2N_f)\,, \tag{2}$$

$$c = \frac{1}{24\pi^2}\left(34N^2 - 10NN_f - 3\,\frac{N^2-1}{N}\,N_f\right). \tag{3}$$

The two-loop solution $\alpha(q)$ to the renormalization group equation can be written in the form

$$\frac{1}{\alpha(q)} = b\ln\!\left(\frac{q}{\Lambda}\right) + \frac{1}{\alpha_*}\,\ln\!\left(\frac{\alpha(q)}{\alpha_* - \alpha(q)}\right), \tag{4}$$

where Λ is an intrinsic scale, and where it has been assumed that $\alpha(q) \le \alpha_*$. With this choice of scale, $\alpha(\Lambda) \simeq 0.782\,\alpha_*$. Then for $q^2 \gg \Lambda^2$ the running coupling displays the usual perturbative behavior $\alpha(q) \approx 1/[b\ln(q/\Lambda)]$, while for $q^2 < \Lambda^2$ it approaches the fixed point α_*.

Analytic studies have indicated that there is a critical value, α_c, of the gauge coupling such that if $\alpha > \alpha_c$ then chiral symmetry breaking takes place. In the presence of an IR fixed point α_*, the chiral transition takes place when α_* reaches α_c, which happens when N_f decreases to a certain critical value N_f^c [5]. Below N_f^c the fermions are massive, the fixed point is only approximate, and we expect confinement to set in at momentum scales on the order of the fermion mass. The two-loop Cornwall-Jackiw-Tomboulis (CJT) potential [9] relates the critical coupling to the quadratic Casimir of the fermion representation, and for fundamental representations gives

$$\alpha_c \equiv \frac{\pi}{3C_2(R)} = \frac{2\pi N}{3(N^2 - 1)}. \tag{5}$$

This leads to the estimate [5]

$$N_f^c = N\left(\frac{100N^2 - 66}{25N^2 - 15}\right). \tag{6}$$

The order parameter (the dynamical fermion mass) vanishes continuously as $N_f \to N_f^c$ from below [5]. Thus for N_f just below N_f^c, the dynamical mass of the technifermion will be small compared to the intrinsic scale Λ. In such a near-critical theory, the dynamical mass (as a function of Euclidean momentum p) falls off approximately like $1/p$ up to scales of order Λ, rather than the perturbative (QCD-like) $1/p^2$. This is due to the fact that the coupling is near the (approximate) IR fixed point, so it evolves slowly (walks) in this regime [5].

Even though no obvious small parameter is involved, an estimate of the next order term in the loop expansion describing the chiral symmetry breaking indicates that the correction is relatively small (less than 20%) [5]. Similarly, the next order term (in the $\overline{\mathrm{MS}}$ scheme) in the β function is also about 20% of the first or second order terms when $\alpha_* \simeq \alpha_c$. We will assume here that the estimates described above provide an approximate description of the IR fixed point and the chiral phase transition.

To implement these ideas we take the PTC group to be $SU(2)_{\mathrm{PTC}}$, and assume that there are four electroweak doublets of technifermions ($N_f = 8$). The motivation for four doublets is that this corresponds to one complete family of technifermions, i.e., techniquarks $Q = (U, D)$ and technileptons $L = (N, E)$. We assume two technicolors in order to keep S as small as possible. Given this theory, Eq. (6) predicts $N_f^c \simeq 7.9$, so we might expect a conformal IR fixed point, leaving the electroweak symmetry intact.

Note, however, that the difference between α_* and α_c in this theory is of order 0.1, which is also the strength of the QCD coupling at the weak scale. Thus it is quite possible that the inclusion of QCD is sufficient to produce chiral symmetry breaking in the color singlet channel for the techniquarks. Suppose, for example, that there is some unification scale above Λ for all gauge couplings. As we evolve the couplings to the IR, the QCD and PTC interactions grow. Below Λ the PTC coupling approaches its fixed point value α_*, which by itself is slightly subcritical for chiral symmetry breaking. Eventually the QCD coupling grows to be of order 0.1. The scale at which the combined interactions reach criticality determines the techniquark dynamical mass.

