Heat Transport Scaling in Turbulent Rayleigh-Bénard Convection: Effects of Rotation and Prandtl Number

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We report experimental measurements of turbulent heat transport in rotating Rayleigh-Bénard convection. The fluid was water with Prandtl number $3 < \sigma < 7$. Heat transport and local temperature measurements were made for Rayleigh numbers $2 \times 10^5 < \mathrm{Ra} < 5 \times 10^8$ and Taylor numbers $0 \le \mathrm{Ta} \le 5 \times 10^9$. For fixed convective Rossby numbers Ro between 0.1 and 1.5, the Nusselt number N scaled closely as the $2/7$ power of Ra but had very little variation with the Prandtl number σ and only a moderate increase with increasing rotation rate. Substantial disagreement is found with existing scaling theories. [S0031-9007(97)04102-1]

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During the past decade, turbulence in Rayleigh-Bénard (RB) convection has become one of the premier testing grounds for concepts of turbulent boundary-layer scaling [1–4]. Adding rotation to turbulent convection addresses a variety of other issues of fundamental importance to many geophysical and astrophysical flows, from ocean deep convection to the convective motions in the sun and in planetary atmospheres. Rotation has many influences on convection including Ekman pumping and Ekman layers associated with differential rotation between boundaries and the interior flow, modification of thermal plumes into thermal vortex structures and the resulting vortex-vortex interactions, and reduction of horizontal length scales [5,6]. Thus, the introduction of rotation could potentially change the picture of turbulent RB convection developed for the nonrotating case. For example, the Coriolis force is often believed, on the basis of the Taylor-Proudman theorem, to produce a more two-dimensional flow for rapidly rotating systems. Recent numerical simulations, however, have shown that for turbulent flows such intuition is not always justified [7]. In addition, rotation provides a number of convenient ways to test some of the concepts of boundary-layer theory in the laboratory which are not easily accessible without rotation. For example, it is difficult to vary the ratio of horizontal to vertical scales without perturbing background heat transport contributions. For nonrotating convection, this ratio is set by the geometric diameter-to-height ratio $\Gamma = D/h$. In rotating convection, however, the horizontal length scale decreases with rotation rate [8,9], so one can effectively scan Γ without changing the cell conditions. Overall, laboratory investigations of turbulent rotating convection provide an excellent testing ground for boundary-layer concepts and is a fascinating and important system in its own right.

Turbulent RB convection in the presence of rotation is characterized by four dimensionless parameters: the Rayleigh number Ra proportional to the temperature difference across the fluid layer of height h, the Taylor number $\mathrm{Ta} = (4\pi f h^2/\nu)^2$ where f is the rotation frequency and ν is the kinematic viscosity, the Prandtl number $\sigma = \nu/\kappa$ where κ is the thermal diffusivity, and the aspect ratio Γ. The onset of convection varies with rotation and for high dimensionless rotation rates the bulk critical Rayleigh number Ra_c scales as $\mathrm{Ta}^{2/3}$. The heat transport is measured by the Nusselt number N which is the total heat transport normalized by the thermally diffusive component. Previously, there have been several experimental measurements of N [10–12] with rotation, but none have determined scaling relations for N as a function of rotation. The main reason for this is that measurements at fixed rotation rate, i.e., constant Ta, appear to asymptote to the nonrotating value of N [12] at high Ra. The probable cause of this behavior is that at fixed Ta larger Ra makes buoyancy relatively more important so that at high enough Ra rotation becomes irrelevant. To address this issue, it is useful to define another parameter for rotating convection which is a combination of the control parameters Ra, Ta, and σ. The *convective* Rossby number $\mathrm{Ro} = \sqrt{\mathrm{Ra}/(\sigma \mathrm{Ta})}$ is a ratio of the rotation period to the buoyancy free-fall time, and it was shown numerically that N scales with Ra as a $2/7$ power law for $\mathrm{Ro} = 0.75$ and for a fluid with $\sigma = 1$ [5]. This result indicates that at fixed Ro the effects of rotation relative to buoyancy are approximately constant and that maintaining Ro fixed would be a good method to test experimentally for heat transport scaling in rotating convection.

