Controlling the Decoherence of a "Meter" via Stroboscopic Feedback

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We propose a simple modification of the experimental scheme employed by Brune *et al.* [Phys. Rev. Lett. **77**, 4887 (1996)] for the generation and detection of a Schrödinger cat state, in which the decoherence of the cat state can be significantly slowed down using an appropriate feedback. [S0031-9007(97)04106-9]

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Decoherence is the rapid destruction of the phase relation between two quantum states of a system caused by the entanglement of these two states with two different states of the environment [1]. The progressive decoherence of a mesoscopic Schrödinger cat has been observed for the first time in the experiment of Brune *et al*. [2], where the linear superposition of two coherent states of the electromagnetic field in a cavity with classically distinct phases has been generated and detected. In this Letter we describe a scheme in which decoherence due to spontaneous emission can be significantly mitigated by coherent feedback.

In Ref. [2], a Schrödinger cat state for the microwave field in a superconducting cavity C has been generated using circular Rydberg atoms crossing a cavity prepared in a coherent state. All the atoms have an appropriately selected velocity and the relevant levels are two adjacent Rydberg states which we denote as $|g\rangle$ and $|e\rangle$. The atoms are initially prepared in the state $|e\rangle$. The high-Q superconducting cavity is sandwiched between two low-Q cavities R_1 and R_2, in which classical microwave fields can be applied and which are resonant with the transition between the state $|e\rangle$ and the nearby lower circular state $|g\rangle$. The intensity of the field in the first cavity R_1 is then chosen so that, for the selected atom velocity, a $\pi/2$ pulse is applied to the atom as it crosses R_1. As a consequence, the atomic state before entering the cavity C is $|\psi_{\rm atom}\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$.

The high-Q cavity C is off resonance with respect to the $e \to g$ transition. However we will allow for a strong field pulse to shift the excited state into resonance with the cavity C. In the off-resonant case, the atom and the field cannot exchange energy but only undergo dispersive frequency shifts depending on the atomic level [3]. The field undergoes an equal and opposite phase shift for the ground and excited states. However as only the relative phase of the two components matters we may attribute all the phase shift to the excited state component and assume no phase shift for the ground state component. We shall assume for simplicity that the atom velocity can be chosen so that the phase shift, $\phi = \pi$. For the generation of a cat state one has to correlate each atomic state to a *superposition* of coherent states with different phases, and this is achieved by submitting the atom to a second $\pi/2$ pulse in the second microwave cavity R_2, so that the state becomes $|\psi'_{\rm atom+field}\rangle = (|e\rangle|\alpha-\rangle + |g\rangle|\alpha+\rangle)/\sqrt{2}$, where we have defined the even (+) and odd (−) Schrödinger cat states as $|\alpha_\pm\rangle = N_\pm^{-1}(|\alpha\rangle \pm |-\alpha\rangle)$, where $N_\pm^2 = 2(1 \pm e^{-2|\alpha|^2})$. This shows that an even or an odd coherent state is generated in the cavity according to the fact the atom is detected in the level $|g\rangle$ or $|e\rangle$, respectively.

To detect the cat state in the cavity we inject a second excited atom, the probe atom, into the system. As shown in [4], the conditional dynamics for the probe atom is defined by the transformation

$$|e\rangle|\alpha_-\rangle \to -|e\rangle|\alpha_-\rangle\,, \qquad |e\rangle|\alpha_+\rangle \to |g\rangle|\alpha_+\rangle\,. \quad (1)$$

This controlled-not dynamics (the atomic state flips only for an odd cat state in the cavity) is an effect of the $\phi = \pi$ phase shift per photon. When there is an even cat, the cavity C does not change the atomic state and the two $\pi/2$ pulses sum up to a single π pulse. In the case of an odd cat, the e component of the atomic state changes sign and the two $\pi/2$ pulses cancel each other. The atomic state of the probe atom is precisely correlated with either an even or odd cat state so that if the probe atom is found in the excited state, the state of the field prepared by the first atom was indeed an odd cat state.