At momentum scales below the techniquark mass, there is no longer an approximate IR fixed point, and the PTC coupling grows. We therefore expect that at a somewhat smaller scale, chiral symmetry breaking will also occur for the technileptons. The magnitude of the splitting is difficult to estimate reliably since it involves the running of the TC coupling in the near-critical regime, and will therefore be very sensitive to the 20% uncertainties discussed above. In this paper, we will assume that the splitting is sizable. The electroweak scale will then be set dominantly by the techniquark dynamical mass, with a smaller contribution from the technileptons. At momentum scales below these masses, the QCD coupling increases from its value of approximately 0.1, eventually reaching confinement strength at Λ_{QCD}.

To be more explicit, we first note that the inclusion of the QCD coupling α_s modifies the PTC fixed point behavior. There is an additional term, $(2/\pi^2)\alpha^2\alpha_s$, in the two-loop PTC β function. Therefore the IR fixed point is only a quasifixed point (for small α_s and thus for momentum scales large compared to Λ_{QCD}) given by

$$\hat{\alpha}_* = -\frac{b}{c} + \frac{2\alpha_s}{c\pi^2}. \tag{7}$$

We next note that QCD effects also modify the two-loop CJT criterion for chiral symmetry breaking (this is equivalent to the big MAC analysis of Ref. [10]) to be

$$\frac{16}{9}\alpha_s(\mu) + \alpha(\mu) = \alpha_c = \frac{4\pi}{9}, \tag{8}$$

where μ is the techniquark chiral symmetry breaking (electroweak) scale. If the PTC coupling is near its IR quasifixed point, then $\alpha(\mu) \approx \hat{\alpha}_*$.

At lower momentum scales, the technifermions decouple and the evolution of α_s is determined by its one-loop renormalization group equation, with the ultraviolet boundary condition given by solving Eqs. (7) and (8). Thus for $\Lambda \gg \mu > \Lambda_{\mathrm{QCD}}$, the electroweak scale μ is related to Λ_{QCD} by

$$\mu = \Lambda_{\mathrm{QCD}} \exp\left(\frac{44}{45 b_{\mathrm{QCD}}(\alpha_c - \alpha_*)}\right), \tag{9}$$

where [from Eq. (2)] $b_{\mathrm{QCD}} = 21/6\pi$. Thus in this theory, the electroweak scale can be computed in terms of the QCD confinement scale. Of course, this result is exponentially sensitive to small errors in the estimate of α_c, and a numerically reliable calculation of μ may require nonperturbative methods. Equation (9) predicts $\ln(\mu/\Lambda_{\mathrm{QCD}})$ to within approximately 20% of the experimental value.

We now consider the effect of the near-critical PTC dynamics on electroweak physics, first discussing vacuum alignment and the S parameter. S was estimated for a one-family $SU(2)_{TC}$ model in Ref. [11]. However, there it was assumed that the TC dynamics was essentially QCD-like, and that strong ETC effects explicitly broke the global flavor symmetry from SU(16) [since there are 8 flavors, but there is no distinction between **2**'s and $\bar{\mathbf{2}}$'s in SU(2)] to $SU(8)_L \times SU(8)_R$, which was then spontaneously broken to $SU(8)_V$. This assumption was important for producing the correct vacuum alignment [12].

Here we are not relying on assumptions about ETC dynamics, but on the combined effect of QCD and near-critical PTC. The effect of QCD is twofold. First, the SU(16) symmetry is explicitly broken down to $SU(3)_c \times SU(2) \times SU(2) \times SU(4) \times U(1)_Q \times U(1)$, where $U(1)_Q$ corresponds to techniquark number. This symmetry is then spontaneously broken, first to $SU(3)_c \times SU(2)_V \times SU(4) \times U(1)_Q$ by the techniquark condensate, and finally to $SU(3)_c \times SU(2)_V \times Sp(4) \times U(1)_Q$ by the technilepton condensate.

