Anomalous Dynamics of a Single Ion in an Optical Lattice

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We have investigated the dynamics of a single ion in a one-dimensional optical lattice by tracing its position through fluorescent photons. A $^{24}Mg^{+}$ ion was radially confined in a two-dimensional radio-frequency trap, while an optical lattice superimposed on a weak electric potential was applied along the free axis. By decreasing the optical potential depth, a change of the transport characteristics from diffusive to quasiballistic was observed. Corresponding to this change, the distribution of the ion position showed an excess fluctuation reminiscent of Lévy walks. A statistical analysis of these fluctuations confirmed a significant deviation from the Gaussian diffusion law. [S0031-9007(97)04069-6]

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Laser cooling of atoms has opened up many new fields, including the opportunity to study random walks [1–6]. The first impressive example was found in the Brownian motion of ultracold atoms in an optical molasses [1], where atoms were cooled and confined by photon recoil to form a viscous medium. The cooling and confinement rely on velocity- or position-dependent absorption of laser photons followed by spontaneous emission. The complexity in the absorption process gives diversity to the random walks of laser-cooled atoms; Brownian as well as anomalous diffusion such as Lévy flights [2,7] can be observed, depending on the cooling technique. Lévy flights have generated broad interest over a cross disciplinary field in recent years [8,9], because of their strange kinetics, self-similarity, and fractal nature. In laser cooling, Lévy statistics have been employed to analyze the atomic momentum distribution in velocity selective coherent population trapping [2]; also Lévy walks are predicted to occur in atomic transport in optical lattices [6].

This Letter presents the first study of the single ion dynamics in a one-dimensional (1D) optical lattice superimposed on a weak harmonic potential. The experiment measured the trajectory of the ion via its reemitted photons using a position-resolved, single-photon, detection technique. This single particle observation allows us to see the microscopic processes in the atomic transport, where an anomalous diffusion process such as Lévy walks can best manifest its unique features. The resultant macroscopic effects, the change of the spatial diffusion constant and the existence of long range correlations, are analyzed by applying statistical techniques to the obtained trajectories.

A red-detuned optical lattice with polarization gradients has been successfully used to realize sub-Doppler temperatures with the help of the Sisyphus effect [10]. It can produce atomic samples with kinetic energies of less than half the optical potential depth, leading to a localization of atoms at the minimums of the potential [11,12]. Since the atoms are trapped in the intensity maximums, the atomic lifetime in a specific well is limited because energy fluctuations due to spontaneous emission occasionally allow the atom to accumulate enough energy to overcome the potential barrier. This causes random walks of the atoms between lattice sites [3]. To date these atomic transports have been studied only in relatively deep optical potential wells, where atoms show Brownian (Gaussian) diffusion [5] because the Sisyphus damping force together with the lattice periodicity strongly limit the random walk step size.

For shallower optical potentials, it has been predicted that these energy fluctuations lead to long flights of an atom that can reach over many wavelengths before it is eventually trapped again, since the Sisyphus damping force is negligible for an atom with a large momentum which is far outside the momentum capture range p_c [6,10,13]. The resultant atomic trajectory is reminiscent of random walk phenomena called Lévy walks [8]. Macroscopically this results in a breakdown of the Gaussian diffusion law and causes a divergence of the spatial diffusion constant. Marksteiner *et al.* [6] discussed the occurrence of this anomalous diffusion in terms of the slowly decaying momentum correlation due to the weak damping force. However, this paper will discuss it using energy correlations. The superimposed weak harmonic potential allows us to observe the potential energy of the ion from its spatial distribution measured with an appropriate averaging time. Once the ion acquires enough kinetic energy to significantly exceed the barrier, it becomes insensitive to the Sisyphus damping and oscillates in the superimposed potential. The total energy is then virtually unchanged for some time period, until the ion is captured by one of the optical potentials with the help of Sisyphus damping. This results in anomalous fluctuations of the ion's kinetic energy with long range correlations whose origin is attributed to the existence of p_c, as discussed with spatial Lévy walks [6]. In order to interpret the experimental results, we performed Monte Carlo simulations

that calculate the classical trajectory of an ion in our total 1D potential [14,15].