The Schrödinger cat state undergoes a very fast decoherence process [5] caused by the inevitable presence of dissipation in the superconducting cavity, which is described by the following density matrix:

$$\rho(t) = \frac{1}{N_\pm^2}\Big[|\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}| + |-\alpha e^{-\gamma t/2}\rangle\langle-\alpha e^{-\gamma t/2}| \pm e^{-2|\alpha|^2(1-e^{-\gamma t})} \times \big(|-\alpha e^{-\gamma t/2}\rangle\langle\alpha e^{-\gamma t/2}| + |\alpha e^{-\gamma t/2}\rangle\langle-\alpha e^{-\gamma t/2}|\big)\Big], \quad (2)$$

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where γ is the cavity decay rate and where the plus (minus) sign corresponds to the even (odd) coherent state. Decoherence is governed by the factor $\exp[-2|\alpha|^2(1 - e^{-\gamma t})]$, which for $\gamma t \ll 1$ becomes $\exp[-2|\alpha|^2\gamma t]$, implying therefore that the interference terms decay to zero with a lifetime $t_{\mathrm{dec}} = (2\gamma|\alpha|^2)^{-1}$.

A greater insight into the decoherence process is obtained by considering the conditional evolution when the decay channel is monitored. This leads to the quantum trajectory picture of a decaying cavity field [6]. If a photon is lost from a cat state, the character of the cat flips from even to odd and vice versa. Between photon emission events, the amplitude of each component of the cat simply decays at half the cavity decay rate, but the character of the cat does not change. Thus by detecting whether or not the cat changes its character we can know if a photon has been lost from the cavity. This knowledge may then be used to feedback on the cavity field to try to return the field to the desired state.

In the experiment of Brune *et al.* [2] the progressive decoherence of the cat state has been observed for the first time. This was achieved by sending a second atom, with the same velocity, through the same arrangements of cavities, after a time delay T. The probability of detecting the second atom in the e or g state is [2]

$$P_{\substack{g\\e}} = \frac{1}{2}\left(1 \pm \mathrm{Re}\{\mathrm{Tr}[e^{-i\pi a^\dagger a}\rho(T)]\}\right). \qquad (3)$$

If one inserts in (3) the explicit expression of $\rho(T)$ given by (2), one gets the four conditional probabilities P_{ij}, $(i, j = e \text{ or } g)$, of detecting the second atom in the state j after detecting the first atom in the state i and which give a satisfactory description of the decoherence process of the cat state. Let us consider, for example, the case of two successive detections of the circular Rydberg state e: in this case the detection of the first atom projects the microwave field in the superconducting cavity in an odd coherent state and the corresponding conditional probability is given by

$$P_{ee}(T) = \frac{1}{2}\left[1 - \frac{e^{-2|\alpha|^2 e^{-\gamma T}} - e^{-2|\alpha|^2(1-e^{-\gamma T})}}{1 - e^{-2|\alpha|^2}}\right]. \qquad (4)$$

The dependence of this conditional probability upon the time delay between the two atom crossings gives a clear description of the cat state decoherence. In fact, if there is no dissipation in the cavity, i.e., $\gamma T = 0$, it is $P_{ee} = 1$ and this perfect correlation between the atomic state and the cavity state is the experimental signature of the presence of an odd coherent state in the high-Q cavity. As long as $\gamma \neq 0$, the conditional probability decreases for increasing delay time T. At a first stage one has a decay to the value $P_{ee} = 1/2$ in the decoherence time $t_{\mathrm{dec}} = (2\gamma|\alpha|^2)^{-1}$; this is the decoherence process itself, that is, the fast transition from the quantum linear superposition state to the statistical mixture describing a *classical* superposition of fields with opposite phases. At larger delays T, the plateau $P_{ee} = 1/2$ turns to a slow decay to zero because the two coherent states of the mixture both tend to the vacuum state and start to overlap, due to field energy dissipation.

In the present paper, we propose a modification of the experiment of Brune *et al.* [2], in which the cat decoherence is not simply monitored but also controlled in an active way. In particular we show that by using an appropriate feedback scheme, it is possible to slow down significantly the decoherence process. To fix the ideas, we shall consider only the case in which the experimentally studied quantity is the conditional probability P_{ee}.

