

## Reconciling High-Scale Left-Right Symmetry with Supersymmetry

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(Received 9 April 1997)

We construct the minimal supersymmetric left-right theory and show that at the renormalizable level it requires the existence of an intermediate  $B - L$  breaking scale. The subsequent symmetry breaking down to the minimal supersymmetric standard model automatically preserves  $R$  symmetry. Furthermore, unlike in the nonsupersymmetric version of the theory, the see-saw mechanism takes its canonical form. The theory predicts the existence of a triplet of Higgs scalars much lighter than the  $B - L$  breaking scale. [S0031-9007(97)03954-9]

PACS numbers: 12.60.Jv, 11.30.Fs, 12.60.Cn

There is no doubt that the minimal supersymmetric standard model (MSSM) has become the most popular extension of the standard model (SM). However, one of the most appealing features of the SM is lost in its supersymmetric counterpart: automatic conservation of baryon and lepton numbers. In supersymmetry (SUSY), unless some mechanism of suppression is found, baryon number violation, as is well known, is catastrophically fast.

It turns out that another popular extension of the standard model, the Left-Right (L-R) symmetric theory [1] offers a natural solution to this MSSM problem. The  $B-L$  symmetry, which is a part of L-R models, automatically forbids all the baryon and lepton number violating operators [2]. L-R theories are interesting in their own right, for among other appealing features, they offer a simple and natural explanation of the smallness of neutrino mass through the so-called see-saw mechanism [3,4].

In view of this, it becomes important to systematically study L-R supersymmetric theories, in order to arrive at a realistic minimal supersymmetric left-right model (MSLRM). Up to now, the only serious attempt in this direction is the work of Kuchimanchi and Mohapatra [5] which showed that in the minimal version of the theory no spontaneous symmetry breaking takes place [6]. Furthermore, when this is cured through the introduction of a parity-odd singlet, the soft SUSY breaking terms inevitably lead to the breaking of electromagnetic charge invariance. This is true at least for a scale of L-R symmetry breaking  $M_R$  above 10 TeV. In this Letter we stick to the physically motivated assumption of  $M_R$  being much larger than the scale of supersymmetry breaking  $M_S$  taken to be not far from the electroweak scale:  $M_S \approx M_W$ . We show that this problem disappears if one allows for an intermediate  $B - L$  breaking scale. Furthermore, the physically unappealing singlet becomes redundant.

The most important result of our study is that *at low energies the model reduces to the MSSM with an exact  $R$  parity*. This is a clear and testable prediction which follows from the underlying gauge symmetry and the pattern of symmetry breaking. Since in the MSSM there

is no control over  $R$  parity, we find it useful to provide an example of a theory that makes this precise prediction. Recently a connection between  $R$  parity and  $U(1)_{B-L}$  has been stressed [7]. A phenomenologically interesting feature of the theory is the possibility of a low-lying  $B - L$  scale,  $M_{BL} \gtrsim 1$  TeV.

Furthermore, the see-saw mechanism in this theory takes its canonical form  $m_\nu \approx m_D^2/M_{BL}$  (where  $m_D$  is the neutrino Dirac mass term), as opposed to the non-supersymmetric version of L-R models or  $SO(10)$  grand unified theories (GUTs). Namely, despite its generic see-saw form, the neutrino mass in ordinary L-R theories depends unfortunately on the unknown parameters of the Higgs potential.

Another important prediction of the theory regards the Higgs masses: one finds an  $SU(2)_L$  triplet with a mass of the order of  $M_{BL}^2/M_R$  (or  $M_S$ , depending which scale is bigger). This could provide a crucial test for the theory with low  $M_{BL}$ .

*The minimal model: a brief review.*—For the sake of self-consistency, and in order to pave the way for the realistic model, we first review briefly the minimal model.

The so-called minimal supersymmetric left-right model is based on the gauge group  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . It contains three generations of quark and leptonic chiral superfields with the following transformation properties:

$$\begin{aligned} Q &= (3, 2, 1, 1/3), & Q_c &= (3^*, 1, 2, -1/3), \\ L &= (1, 2, 1, -1), & L_c &= (1, 1, 2, 1), \end{aligned} \quad (1)$$

where the numbers in the brackets denote the quantum numbers under  $SU(3)_c$ ,  $SU(2)_L$ ,  $SU(2)_R$ , and  $U(1)_{B-L}$ , respectively. The Higgs sector consists of

$$\begin{aligned} \Phi_i &= (1, 2, 2, 0), & (i = 1, 2), \\ \Delta &= (1, 3, 1, 2), & \bar{\Delta} = (1, 3, 1, -2), \\ \Delta_c &= (1, 1, 3, -2), & \bar{\Delta}_c = (1, 1, 3, 2). \end{aligned} \quad (2)$$

The number of bidoublets is doubled in order to achieve a nonvanishing Cabibbo-Kobayashi-Maskawa quark mixing

matrix, and the number of triplets is doubled for the sake of anomaly cancellations.

