Cavity-Induced Interference Pattern with Dark Center from Two Fluorescing Atoms

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We show that the resonance fluorescence from a symmetric system of two incoherently pumped atoms coupled to a cavity mode produces interference fringes that possess a minimum on the symmetry plane. This unique deviation from Young-type interferences can be explained intuitively by the process of stimulated emission and reabsorption, leading to a cavity-induced anticorrelation of the atomic dipoles. Detailed consideration of an optical pumping scheme reveals the surprising role of cavity damping. We propose an experiment utilizing a two-level system in In^+. [S0031-9007(97)04211-7]

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In a recent experiment by Eichmann *et al.* [1] it was demonstrated that Young-type interferences may be observed in the light scattered by a pair of trapped ions which are weakly and coherently driven. The ions elastically scatter the driving field and, therefore, act as point sources of coherent light similar to the slits in Young's original experiment. The case of a strong field has been investigated by Kochan *et al.* [2], who established that the reduction of fringe visibility, arising from the increasing dominance of inelastic scattering processes, may be partially recovered by coupling the two atoms to a cavity mode. We shall report here an even stronger cavity-induced modification of the interference pattern that occurs when the coherent driving field is replaced by an incoherent pump. In stark contrast to the Young-type interference patterns resulting from coherent excitation [1–3], we find for the first time that an intensity *minimum* can occur at line center despite the setup being entirely symmetric. This is an intrinsically quantum-mechanical effect with no classical analog and arises from the destructive quantum interference of the two paths coupling the antisymmetric Dicke state [4] to the atomic ground state via the cavity.

In this Letter, we study a pair of incoherently pumped two-level atoms coupled to a single cavity mode. By allowing the atomic excitation rate to be greater than the decay rate, we generalize the system to the simplest theoretical model of a two-atom laser. For the two-level model of a one-atom laser, the statistical [5,6] and spectral [7] properties have been studied in detail. Furthermore, the introduction of a pump operator has permitted the treatment of more realistic, multilevel schemes [8]. The culmination of this effort has been the first proposal for an experimental realization of a one-atom laser in the form of the ion-trap laser [9]. These theoretical proposals, together with recent advances in the trapping of two ions [1,10], suggest that it is now sensible to investigate microscopic laser systems utilizing more than one atom [11]. Here, we focus on the far-field intensity pattern of the fluorescence from a two-atom laser below and above threshold. The interference fringes are of particular significance as they provide a measure of atom-atom correlations. For weak pumping, the fringes are shown to possess a minimum in the center, i.e., on the symmetry plane perpendicular to the line connecting the atoms. On the other hand, a maximum is found at line center in the laser regime. These effects are explained using an intuitive picture of stimulated emission and reabsorption as well as more rigorously in terms of an optical pumping scheme and quantum interference.

The two-atom laser model under consideration, as shown in Fig. 1, is described by the master equation

$$\frac{\partial}{\partial t}\rho = \frac{1}{i\hbar}[H,\rho] + L_{\text{atom}}\rho + L_{\text{field}}\rho$$

for the atom-field density operator ρ with the Tavis-Cummings Hamiltonian [12] in the case of resonant interaction

$$H = -\hbar g_1(a\sigma_1^\dagger + a^\dagger\sigma_1) - \hbar g_2(a\sigma_2^\dagger + a^\dagger\sigma_2)$$

and the Liouville operators

$$L_{\text{atom}}\rho = -\frac{R_{AB}}{2}\sum_{k=1}^{2}(\sigma_k^\dagger\sigma_k\rho + \rho\sigma_k^\dagger\sigma_k - 2\sigma_k\rho\sigma_k^\dagger) - \frac{R_{BA}}{2}\sum_{k=1}^{2}(\sigma_k\sigma_k^\dagger\rho + \rho\sigma_k\sigma_k^\dagger - 2\sigma_k^\dagger\rho\sigma_k),$$

$$L_{\text{field}}\rho = -\frac{A}{2}(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger)$$

describing atomic relaxation and field damping, respectively. We have introduced the atomic decay rate R_{AB}, the pump rate R_{BA}, the cavity decay rate A, the photon annihilation (creation) operator a ($a^\dagger$), and the atomic lowering operators $\sigma_1 = |B_1\rangle\langle A_1| \otimes 1$ and $\sigma_2 = 1 \otimes |B_2\rangle\langle A_2|$