Applying a feedback loop to a quantum system means subjecting it to a series of measurements and then using the result of these measurements to modify the dynamics of the system. Wiseman and Milburn have developed a quantum theory of continuous feedback [7]. This theory has been applied in Ref. [8] to show that an appropriate continuous feedback loop can be used to slow down the decoherence of a Schrödinger cat in an optical cavity. In the Brune *et al.* experiment [2] it is not possible to monitor continuously the state of the radiation in the cavity, since the involved field is in the microwave range and there are not good enough detectors in this wavelength region. In this case, continuous measurement can be replaced by a series of *repeated* measurements, performed by off-resonance atoms crossing the superconducting cavity one by one with a time interval T. As a consequence, one could try to apply a sort of "discrete" feedback scheme modifying in a "stroboscopic" way the cavity field dynamics according to the result of the atomic detection.

We will consider only the case where atomic detection of the first atom prepares an odd cat state. From Eq. (1), we see that the state of the probe atom is correlated with an even or odd cat, and may thus be used to determine if the cat has undergone a flip from odd to even by photon emission. The feedback loop must supply the cavity with a photon whenever the probe atom is found in state g, while it has to do nothing when the atom is detected in the e state. This can be realized with a switch to Stark shift a subsequent atom onto resonance with respect to the radiation mode in the superconducting cavity whenever the probe atom is detected in the g state after crossing the cavity. The on-resonance atom can now deposit a single photon in the cavity. We will determine the time evolution of P_{ee} in the presence of feedback.

The time evolution of the microwave field in the high-Q cavity can be described by the transformation from the state just before the crossing of a nonresonant Rydberg atom to the state of the radiation mode before the next nonresonant atom crossing. This transformation is given by the composition of two successive mappings $\rho' = \Phi(\rho) = \Phi_{\mathrm{diss}}(\Phi_{\mathrm{fb}}(\rho))$, where Φ_{fb} describes the effect of the interaction with the nonresonant probe atom followed by the conditional effect of the resonant feedback atom. The operation Φ_{diss} describes instead the dissipative evolution of the field mode during the time interval T between measurement and feedback steps, and it is characterized by the energy relaxation rate γ.

To construct Φ_{fb} we must first determine the conditional state of the cavity, given the state of the probe atom. These are given by

$$\rho_{\substack{g \\ e}} = \frac{1}{4}[e^{-i\pi a^\dagger a}\rho e^{i\pi a^\dagger a} + \rho \pm e^{-i\pi a^\dagger a}\rho \pm \rho e^{i\pi a^\dagger a}]. \tag{5}$$

Second, we need to determine the change in the conditional state when a resonant atom is injected in the excited state. The feedback mechanism acts only if the atom has been found in g, and corresponds to injecting a resonant excited state atom [9]. The details will be presented elsewhere. The effect of the nonunit efficiency of the atomic detectors η, which is of the order of $\eta = 0.4$ in the actual experiment, must also be included. Combining all the operations, we derive the explicit expression of the feedback operator Φ_{fb}:

$$\begin{aligned}\Phi_{\mathrm{fb}}(\rho) = {} & \eta\rho_e + \eta\cos(\mu\sqrt{aa^\dagger})\rho_g\cos(\mu\sqrt{aa^\dagger}) \\ & + \eta a^\dagger \frac{\sin(\mu\sqrt{aa^\dagger})}{(aa^\dagger)^{1/2}}\rho_g\frac{\sin(\mu\sqrt{aa^\dagger})}{(aa^\dagger)^{1/2}} a \\ & + (1-\eta)[\rho_e + \rho_g],\end{aligned} \tag{6}$$

where $\mu = \Omega\tau$, with Ω denoting the resonance Rabi frequency and τ is the atom-field interaction time. The probability of releasing the photon within the high-Q cavity is maximized when the sine term in (6) is maximum. In the case of the Schrödinger cat state studied here this essentially corresponds to the condition $\mu|\alpha| = \pi(m + 1/2)$ (m integer) and it can be obtained with an appropriate selection of the velocity of the feedback atom. Here we assume that the feedback resonant atoms come from a second source and that their state is not detected after exiting the cavities. In writing this expression we have implicitly assumed that not only the off-resonant atom time of flight, but also the feedback loop delay time, is much smaller than the typical time scales of the system and that they can be neglected. This means considering only Markovian feedback and this simplifies considerably the discussion [7]. The operator Φ_{diss} describing the dissipative time evolution between two successive atom crossings can be obtained from the exact evolution of a cavity in a standard vacuum bath [10].