We used an rf quadrupole ring trap [16] to store a single $^{24}Mg^{+}$. It gave a radial confinement with a secular frequency of 900 kHz. Along the axis of the quadrupole field the ion was confined by a weak electric potential with an oscillation frequency of $\omega/2\pi = 13$ kHz. A periodic optical potential was produced tangential to the trap axis with a pair of counterpropagating, crossed-linear-polarized, laser beams, which were slightly red detuned from the $^2S_{1/2} - ^2P_{3/2}$ transition of $^{24}Mg^{+}$ at $\lambda = 280$ nm (natural linewidth $\Gamma/2\pi = 43$ MHz). A sketch of the total 1D potential is depicted in Fig. 1(a).

The experiment concentrated on the axial motion of the ion as measured by its displacement $x(t_i)$ from the minimum of the electric potential. The ion position was determined by detecting its fluorescent photons which pass through a microscope lens with a numerical aperture (NA) of NA $= 0.28$, and are then projected onto a single-photon-counting position analyzer with a time resolution of 10 μs. The total photon detection efficiency was 10^{-4}. The effective position resolution was estimated experimentally to be ~ 3 μm, due mainly to the thermal motion of the ion in the focal direction. A single measurement recorded the position and time of arrival of 2^{16} photons as the series $\{x(t_i)\}$. This required total observation times $T_{\rm obs}$ of up to a few minutes. The signal to noise ratio was 10^2–10^3, limited mainly by the presence of fluorescence light scattered from the surface of the trap electrodes. An example of an ion trajectory is shown in Fig. 1(b). The analysis of the ion trajectories employed, for different time scales, two kinds of statistical techniques: an autocorrelation function and a random walk analysis.

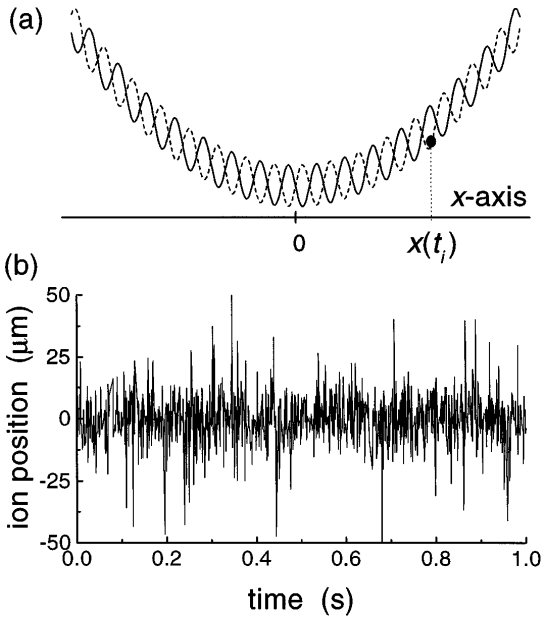

FIG. 1. (a) A schematic diagram of the 1D potential: an optical lattice superimposed on a weak electric potential. The light shifted Zeeman substates $m = \pm 1/2$ produce the potentials shown as solid and dotted lines. (b) An example of an ion trajectory measurement. $x(t_i)$ denotes the ion's displacement from the center of the electric potential at time t_i.

A position-autocorrelation function $\langle x(0)\,x(\tau)\rangle$ was used to analyze the spatial diffusion of an ion in the short time scale. It was calculated from $\phi(\tau)$, the average of the squared displacements between position detections separated by a time delay τ. $\phi(\tau)$ is thus defined as

$$\langle [x(t+\tau) - x(t)]^2 \rangle = 2\langle x^2 \rangle - 2\langle x(0)\,x(\tau)\rangle\,, \qquad (1)$$

where the angle brackets denote averaging over time t. In this manner the contribution of stray photons and any position uncertainty in the optical imaging are averaged out and result in a constant offset.