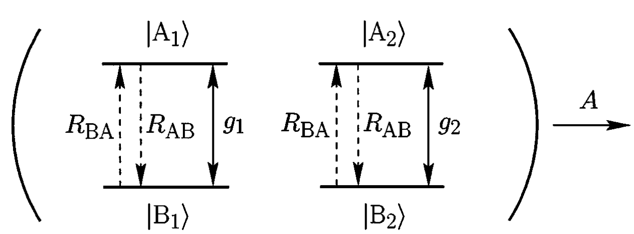

FIG. 1. Schematic representation of the two-atom laser.

for the first and second atoms, respectively. The interatomic spacing is assumed to be much larger than the transition wavelength so that cooperative effects [4,10] can be ignored. Furthermore, we consider the atoms to be well localized and neglect their external motion. Unless stated otherwise, we restrict the discussion to the case $g_1 = g_2 = g$.

For the remainder we focus on the fringe contrast factor

$$C = \frac{\langle \sigma_1^\dagger \sigma_2 + \sigma_2^\dagger \sigma_1 \rangle}{\langle \sigma_1^\dagger \sigma_1 + \sigma_2^\dagger \sigma_2 \rangle}$$

in the steady state. We stress that $|C|$ is simply the fringe visibility of the far-field intensity pattern

$$|C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}.$$

The sign of C, however, contains additional information about the position of the interference maxima with respect to the line center. For positive values of C, the intensity of the fluorescent light displays a maximum on the symmetry plane; for negative C, there is a minimum. In this way the nature of the atom-atom correlations may be extracted from the contrast factor. All symmetric two-atom systems studied so far [1–3] display a positive contrast factor C, indicating that the atomic dipoles are always oriented in the same sense. However, this is not true for the present atom-field system. In Fig. 2, we establish that strong anticorrelations, corresponding to a negative contrast factor, may exist.

We now present an intuitive physical picture for the occurrence of cavity-induced anticorrelation with the help of Fig. 3. For incoherent pumping, the atoms are correlated solely through their interaction with the cavity mode. Furthermore, for a weak pump, the resonator field will be approximately in the vacuum state and the two atoms only weakly excited with most of their population remaining in the ground state. We now consider an event in which one of the atoms, due to its small excited-state population, undergoes a stimulated emission process. As a result, a single excitation is deposited into the cavity mode. There now exists a likelihood that the second atom absorbs the photon because of the high probability that it initially occupies the ground state. Thus, for the case of weak pumping, we would expect the atomic dipoles to be anticorrelated leading to a minimum at line center. This heuristic argument predicts that the anticorrelation should survive for pump strengths sufficiently small such that there is no atomic inversion and only a small number of cavity photons. By comparing Fig. 2 with Figs. 4 and 5, depicting, respectively, the steady-state atomic inversion and mean photon number, we find that this is indeed the case.

For stronger pump strengths, we find a positive fringe contrast factor, indicating that the atomic dipoles have the same orientation. This is a consequence of the pump being sufficiently strong to invert the atoms. Similar to the behavior found in studies of the one-atom laser [5,6], this permits the accumulation of a coherent cavity field. Therefore, the interference pattern will be comprised not only of inelastically scattered pump radiation, but also of elastically and inelastically scattered light from the coherent cavity field. The occurrence of elastic scattering processes naturally leads to an intensity maximum at line center, as found in previous studies [1–3]. With this heuristic argument based on population inversions and cavity photon statistics, we would expect that the maximum at line center will persevere provided that the pump strength has not crossed the second lasing threshold at which lasing terminates [5,9]. For pump strengths beyond the second threshold, the contrast factor will approach zero due to the diminished contribution of elastic scattering. This may be confirmed by inspection of Fig. 4, in which we identify the lasing regime as the section of the curve where the inversion has a linear relationship with pump strength. Alternatively, the thresholds for lasing can be inferred from the laser peak in Fig. 5.