The general expression of the transformation Φ describing the transition from the state of the cavity field at time nT, i.e., just before the injection of the nth off-resonant probe atom, to the state at time $(n+1)T$, is written for density matrix elements in the following way ($\langle n|\Phi(\rho)|n+p\rangle = \rho'_{n,n+p}$):

$$\begin{aligned}\rho'_{n,n+p} = {} & \sum_{k=0}^{\infty}\Bigg\{\frac{c_{n,k}c_{n+p,k}}{4}[\eta s_-(n,k)^2 + 4(1-\eta) + \eta s_+(n,k)^2\cos(\mu\sqrt{n+k+1})\cos(\mu\sqrt{n+p+k+1})] \\ & + \eta\frac{c_{n,k+1}c_{n+p,k+1}}{4}s_+(n,k)^2\sin(\mu\sqrt{n+k+1})\sin(\mu\sqrt{n+p+k+1})\Bigg\} \\ & \times \rho_{n+k,n+p+k} + \eta\frac{c_{n,0}c_{n+p,0}}{4}\sin(\mu\sqrt{n})\sin(\mu\sqrt{n+p})s_-(n,0)^2\rho_{n-1,n+p-1},\end{aligned} \tag{7}$$

where

$$c_{n,k} = \sqrt{\frac{(n+k)!}{n!k!}e^{-n\gamma T}(1-e^{-\gamma T})^k}$$

and $s_\pm(n,k) = 1 \pm (-1)^{n+k}$. Equation (7) gives the stroboscopic time evolution of the microwave field in the superconducting cavity in the presence of the proposed feedback mechanism. This dynamics can be experimentally monitored from the reconstruction of the probability of detecting the off-resonance atoms in the state e, $P_e(nT)$, using Eq. (3) evaluated at times nT. The time evolution of this probability is plotted in Fig. 1, where an initial odd coherent state with $|\alpha|^2 = 3.3$ (just the value corresponding to that of the actual experiment of Ref. [2]) is considered. The full line refers to the no feedback case ($\mu = 0$), that is, the theoretical prediction of Eq. (4), the dashed line refers to $\mu = \pi/6$ and $\gamma T = 0.02$, the dotted line to $\mu = \pi/2$ and $\gamma T = 0.02$, horizontal crosses to $\mu = \pi/2$ and $\gamma T = 0.2$, and diagonal crosses to $\mu = \pi/6$ and $\gamma T = 0.2$. All the curves refer to the realistic case of a detection efficiency $\eta = 0.4$.

The comparison between the curves in the presence of feedback and that in absence of feedback is impressive: the decay of this probability can be not only slowed down, but also partially inhibited in the sense that the asymptotic value of P_e becomes nonzero.

However, the fact that P_e can be kept very close to one for an indefinite time does not mean that the initial odd cat

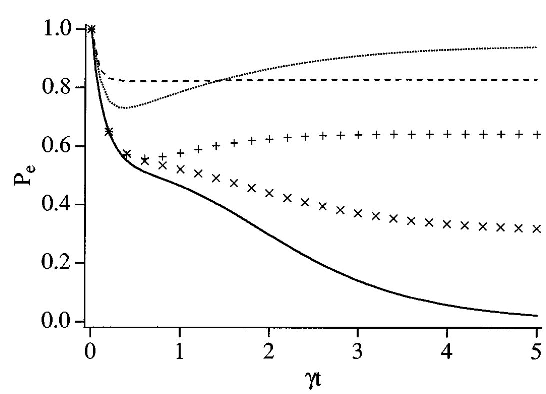

FIG. 1. Time evolution of the probability of detecting the off-resonant atoms in state e in the case when $|\alpha|^2 = 3.3$ and the detection efficiency is $\eta = 0.4$. Full line: $\mu = 0$ (no feedback case); dashed line: $\mu = \pi/6$ and $\gamma T = 0.02$; dotted line: $\mu = \pi/2$ and $\gamma T = 0.02$; horizontal crosses: $\mu = \pi/2$ and $\gamma T = 0.2$; diagonal crosses: $\mu = \pi/6$ and $\gamma T = 0.2$.