The gauge symmetry is augmented by a discrete parity or left-right symmetry under which the fields transform as

$$\begin{aligned} Q &\leftrightarrow Q^*, & L &\leftrightarrow L^*, & \Phi_i &\leftrightarrow \Phi_i^\dagger, \\ \Delta &\leftrightarrow \Delta_c^*, & \bar{\Delta} &\leftrightarrow \bar{\Delta}_c^*. \end{aligned} \quad (3)$$

The minimal model suffers from an incurable disease: *it cannot break parity spontaneously* [5]. One possible way out is to add a parity-odd singlet [8] which in our opinion is not so appealing. Moreover, although now parity could be spontaneously broken, it turns out that the same happens to the electromagnetic charge.

In this theory, as Kuchimanchi and Mohapatra show [5], the vacuum manifold contains a circle parametrized by an angle  $\theta$

$$\begin{aligned} \langle \Delta^c \rangle &= d \begin{pmatrix} 0 & \sin \theta \\ \cos \theta & 0 \end{pmatrix}, \\ \langle \bar{\Delta}_c \rangle &= \bar{d} \begin{pmatrix} 0 & \cos \theta \\ \sin \theta & 0 \end{pmatrix}, \end{aligned} \quad (4)$$

where  $d = \bar{d}$  in the absence of soft SUSY breaking terms. The problem appears when these terms are switched on, since in general the soft mass terms for  $\Delta_c$  and  $\bar{\Delta}_c$  will be different, whereas left-right symmetry was forcing them to be equal in the original superpotential valid at the scale of parity breaking  $M_R$ . In other words, at the scale of SUSY breaking  $M_S$  the world is not left-right symmetric anymore. Thus  $d = \bar{d}$  no longer holds, and it can be shown that the minimum corresponds to  $\theta = \pi/4$ , which breaks electromagnetic charge invariance. Notice that there is no hope that we live in the false

charge-preserving vacuum, due to the original continuous degeneracy. Our vacuum falls classically (without need for quantum tunneling) into the true charge-breaking one.

To avoid this, one could resort to the use of nonrenormalizable higher-dimensional terms as suggested in [9]. We prefer in what follows to focus on the phenomenologically attractive possibility of an intermediate  $B - L$  breaking scale.

*The  $B - L$  route.*—The idea here, often discussed in the context of ordinary L-R models, is to break  $SU(2)_R$  down to its subgroup  $U(1)_R$  while preserving  $B - L$ . This is achieved by including two new Higgs superfields  $\Omega$  and  $\Omega_c$  with the following quantum numbers [5]:

$$\Omega = (1, 3, 1, 0), \quad \Omega_c = (1, 1, 3, 0), \quad (5)$$

where under parity  $\Omega \rightarrow \Omega_c^*$ .

*What is new, however, is the fact that there is no need for the parity-odd singlet  $\Sigma$ .* This in our opinion is an important result and it tells us that in a sense this model is a realistic MSLRM at least at the renormalizable level. Furthermore, the vacuum expectation value (VEV) of the triplet  $\Omega_c$  splits the masses of the  $SU(2)_L \times U(1)$  Higgs doublets in the bidoublets  $\Phi$ , allowing thus for the MSSM at low energies.

We now show that parity can be broken spontaneously and at the same time electromagnetic charge is automatically preserved. The effect of introducing the  $B - L$  neutral triplets  $\Omega, \Omega_c$  is best appreciated by first considering the extremization of the potential at high scales  $M_R \gg M_S, M_W$ , where the effect of the soft breaking terms is negligible so that the potential has the form it takes for a supersymmetric gauge theory with superpotential

$$\begin{aligned} W_{LR} &= \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L_c + \mathbf{h}_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q_c + i\mathbf{f} L^T \tau_2 \Delta L + i\mathbf{f}^* L^{cT} \tau_2 \Delta_c L_c + m_\Delta \text{Tr} \Delta \bar{\Delta} + m_\Delta^* \text{Tr} \Delta_c \bar{\Delta}_c \\ &+ \frac{m_\Omega}{2} \text{Tr} \Omega^2 + \frac{m_{\Omega_c}^*}{2} \text{Tr} \Omega_c^2 + \mu_{ij} \text{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j + a \text{Tr} \Delta \Omega \bar{\Delta} + a^* \text{Tr} \Delta_c \Omega_c \bar{\Delta}_c + \alpha_{ij} \text{Tr} \Omega \Phi_i \tau_2 \Phi_j^T \tau_2 \\ &+ \alpha_{ij}^* \text{Tr} \Omega_c \Phi_i^T \tau_2 \Phi_j \tau_2, \end{aligned} \quad (6)$$