QCD is essential in ensuring that the chiral symmetry breaking for the techniquarks produces the correct vacuum alignment [12]. Assuming that spectral density functions of an SU(2) technicolor theory are similar to those of QCD, it has been shown that the chiral symmetry breaking of the technileptons [$SU(4) \rightarrow Sp(4)$] will break electromagnetism rather than $SU(2)_L$ [12]. Here, however, the chiral symmetry breaking scale for the technileptons, $\mathcal{O}(4\pi f_L)$, is below that of the techniquarks, $\mathcal{O}(4\pi f_Q)$. If the splitting of scales is sizable enough, then at the scale where the technileptons condense, the techniquarks will have already broken $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. It can then be argued that electromagnetism remains unbroken since the $SU(2)_L$ gauge boson contribution to the vacuum energy is cut off in the IR by the W and Z masses. In general, the analysis is complicated by the fact that the spectral density functions for a PTC theory may bear little resemblance to their QCD analogs. In what follows, we will assume that correct vacuum alignment is achieved.

The techniquark symmetry breaking produces the three Goldstone bosons required for the W and Z masses and an additional techniaxion common to one-family models. The technilepton symmetry breaking $SU(4) \rightarrow Sp(4)$ produces five pseudo-Nambu-Goldstone bosons (PNGB's). Three have electroweak quantum numbers and mix with the techniquark Goldstones. We anticipate that the combination orthogonal to the longitudinal gauge bosons, composed primarily of technileptons, will receive a large mass from new, high-energy interactions (unspecified here) that explicitly break the SU(4) symmetry. The other two technilepton PNGB's are $SU(2)_L$ and QCD singlets, and remain massless without the new SU(4)-breaking interactions. The techniaxion must also rely on new, high-energy interactions to provide a mass. In this case, the interactions must explicitly break the U(1) symmetry.

The approximate SU(16) symmetry of the model implies that there are 110 additional (colored) scalars, which would be conventional PNGB's if $SU(2)_{PTC}$ were strong enough by itself to spontaneously break SU(16). Their masses could then be computed perturbatively in α_s [12] and would be of $\mathcal{O}(\sqrt{\alpha_s}\,4\pi f_Q)$. Here, with QCD required to help with the breaking, the computation of their masses is subtle, since as $\Lambda_{QCD} \rightarrow 0$ there is no chiral symmetry breaking, and therefore no Nambu-Goldstone bosons. It is plausible, however, to assume that if we consider fluctuations around the broken vacuum, perturbative QCD effects still lead to masses of $\mathcal{O}(\sqrt{\alpha_s}\,4\pi f_Q)$ for these scalars. They could also receive even larger masses from the new, high-energy interactions discussed above.

Of the 110 colored scalars, 56 have already been considered in Ref. [11], where they were true PNGB's. Their contribution to S, along with that of the colorless PNGB's discussed above was estimated to be no larger than 0.6, with the contribution becoming smaller as their masses approach $4\pi f_Q$. Of the 54 new colored scalars, the $SU(3)_c$ triplet leptoquarks and diquarks produce a one-loop [2] contribution to S which we estimate to be less than $\mathcal{O}(0.1)$. Turning to contributions to S arising at scales $4\pi f_Q$ and above, it was noted in Ref. [11] that if there is a sizable mass splitting between techniquarks and technileptons, as could arise in the present theory, and a splitting between the technielectron and technineutrino, then the technileptons could also give a negative contribution to S. While the model described so far has no mechanism for the splitting of the technielectron and technineutrino, the smaller technilepton masses will be more sensitive than the techniquark masses to the new, high-energy interactions necessary to give mass to the quarks and leptons.

Putting all this together, our crude estimates, including those of Ref. [11], suggests that there is a significant range of parameter space where the full contribution to S in this model may lie below the 95% confidence experimental upper limit of approximately 0.17. Of course, a truly reliable estimate of S is not yet available in a non-QCD like technicolor theory, and this is especially true of this model with its complex spectrum of techniparticles.