Figure 2 shows the shape of $\phi(\tau)$ for different optical potential depths $U_{\rm opt} = -\frac{2}{3}\hbar\Delta s_0$ [10], where $s_0 = \frac{I/I_{\rm sat}}{1+(2\Delta/\Gamma)^2}$, $I_{\rm sat} = \frac{2\pi^2\hbar c\Gamma}{3\lambda^3}$, Δ, and I are the saturation parameter, the saturation intensity of the transition, the laser detuning, and the intensity, respectively. The potential depth is given in units of photon recoil energy $E_R = \frac{\hbar^2 k^2}{2m}$, with $E_R/h = 106$ kHz. For deep optical potentials where a slow rise in $\phi(\tau)$ is seen, a spatial diffusion constant can be defined by the slope of $\phi(\tau)$ at $\tau = 0$, i.e., $2D_X \equiv \phi'(0)$ [17]. The spatial diffusion of an ion in this short time period is so small that the effect of the superimposed electric potential is negligible and the diffusion constant can be compared with a pure 1D lattice [6]. We observed a minimum diffusion constant of 6×10^{-8} m^2/s or $23\hbar/m$ at $U_{\rm opt} \sim 300E_R$. The diffusion constant increased by a factor of 10, roughly inversely proportional to $U_{\rm opt}$, in the range $200E_R$ to $100E_R$, and eventually diverged. These observations agree with our Monte Carlo simulation within a factor of 2 [15].

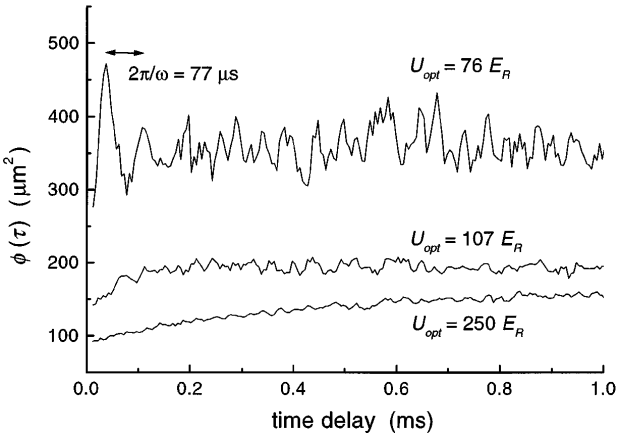

FIG. 2. The squared position difference $\phi(\tau)$ as a function of the time delay τ. The optical potential depth for each curve is indicated in the figure, obtained by changing the laser detuning from $\Delta = -3.2\Gamma$ ($U_{\rm opt} = 250E_R$) to -10.8Γ ($76E_R$) with a fixed laser intensity of $I = 12I_{\rm sat}$.

As the optical potential depth is decreased, the slope of the initial increase in $\phi(\tau)$ becomes steeper until the function eventually displays a sinusoidal oscillation at the frequency ω of the electric potential. This clearly shows the atomic transport changing from a diffusive motion between lattice sites to a quasiballistic motion in the electric potential. The quasiballistic motion implies that the optical potential is too shallow to confine the ion and that harmonic oscillator states exist on a longer time scale. In this unlocalized regime the average kinetic energy of the ion can be estimated from its time-averaged spatial distribution in the electric potential.

In this parameter range, excess long-lived fluctuations in the ion's position distribution were found. In order to evaluate these fluctuations we divide up the total observation time $T_{\rm obs}$ into 2^{11} intervals and calculate a mean square average value of the ion's displacement $u(t_n) \equiv \frac{1}{N_n}\sum_{t_n<t_i<t_{n+1}} x(t_i)^2$ for each interval, where N_n is the number of sampling points and $t_n = n\frac{T_{\rm obs}}{2^{11}}$ the starting time of the nth interval. On the average, each interval corresponds to more than 10^2 oscillation periods in the electric potential and contains $\sim 2^5$ position detections. Monte Carlo simulations have shown that, even with a deep optical potential, the potential energy $U_n = \frac{1}{2}m\omega^2 u(t_n)$ determined from the position distribution of the ion in the electric potential can still be associated approximately with its kinetic energy, as long as the averaging time is much larger than both the ion's localization time in an optical well and the oscillation period of the electric potential. Figure 3(a) shows the temporal behavior of U_n for two different optical potential depths. For a shallow optical potential, large fluctuations are observed, which are reminiscent of Lévy walks [7,8] in the sense that there is no definite characteristic time scale for the appearance of these energy jumps.