We have made precise measurements of N as a function of Ra, σ, and Ta numbers to obtain

$$N = A(\sigma, \mathrm{Ro})\mathrm{Ra}^{\alpha(\sigma,\mathrm{Ro})}. \qquad (1)$$

Our measurements of heat transport were improved by our ability to rotate the system. Rotation was used to suppress convection, thereby enabling an accurate measurement of the background thermal conductivity. Without rotation, we find $\alpha = 0.286 \pm 0.004$, consistent with the $2/7$ power law obtained in other experiments using water [13–15], and $A(\sigma = 6.7) = 0.164 \pm 0.006$. Despite the

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fundamental changes in flow structure resulting from rotation, the scaling exponent hardly changed, $0.268 < \alpha < 0.287$, over a range of $0 \leq \mathrm{Ta} < 10^{10}$. Over that same range, A increased with rotation, corresponding to an enhancement in N at equal Ra of about 20%. If interpreted purely on the basis of the increase in the effective Γ with rotation, a factor of between 3 and 5, the increase in N is opposite to the significant decrease predicted by the scaling theory of Shraiman and Siggia [3]; their $\Gamma^{-3/7}$ scaling yields a decrease in N by about a factor of 2. If, on the other hand, the important quantity for the heat transport scaling is the geometric aspect ratio rather than the ratio of horizontal to vertical scales, then important discrepancies with Shraiman and Siggia remain as we find no evidence for large scale circulation spanning the container [16]. In addition, we have measured the dependence of A on σ over a narrow range $3 < \sigma < 7$ and found it to be a much weaker function of σ than predicted by scaling theories [2–4] which show a $2/7$ power-law scaling of N with Ra.

The experimental apparatus is similar to one used previously for studies of rotating convection [12]. The convection cell consisted of anodized-aluminum top and bottom plates and Plexiglas sidewalls with a square cross section to facilitate visualization from the side. A small hole in the center of the top plate allowed a thermistor attached to a rigid rod to be positioned vertically using a stepping-motor-driven translation stage. The cell height was $h = 9.4$ cm and the interior lateral dimensions of one of the sides was $l = 7.3$ cm giving an aspect ratio of $\Gamma = l/h = 0.78$. Each plate's average temperature was measured with a set of four thermistors embedded in the top or bottom plate. Heat was supplied to the cell using a film heater on the bottom plate, and the top-plate temperature was regulated at 21.5 °C with rms fluctuations of 0.01 °C. The Prandtl number for water at the cell mean temperature used for most of the measurements was 6.3. More details of the apparatus will be presented elsewhere.

The parameter space for rotating convection is shown in Fig. 1 which illustrates some of the complications involved in interpreting heat transport results in that space. Previous measurements of N as a function of Ra [12] for $\mathrm{Ra} < 2 \times 10^7$ at fixed rotation rate showed that N was enhanced by rotation for some intermediate values of Ra but that N approached its nonrotating value at higher Ra. At fixed Ta, however, the influences of rotation and buoyancy change with Ra so that at high enough Ra buoyancy will dominate over rotation. This idea is consistent with the reported measurements. Numerical simulations of rotating convection [5,17] have demonstrated that if the convective Rossby number Ro is fixed, the relative strength of rotation is kept constant and scaling of N with Ra is observed with the same exponent $2/7$ as for nonrotating convection. In Fig. 1, we show lines of constant Ro in the space of Ra and Ta. Also shown for comparison is the linear stability line for the onset of bulk convection [18] and a line indicating the onset of turbulent scaling. Within this regime, we keep