state can be preserved almost perfectly, because the quantity P_e gives only a partial information on the state of the radiation mode within the cavity [see Eq. (3)]. Perfect cat state "freezing" can be realized only in cavities with an infinite Q; the proposed feedback scheme inevitably modifies the initial state, even in the ideal conditions of perfect detection efficiency $\eta = 1$ and continuous feedback $\gamma T \approx 0$. In fact our model can preserve for an infinite time the initial photon number distribution only at best. But it causes a kind of phase diffusion, because the photon left in the cavity by the resonant atom has no phase relationship with those in the cavity. To state it in other words, our feedback scheme protects very well the relative phase of the coefficients of the two components of the initial cat state (which is π for the odd cat state) generating at the same time the diffusion of the phase of the two coherent states. The phase diffusion however is unconventional and slower than usual phase diffusion. This is still a relevant result because it shows how quantum coherence can be partially protected, only making a slight modification of the beautiful experiment of [2]. This is clearly shown by Figs. 2(a) and 2(b), where the Wigner function of the cavity state after a time $t = 0.44/\gamma$ ($t \sim 3t_{\mathrm{dec}}$) for the same initial odd coherent state with $|\alpha|^2 = 3.3$ considered in Fig. 1, is plotted. Figure 2(a) refers to the feedback case with $\mu = \pi/6$, $\gamma T = 0.02$, and $\eta = 0.4$, while Fig. 2(b) shows the situation in absence of feedback. The figures clearly show the effectiveness of our scheme: since $t \sim 3t_{\mathrm{dec}}$, the state in absence of feedback has become a mixture of two coherent states with opposite phases, and the oscillations associated to quantum coherence have essentially disappeared. On the contrary, the state evolved in presence of feedback is almost indistinguishable from the initial one and the interference oscillations are still very visible. Figure 2(a) also shows that the unconventional, feedback-induced phase diffusion is actually very slow, since its effects are not yet visible after $t \sim 3t_{\mathrm{dec}}$; moreover we have also checked that the rotationally invariant stationary state is not reached even after ten decoherence times.

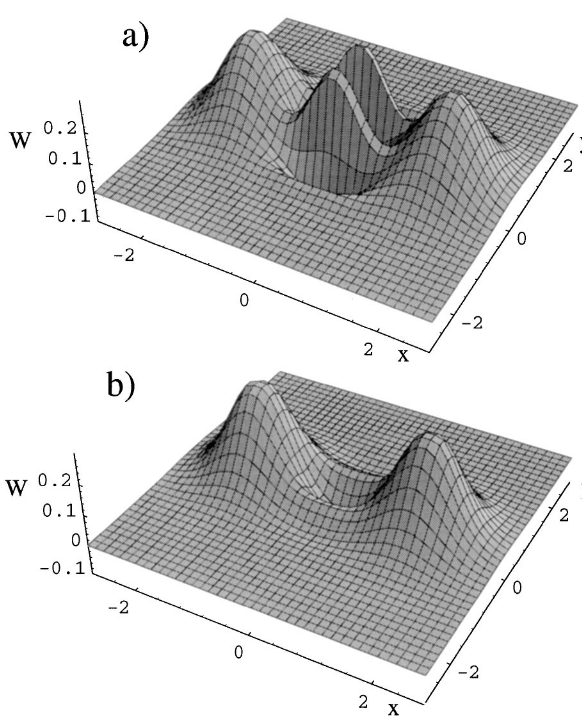

FIG. 2. Wigner function for an initial odd coherent state with $|\alpha|^2 = 3.3$ after an elapsed time $t = 0.44/\gamma$. (a) Evolution in presence of feedback with $\mu = \pi/6$, $\gamma T = 0.02$, $\eta = 0.4$; (b) no feedback case.

Here we have assumed that it is possible to send *exactly* one atom at a time in the cavity, while in [2] atomic pulses with an average number $\bar{n}$ less than one are used. Essentially, this is equivalent to having, in our model, an effective quantum efficiency $\eta_{\mathrm{eff}} = \eta \bar{n}$. Nonetheless, the performance of the feedback scheme could be improved with respect to that shown by the figures, where we have preferred to be as close as possible to the actual experimental values. In fact one could use more efficient atomic detectors and, above all, one could make the time interval between two successive detections T as small as possible. This is the most relevant parameter (see also Fig. 1) since decoherence can be better inhibited if one can "check" the cavity state, and eventually try to restore it, as soon as possible.

The scheme proposed here could also be useful for the use of cavity QED systems for quantum information processing. Within this context, most of the proposals that have already appeared adopt quantum error correction techniques [11] to oppose to decoherence. These proposals are difficult to realize experimentally, while here we propose a physical control of decoherence which can be implemented in an already performed experiment.

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