It is worth noting that the laser peak of Fig. 5 is a factor of 4 greater for the two-atom laser than for the

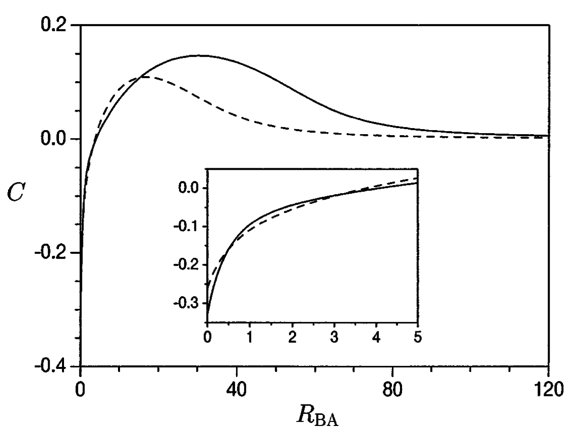

FIG. 2. Fringe contrast factor C versus pump rate R_{BA} for $R_{AB} = 1$ and $g = 3$. The symmetric case $g_1 = g_2 = g$ (solid curve) is compared to the asymmetric case $g_1 = g$ and $g_2 = g/2$ (dashed curve). All rates are in units of A. The inset focuses on the weak-pump regime.

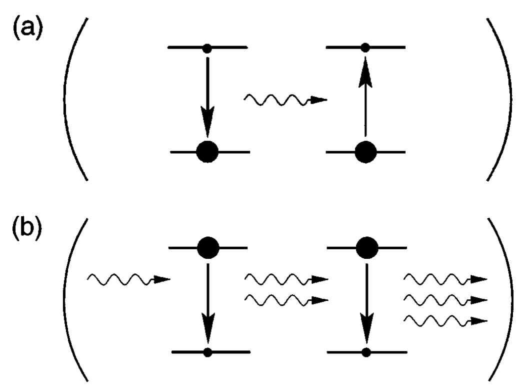

FIG. 3. Intuitive explanation for the (a) negative and (b) positive atom-atom correlation.

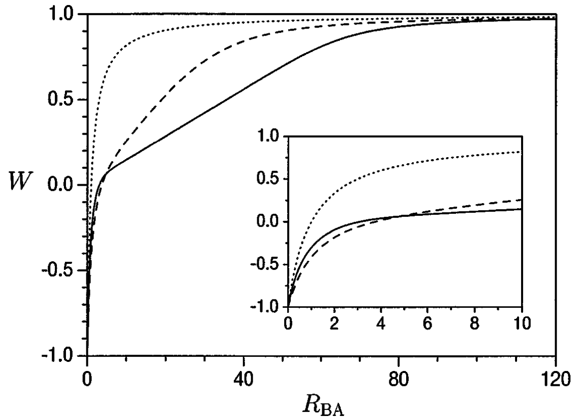

FIG. 4. Atomic inversion W versus pump rate R_{BA} for $R_{AB} = 1$ and $g = 3$. The two-atom laser with $g_1 = g_2 = g$ (solid curve) is compared to the one-atom laser (dashed curve) and to a single atom not coupled to a cavity field (dotted curve). All rates are in units of A. The inset focuses on the weak-pump regime.

one-atom laser, similar as in Ref. [11]. This increase is related to the extra depletion of pump-induced atomic inversion, relative to the one-atom case, as observed by comparing the solid and dashed curves in Fig. 4. Closer inspection of Fig. 4 (see inset) reveals that for weak pumping less excited population is depleted with two atoms than with one. This is a natural consequence of the reabsorption process sketched in Fig. 3(a). One might be tempted to think that there is also an anticorrelation when the atoms are weakly excited by a thermal cavity field instead of an incoherent pump. In such a case, however, the excitation process itself correlates the atomic dipoles and this correlation cannot be compensated by the emission-and-reabsorption process described above.