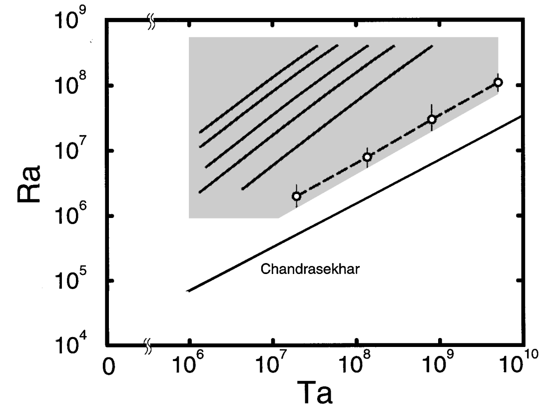

FIG. 1. Parameter space diagram Ra vs Ta. Measurements at fixed Ro were in the shaded area in which the five solid lines correspond to different values of Ro (from right to left, 0.30, 0.52, 0.75, 1.15, and 1.49). Symbols (∘) set the lower bound above which N exhibited approximate scaling with Ra. Theoretical prediction for onset of bulk convection under rotation is also shown [18].

Ro constant and are able to adjust its value through 1 which we find marks reasonably well the crossover from rotation-free to rotation-dominated behavior.

Over two decades of power-law scaling of N with Ra were observed when Ro was fixed. In Fig. 2, N is plotted versus Ra showing the scaling of the heat transport for several different values of Ro including the nonrotating case ($\mathrm{Ro} = \infty$) in the range $5 \times 10^6 < \mathrm{Ra} < 5 \times 10^8$. The curves for different Ro are quite parallel and close to a $2/7$ power law. For the nonrotating case, $\alpha = 0.286 \pm 0.004$ and $A = 0.164 \pm 0.006$ from a fit over the range $4 \times 10^7 < \mathrm{Ra} < 5 \times 10^8$. The lower end of this range was dictated by the "soft-to-hard" turbulence transition indicated by the change from Gaussian to exponential probability distribution functions (PDF). This transition was first observed in helium-gas convection and later in water [2,14]. From these and similar data, the scaling exponent α and the coefficient A as a function of Ro were obtained. First, however, because σ varies with temperature

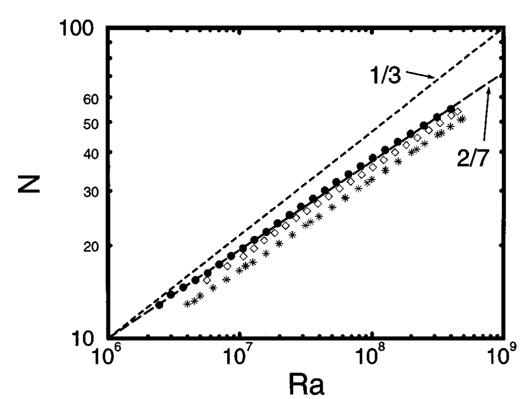

FIG. 2. N vs Ra at fixed Ro: 0.30 (•), 0.75 (◇), and ∞ (∗). The short dashed line and the long dashed line represent the $1/3$ and $2/7$ power laws, respectively.

and consequently with Ra because the top plate was maintained at fixed temperature, it was necessary to measure the variation of N with the mean-cell temperature. This also allowed us to test, in a narrow range but with high accuracy, the dependence of N on σ. Recently comparisons between results using very different fluids (mercury with $\sigma = 0.025$ and water with $\sigma \approx 6$) in the same convection cell show an increasing N with σ [15] between low and intermediate σ. To augment these results closely spaced measurements in a single cell are necessary for a quantitative test of the predicted scaling of N with σ. Because A is very sensitive to changes in α, we computed it assuming $\alpha = 2/7 = 0.286$. In Fig. 3, the dependence of A on σ is shown with and without a small correction for the temperature dependence of the background conductivity. Both data sets show very little dependence of A on σ. The $\alpha = 2/7$ scaling theories of Castaing *et al.* [2] and of Shraiman and Siggia [3] both predict a $\sigma^{-1/7}$ scaling [4]. Such a dependence is shown in the plot and clearly disagrees with the experimental data. A σ-independent A was predicted for $\sigma > 0.1$ but with a scaling exponent of $1/3$ [19]. Thus, the σ dependence of N in combination with the $2/7$ power law that we measure is unexplained by present scaling theories.