with  $\mathbf{h}_{q,l}^{(i)} = \mathbf{h}_{q,l}^{(i)\dagger}$ ,  $\mu_{ij} = \mu_{ji} = \mu_{ij}^*$ ,  $\alpha_{ij} = -\alpha_{ji}$ ,  $\mathbf{f}$  and  $\mathbf{h}$  are symmetric matrices, and generation and color indices are understood.

Supersymmetry implies  $F$  flatness conditions given by the following equations for the scalar fields:

$$\begin{aligned} F_{\bar{\Delta}} &= m_\Delta \Delta + a \left( \Delta \Omega - \frac{1}{2} \text{Tr} \Delta \Omega \right) = 0, \\ F_{\bar{\Delta}_c} &= m_\Delta^* \Delta_c + a^* \left( \Delta_c \Omega_c - \frac{1}{2} \text{Tr} \Delta_c \Omega_c \right) = 0, \\ F_{\Delta} &= m_\Delta \bar{\Delta} + i\mathbf{f} L L^T \tau_2 + a \left( \Omega \bar{\Delta} - \frac{1}{2} \text{Tr} \Omega \bar{\Delta} \right) = 0, \\ F_{\Delta_c} &= m_\Delta^* \bar{\Delta}_c + i\mathbf{f}^* L_c L_c^T \tau_2 + a^* \left( \Omega_c \bar{\Delta}_c - \frac{1}{2} \text{Tr} \Omega_c \bar{\Delta}_c \right) = 0, \\ F_{\Omega} &= m_\Omega \Omega + a \left( \bar{\Delta} \Delta - \frac{1}{2} \text{Tr} \bar{\Delta} \Delta \right) = 0, \\ F_{\Omega_c} &= m_{\Omega_c}^* \Omega_c + a^* \left( \bar{\Delta}_c \Delta_c - \frac{1}{2} \text{Tr} \bar{\Delta}_c \Delta_c \right) = 0, \\ F_L &= 2i\mathbf{f} \tau_2 \Delta L = 0, \\ F_{L_c} &= 2i\mathbf{f}^* \tau_2 \Delta_c L_c = 0. \end{aligned} \quad (7)$$

In the above drop the  $\Phi$  fields, which must have zero VEVs at  $M_R$ . It is easy to show that  $\langle\Phi\rangle = 0$  is consistent with (7).

We also have to satisfy the  $D$ -flat conditions, namely,

$$\begin{aligned} D_{Ri} &= 2\text{Tr}\Delta_c^\dagger\tau_i\Delta_c + 2\text{Tr}\bar{\Delta}_c^\dagger\tau_i\bar{\Delta}_c \\ &\quad + 2\text{Tr}\Omega_c^\dagger\tau_i\Omega_c + L_c^\dagger\tau_iL_c = 0, \\ D_{Li} &= 2\text{Tr}\Delta^\dagger\tau_i\Delta + 2\text{Tr}\bar{\Delta}^\dagger\tau_i\bar{\Delta} \\ &\quad + 2\text{Tr}\Omega^\dagger\tau_i\Omega + L^\dagger\tau_iL = 0, \\ D_{B-L} &= -L^\dagger L + 2\text{Tr}(\Delta^\dagger\Delta - \bar{\Delta}^\dagger\bar{\Delta}) \\ &\quad + L_c^\dagger L_c - 2\text{Tr}(\Delta_c^\dagger\Delta_c - \bar{\Delta}_c^\dagger\bar{\Delta}_c) = 0. \end{aligned} \quad (8)$$

Here we keep the left-handed fields since we have to show that parity can be broken spontaneously and at the same time we wish to know whether  $R$  parity is broken or not.

Typically in SUSY theories minimization of the  $D$ -term potential [in our case Eq. (8)] leads to a number of flat directions which may be characterized by the set of holomorphic gauge invariants formed from the chiral multiplets [10]. Then, one uses the vanishing of the  $F$  potential (7) in an attempt to determine as much as possible of these holomorphic functions. One can use this elegant method to prove that in this theory a parity-broken minimum leads to a determination of these gauge invariants, therefore lifting the flat directions (again, neglecting the squarks fields as in the MSSM). Because of the lack of space, we leave this analysis for a separate publication [C. S. Aulakh, A. Melfo, A. Rašin, G. Senjanović (to be published)], and instead present here a straightforward analysis that leads to the determination of the vacuum manifold.