A more rigorous and quantitative understanding of cavity-induced anticorrelation may be arrived at by studying the ladder of atom-field states shown in Fig. 6. We

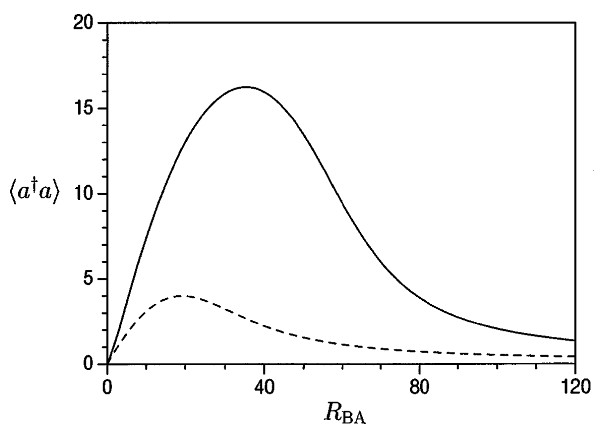

FIG. 5. Mean photon number $\langle a^\dagger a \rangle$ versus pump rate R_{BA} for $R_{AB} = 1$ and $g = 3$. The two-atom laser with $g_1 = g_2 = g$ (solid curve) is compared to the one-atom laser (dashed curve). All rates are in units of A.

first examine the case $R_{AB} = 0$ to highlight the special role of the antisymmetric Dicke state. If the atoms are not pumped ($R_{BA} = 0$), the steady state of the field will be the vacuum and the atoms will in general be in the ground state, i.e., the atom-field system is described by the state vector

$$|\psi_1\rangle = |B_1, B_2\rangle \otimes |0\rangle .$$

Without atomic relaxation, however, the steady-state solution is not unique. Instead, there is a second stationary state, in which the atoms occupy the antisymmetric Dicke state,

$$|\psi_2\rangle = \frac{|A_1, B_2\rangle - |B_1, A_2\rangle}{\sqrt{2}} \otimes |0\rangle = |-, 0\rangle .$$

This can be seen from the equation $H|\psi_2\rangle = 0$, which summarizes the destructive interference of the two probability amplitudes for the deexcitation of the first and second atoms. In the absence of atomic decay, the steady state can thus be any incoherent superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$. The assumption of an exactly vanishing decay rate R_{AB} is, of course, unphysical, and any nonvanishing decay will always lead to the steady state $|\psi_1\rangle$. Nevertheless, as we now show, the state $|\psi_2\rangle$ continues to play an important role in a more general situation. This is of particular relevance since population of the antisymmetric state corresponds to an anticorrelation of the atomic dipoles. Thus, population of this state will contribute to a negative contrast factor.

Figure 6 depicts a schematic representation of the lowest levels and transitions for the atom-field system in terms of the atomic Dicke states $|A_1, A_2\rangle$, $|B_1, B_2\rangle$, $|\pm\rangle = (|A_1, B_2\rangle \pm |B_1, A_2\rangle)/\sqrt{2}$ and the Fock states $|n\rangle$ of the cavity mode. States with the same excitation energy are horizontally aligned. All the coherent and incoherent couplings between the states due to atom-field interaction, atomic decay, pumping, and cavity damping are represented by arrows. Note that the antisymmetric states $|-, n\rangle$ do not couple coherently to any other state. However, they are populated by incoherent processes with the same rates as the symmetric states $|+, n\rangle$.