The scaling of heat transport with rotation shows an increase in N with decreasing Ro (higher rotation) and a power-law scaling close to $2/7$. In Fig. 4, we show the dependence of α, A_α, and $A_{2/7}$ on Ro where A_α is calculated for the corresponding value of α and $A_{2/7}$ assumes $\alpha = 2/7$. There is a small decrease in α with decreasing Ro but the magnitude of the variation, about 10%, is difficult to distinguish from possible systematic errors inherent in fitting power laws over only two decades in Ra. Analogous measurements, i.e., with rotation, in helium gas would be necessary to quantitatively evaluate the significance of this small change in scaling exponent with rotation. On the other hand, the increase in the coefficient A with decreasing Ro shows the enhancement of heat transport by rotation. The change in $A_{2/7}$ of about 25% is a better representation of the total absolute change in N because A_α is affected by the magnitude of α. For all three data sets, Ro ≈ 1 seems to separate scaling consistent with the nonrotating case from rotation-dominated scaling at small Ro. One plausible explanation for the increase in N with increasing rotation is that the vortex structures in rotating convection which are formed out of the boundary layer are more effective in extracting heat from the boundary layer than thermal plumes owing to Ekman suction [5,12]. This runs somewhat counter to the intuition based on the Taylor-Proudman theorem that rotation should suppress convection. Although this intuition applies to the increase in the *onset* of convection where the flow is approximately steady, our results clearly show that care needs to be taken in applying intuition developed for nonturbulent flows to states in the turbulent regime [7].

Another interesting feature of rotating convection is that the effective size of the container, i.e., its effective aspect ratio, varies with rotation rate. Previous measurements for helium-gas convection without rotation showed a decrease in N by a factor of about 1.5 for a 6.7-fold increase in Γ [20]. Rotation provides another means for testing the lateral-size dependence of N without physically changing the container. For example, based on the linear critical wave number [18], the effective aspect ratio increases by

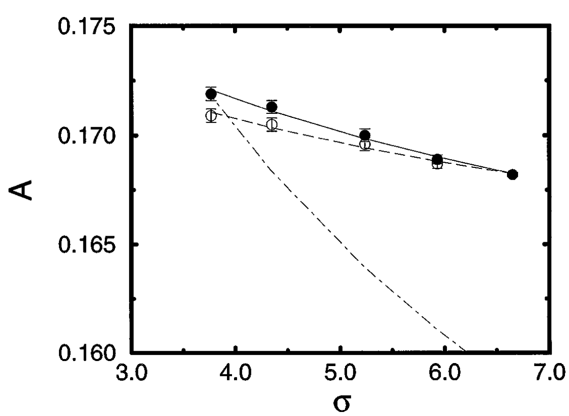

FIG. 3. Scaling coefficient A vs σ without correction ($\bullet$) and with temperature correction ($\circ$). Solid line is power-law fit to the data and the dot-dashed line is the predicted $\sigma^{-1/7}$ dependence.

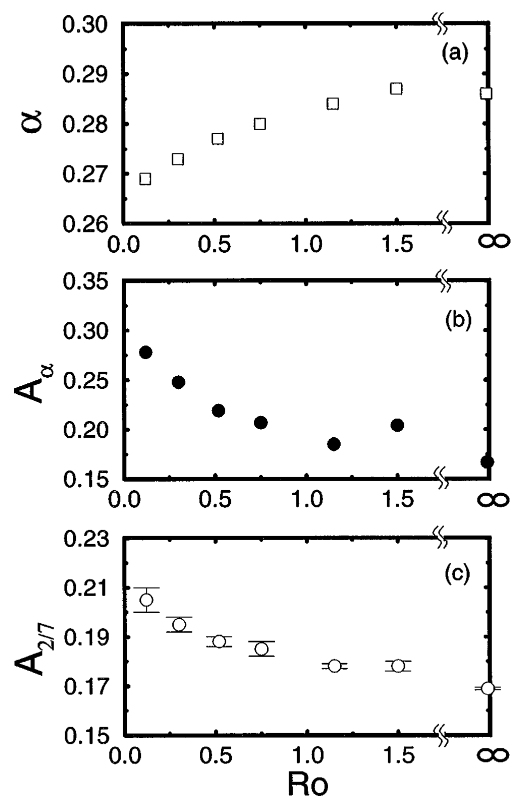

FIG. 4. (a) Scaling exponent α, (b) scaling coefficient A_α, and (c) scaling coefficient $A_{2/7}$ vs Ro. Statistical error bars in (a) and (b) are smaller than the data points but estimated systematic uncertainties are (a) ± 0.005 and (b) ± 0.05.