To characterize the fluctuations quantitatively, we take U_n as the step size of a random walk and evaluate the total random walk distance after n steps starting from n_0, which is given by $s_n = \sum_{k=n_0}^{n_0+n} U_k$. The statistical nature of the random walks can be determined by the power law dependence of the root mean square (rms) fluctuation of s_n,

$$\Delta s_n = \sqrt{\langle s_n^2\rangle_{n_0} - \langle s_n\rangle_{n_0}^2} \propto n^{\alpha}, \qquad (2)$$

where brackets denote expectation values averaging over all possible starting points n_0. A diffusion process is defined as Gaussian for $\alpha = \frac{1}{2}$ and anomalous for $\alpha \neq \frac{1}{2}$. The result of this analysis is shown in Fig. 3(b). When $U_{\rm opt} = 250E_R$, we observed an exponent α of 0.51, which indicates that the diffusion process of $\{U_n\}$ is Gaussian. On the other hand, for $U_{\rm opt} = 85E_R$, α was 0.77 for an averaging time τ_c of up to a second, which is a significant deviation from Gaussian diffusion.

We verified the origin of this anomalous diffusion in the following ways. First, the polarizations of the lattice lasers were changed to remove Sisyphus cooling so that the ion was effected only by far detuned Doppler cooling [18]. Second, to confirm the existence of time correlation in the fluctuation, we randomly shuffled the time order of $\{U_n\}$ and calculated the rms fluctuation Δs_n [9]. Both procedures suppressed the anomalous character and led to Gaussian diffusion with $\alpha = \frac{1}{2}$. The result for the second procedure is indicated by the dashed line in Fig. 3(b). In addition, a histogram of the spatial distribution of the ion showed a broader wing than that of a Gaussian. These pieces of evidence confirm that the origin of the anomalous behavior of $\{U_n\}$ lies in the broad momentum distribution with a long correlation time, which is introduced by the polarization gradient field with a shallow optical potential depth, as discussed in Ref. [6].

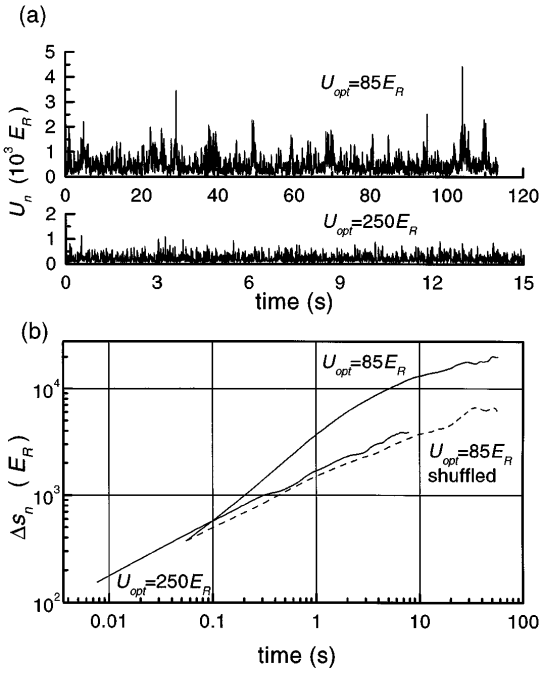

FIG. 3. (a) Temporal fluctuations of the ion's potential energy U_n estimated from the mean square displacement for two different optical potential depths. (b) Log-log plot of the rms fluctuation Δs_n vs time for the respective fluctuations of U_n shown in (a). The dashed line was generated by data shuffling for $U_{\rm opt} = 85E_R$; see text.