We next consider the isospin breaking effects resulting from the interactions (unspecified here—such interactions could be generated by a light composite scalar which may form in the breaking of a chiral gauge theory down to a theory with an IR fixed point [13]) that produce the coupling of the top quark to the techniquark condensate and hence the top quark mass. To accomplish this we first need an estimate of the Goldstone boson decay constant f_Q in the techniquark sector. If the techniquarks and technileptons were degenerate, then using the relation $f_L^2 + 3f_Q^2 = v^2$ we would have $f_Q = f_L = v/2 \simeq 123$ GeV. In the present model the techniquarks are heavier than the technileptons, and for purposes of numerical estimates we will simply take $f_Q = 2f_L$ (i.e., $f_Q \simeq 136$ GeV).

Recent quenched lattice results [14] suggest that the average of the up and down quark masses, $\hat{m}$, is smaller than previously assumed values: $\hat{m} \approx 3.6$ MeV. Calculations with dynamical quarks find even smaller values for $\hat{m}$ [14]. Using the well known relation between the quark condensate $\langle \hat{\bar\psi}\psi \rangle$, $\hat{m}$, and the pion mass, this leads to a new estimate of the QCD condensate,

$$\frac{\langle \bar\psi\psi \rangle}{f_\pi^3} = \frac{m_\pi^2}{2\hat{m} f_\pi} \approx 27 \approx 8\pi \,. \tag{10}$$

Using this relation to estimate the techniquark condensate, we have for the top quark mass

$$m_t \approx \frac{27 f_Q^3}{\Lambda_t^2} \frac{\Lambda_t}{f_Q} \,, \tag{11}$$

where Λ_t is the scale (which can in principle be below the scale where PTC is embedded in a larger gauge group) of the physics that induces the top quark coupling (depending on how this coupling is produced there may be an additional factor of N_c in the condensate factor) to the condensate. The factor Λ_t/f_Q accounts for the high-energy enhancement effects of IR fixed point (walking) dynamics [1]. Using $m_t = 175$ GeV, we find $\Lambda_t \simeq 2.9$ TeV. In making this estimate, we have assumed that $\Lambda \gg \Lambda_t$ so that α stays very close to α_*, and therefore the dynamical techniquark mass falls only like $1/p$, up to momenta of order Λ_t. If this is not the case, then a smaller value of Λ_t will be necessary. We have also taken the walking to start at the scale f_Q, which is an additional uncertainty in the calculation.

In generic models, we expect that these isospin violating interactions which produce m_t will also induce isospin violating, effective four-techniquark interactions, which contribute to $\Delta\rho_*$. To lowest order in the interaction this so-called "direct" contribution can be estimated to be

$$\Delta\rho_*^d \approx \frac{(3 f_Q^2)^2}{v^2 \Lambda_t^2} \,, \tag{12}$$

which is less than 0.5% as long as $\Lambda_t > 3.2$ TeV. In Ref. [11], it was pointed out that there are also potentially negative contributions to $\Delta\rho_*$ arising from PNGB's. To higher order in the isospin violating interactions, there will be additional contributions to $\Delta\rho_*$. These include effects that arise from the mass difference between the U and D techniquarks. In order that they also remain beneath the experimental bound, the U and D must be nearly degenerate [15].

To conclude, we have explored a new framework for technicolor model building which relies on the near-critical behavior of a theory with an approximate IR fixed point. Since QCD effects are crucial for rendering the fixed point only approximate and producing electroweak symmetry breaking, the weak scale is predicted in terms of the QCD scale. The combination of near-critical PTC and QCD can significantly split the techniquarks and technileptons, and provide a framework for reducing the value of S [11]. Whether the electroweak gauge symmetry is broken in the correct pattern, and whether the predicted weak scale and the deviations from the minimal standard model agree with experiment will require further study.

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