about 20 times from zero rotation up to $\mathrm{Ta} = 5 \times 10^9$. This overestimates the horizontal scale reduction in the weakly turbulent and turbulent regimes which has been estimated at about 5 by measurements of average vortex density as a function of Ra and Ta [8,9]. Ignoring other influences of rotation, this feature allows for a test of the lateral-size dependence of heat transport scaling, predicted by Shraiman and Siggia to vary as $\Gamma^{-3/7}$ power and independent of lateral size for the theory of Castaing *et al.* The former result would predict a *decrease* in N by a factor of about 2 between nonrotating convection and the largest Ta. Further, it would completely disrupt the scaling at fixed Ro because Γ increases by a factor of about 3 for $\mathrm{Ro} = 0.3$. Instead N *increases* with increasing rotation and $2/7$ scaling is observed for constant Ro. Although it is possible that the large decrease in N predicted by Shraiman and Siggia in combination with the decrease in horizontal scale caused by rotation could be balanced by an equally large increase in Ekman suction or other enhancements resulting from rotation, this coincidence seems unlikely especially with the observed $2/7$ scaling of N at fixed Ro. Supporting evidence for an increase in effective lateral size comes from temperature PDFs measured in the center of the convection cell. Without rotation, there is a transition from Gaussian to exponential PDFs consistent with previous work using water [14] as illustrated in Figs. 5(a) and 5(b). In sharp contrast, for any value of rotation we used, the PDFs had exponential tails for all accessible Ra. Some examples are shown in Figs. 5(c) and 5(d). Such behavior was also seen in helium-gas convection [20] and in numerical simulations [21] when the aspect ratio was increased. Our observations of exponential PDFs are in contradiction with predictions

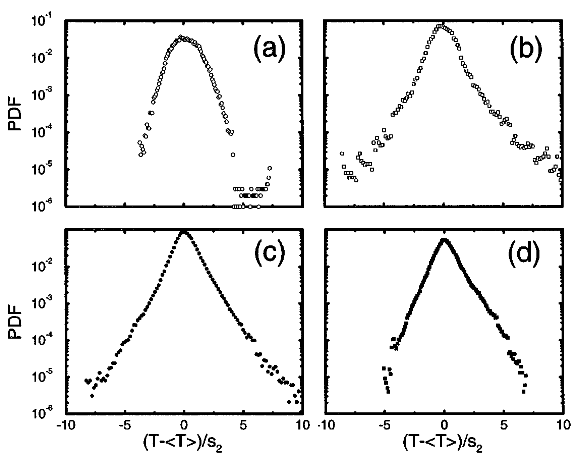

FIG. 5. Temperature PDFs at cell center without rotation ($\mathrm{Ta} = 0$) for (a) $\mathrm{Ra} = 1.1 \times 10^7$ and $\mathrm{Ra} = 4.0 \times 10^8$. With rotation for (c) $\mathrm{Ra} = 1.2 \times 10^7$ and $\mathrm{Ta} = 1.35 \times 10^6$ and (d) $\mathrm{Ra} = 4.0 \times 10^8$ and $\mathrm{Ta} = 5.04 \times 10^9$. Notice the transition from Gaussian (a) to exponential (b) PDFs without rotation. Gaussian PDFs were not observed with rotation. Normalization factor s_2 is the standard deviation of the distribution and $\langle T \rangle$ is the mean temperature.

from numerical simulations [5] that temperature PDFs for rotating convection would have Gaussian tails at all Ra and experimental measurements at much higher $\mathrm{Ra} \approx 10^{11}$ that show Gaussian PDFs [22]. These differences have not been resolved. As our results confirm, the problem of turbulent convection continues to be a great source of interest and of new insights which challenge both theory and experiment.

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