Finally, we studied the change of the exponent α as a function of the optical potential depth, which is shown in Fig. 4. The exponent was determined from the linear part of the slope at short times, as is seen in Fig. 3(b). For a wide range of experimental parameters where polarization gradient cooling was effective the diffusion process was approximately Gaussian. A transition to anomalous diffusion was observed at $U_{\rm opt} \sim 100E_R$. The inset shows the corresponding kinetic energy of the ion estimated from the spatial distribution, $E_K = \frac{1}{4}m\omega^2\phi(\infty)$; one can see the general trend of polarization gradient cooling, i.e., the kinetic energy decreases proportional to $U_{\rm opt}$ and finally

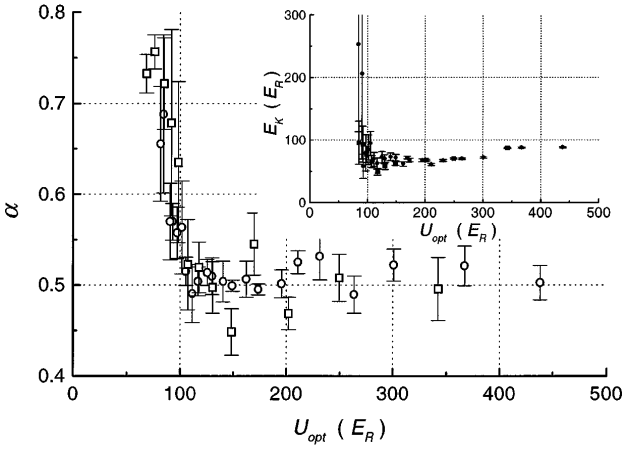

FIG. 4. The exponent α as a function of the optical potential depth U_{opt}. The error bars show the standard deviation calculated from several data sets measured at the same laser parameters. The change from Gaussian ($\alpha = 0.5$) to anomalous diffusion is seen at $U_{\mathrm{opt}} \sim 100E_R$. The inset shows the corresponding kinetic energy of the ion. The circles and squares show the data taken on different days.

diverges [19]. This divergence of the kinetic energy is found to correlate with the onset of anomalous diffusion, which shows that the divergence of the kinetic energy in polarization gradient cooling is related to these anomalous energy fluctuations.

Quantitative comparisons of our results with the existing theories [6] are not straightforward because of the superimposed electric potential, the spatially varying micromotion [20,21], and the damping force due to Doppler cooling. From our Monte Carlo simulations [15] which neglected micromotion and Doppler cooling, the threshold for anomalous diffusion was at $U_{\mathrm{opt}} \sim 50E_R$ with a correlation time as short as $\tau_c \sim 30$ ms for $U_{\mathrm{opt}} = 33E_R$. Our experimental results showed a threshold U_{opt} twice as large and τ_c nearly a hundred times longer for $U_{\mathrm{opt}} = 85E_R$, as is seen in Fig. 3(b). We speculate that these discrepancies can be attributed to the influence of residual micromotion which appears away from the minimum of the electric potential. In the absence of micromotion, the energy correlation should be truncated near the turning point of the external potential because the ion velocity would be close to zero and thus inside the capture range of the Sisyphus cooling [10]. However, due to the influence of micromotion in the experiment [21], the ion is actually heated more than cooled near the turning points of the electric potential. Therefore, the existence of micromotion sustained the energy correlation and allowed us to see the anomalous fluctuations unexpectedly clearly. The energy correlation is finally truncated by Doppler cooling, which limits the average kinetic energy by its equilibrium temperature of $\sim 10^3 E_R$ for our laser parameters of $U_{\mathrm{opt}} = 85E_R$ with $I = 12I_{\mathrm{sat}}$ and $\Delta = -9.7\Gamma$ [18].

In summary, we have measured the spatial diffusion constant and the microscopic process of the divergence of the kinetic energy in polarization gradient cooling by employing a novel single ion tracing technique. The origin of the long-range correlations of the latter is closely related to the anomalous transport of an atom in an optical lattice, recently predicted in Ref. [6]. A better confinement in the radial direction will allow us to measure the ion position with a diffraction limited resolution. An extension of this study to a few ions would be interesting because the strong motional coupling between the ions due to Coulomb interactions can introduce cooperative effects in the appearance of anomalous diffusion.

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