The physics leading to a preferred population of the antisymmetric state, and correspondingly an intensity minimum at line center, is most transparent in the case

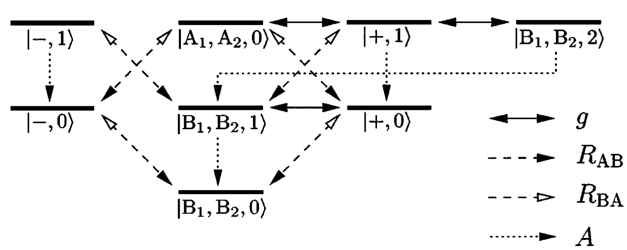

FIG. 6. Schematic representation of the lowest levels and transitions for the total atom-field system with the atomic Dicke states $|\pm\rangle = (|A_1, B_2\rangle \pm |B_1, A_2\rangle)/\sqrt{2}$.

where $R_{BA} \ll R_{AB}, g, A$. In such a regime the steady-state density operator will be an incoherent superposition of the four lowest energy states: $|B_1, B_2, 0\rangle$, $|\pm, 0\rangle$, and $|B_1, B_2, 1\rangle$. A preferred population of the state $|-, 0\rangle$ relative to $|+, 0\rangle$ will now occur for the simple reason that they are both incoherently populated with equal rates, but depopulated with unequal rates. Specifically, $|+, 0\rangle$ has an additional decay channel to the ground state $|B_1, B_2, 0\rangle$ via the state $|B_1, B_2, 1\rangle$, as shown in Fig. 6. The rate of depopulation of $|+, 0\rangle$ through this cavity-induced channel is naturally maximized for large values of g. Furthermore, for a given coupling strength g an optimum value of the cavity decay rate A exists. It is clear that small values of A will result in a low rate of depopulation as the state $|B_1, B_2, 1\rangle$ will be only marginally damped. Large values of A will also result in low rates of depopulation by suppressing the coherence of the transition $|+, 0\rangle \rightarrow |B_1, B_2, 1\rangle$ before any significant exchange of atomic to cavity excitation can occur. Thus we have the surprising result that the anticorrelation between the atomic dipoles is strongest neither for a good cavity ($A \ll g$), nor for a bad cavity ($A \gg g$), but in the intermediate regime.

Having considered the role of the cavity in inducing anticorrelations, we now consider the influence of atomic decay in the weak-pump limit. Atomic decay has the effect of destroying anticorrelations by reducing the difference between the depopulation rates of $|+, 0\rangle$ and $|-, 0\rangle$. Therefore, for increasing R_{AB} the fringe contrast factor approaches zero. On the other hand, C approaches -1 as R_{AB} tends towards zero.

The features described above are manifest in the exact analytical expression for the contrast factor in the weak-pump limit. By solving a judiciously chosen subset of the equations of motion we find

$$C_0 \equiv \lim_{R_{BA} \to 0} C = \frac{-4g^2 A}{4g^2(A + 2R_{AB}) + AR_{AB}(A + R_{AB})}.$$

As may be inferred from Fig. 2, this limit is particularly interesting since it gives for a given system the lowest possible contrast factor. The analytical expression shows that the contrast factor is minimized for $A = g\sqrt{8}$; that is, dissipation can play a constructive role in the generation of quantum correlations.

We finally propose an experimental scheme using two trapped In^+ ions localized in a resonator to obtain interference fringes with a minimum at the center. To continually cool the ions, the cavity mode could be detuned to a motional sideband. By coupling the In^+ transition between $|B\rangle = |5\,^1S_0\rangle$ and $|A\rangle = |5\,^3P_1\rangle$ to a circularly polarized, incoherent pump field Γ and a resonator mode, a two-level system with decay rate $R_{AB} = 2.3$ MHz $+ \Gamma$, pump rate $R_{BA} = \Gamma$, and wavelength $\lambda = 230.6$ nm is selected [13]. For $g = 3$ MHz (corresponding to a mode volume $V = 0.5 \times 10^{-12}$ m^3), $A = 10$ MHz, and $\Gamma = 0.2$ MHz, a negative contrast factor of 37% is obtained. In this case, each atom has a probability of 0.1 to be in the excited state, resulting in a flux of 5×10^5 fluorescence photons per second. The cavity-induced anticorrelation is robust with respect to variations of the parameters.

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