It is obvious from (7) and (8) that the left-handed VEVs can be taken to vanish

$$\langle\Delta\rangle = \langle\bar{\Delta}\rangle = \langle\Omega\rangle = \langle L\rangle = 0. \quad (9)$$

We should mention that in this case  $\langle\Phi\rangle$  must vanish, as can be easily seen by minimizing  $V_F$  and  $V_D$ . Although clearly there is a solution in which the right-handed counterpart fields also have vanishing VEVs, and no symmetry is broken, we now focus on the realistic parity-breaking case.

First notice that (7) gives

$$\text{Tr}\Delta_c^2 = \text{Tr}\Delta_c\Omega_c = 0. \quad (10)$$

By an appropriate  $SU(2)_R$  rotation one may put  $\Delta_c$  in the form

$$\langle\Delta_c\rangle = \begin{pmatrix} 0 & \langle\delta_c^{--}\rangle \\ \langle\delta_c^0\rangle & 0 \end{pmatrix}, \quad (11)$$

where superscripts denote electromagnetic charges

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}. \quad (12)$$

Now (10) gives  $\langle\delta_c^{--}\rangle\langle\delta_c^0\rangle = 0$ , which implies the electromagnetic charge-preserving form for  $\langle\Delta_c\rangle$ . Next, from  $F_{L_c} = 0$ , barring accidental cancellations among different families; and using again (10) and  $D_{B-L} -$

$D_{3R} = 0$ , it is an easy exercise to show that  $\langle L_c\rangle$  vanishes, and that  $\langle\Omega_c\rangle$  and  $\langle\bar{\Delta}_c\rangle$  preserve  $Q_{em}$ . In short

$$\begin{aligned} \langle\Omega_c\rangle &= \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix}, & \langle L_c\rangle &= 0, \\ \langle\Delta_c\rangle &= \begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix}, & \langle\bar{\Delta}_c\rangle &= \begin{pmatrix} 0 & \bar{d} \\ 0 & 0 \end{pmatrix}. \end{aligned} \quad (13)$$

This proves the two important claims we made earlier. First, that the electromagnetic charge invariance of this vacuum is automatic for any value of the parameters of the theory (of course, neglecting as we did the squarks fields). Second, that the symmetry breaking down to the MSSM preserves  $R$  parity since  $\langle L\rangle = \langle L_c\rangle = 0$  generation by generation. This may not be obvious, since the above VEV for  $\Delta_c$  breaks  $B-L$  by two units and gives a Majorana mass to the right-handed neutrino. However, since  $R$  parity can be written as

$$R = (-1)^{3(B-L)+2s} \quad (14)$$

(with  $s$  the particle spin), the  $\Delta$  fields are even under it. Of course, as often happens in supersymmetry, this vacuum is degenerate with the unbroken one. The important point is that now they are not connected continuously.

With the remaining  $D$  and  $F$  equations it is straightforward to find the absolute values of the nonvanishing VEVs

$$\begin{aligned} |w| &= \left| \frac{m_\Delta}{a} \right| \equiv M_R, \\ |d| = |\bar{d}| &= \left| \frac{m_\Delta m_\Omega}{a^2} \right|^{1/2} \equiv M_{BL}. \end{aligned} \quad (15)$$

Notice an interesting property of (15). If we wish to have  $M_R \gg M_{BL}$ , we need  $m_\Delta \gg m_\Omega$ , i.e., a sort of inverse hierarchy of the mass scales. The same situation is encountered in the case of the  $P$ -odd singlet.

The analysis of the Higgs mass spectrum proceeds as usual, with expected results, except for the mass of the  $\Omega$  triplet. Instead of being  $M_{BL}$  as one would imagine naively, it turns out to be of order  $M_{BL}^2/M_R$ .

*Low-energy effective theory and  $R$  parity.*—An important question that must be faced is what happens when the soft supersymmetry breaking terms are turned on. Specifically, one would like to know the fate of  $R$  parity. In order to answer this question we need to have an effective low-energy theory after the heavy fields are integrated out.

It is easy to check that due to the trilinear terms in the supersymmetric potential  $L_c$  can get a VEV only if  $L$  acquires one, and we have  $\langle L_c\rangle \simeq \langle L\rangle M_S/M_R$ . Thus there is no  $R$ -parity violation in the right-handed sector until after it is broken (if at all) by the VEV of the left-handed sneutrino.

We show now that phenomenological considerations prevent this from happening. Notice first that in the limit of infinite  $M_R$ , the MSLRM reduces to the MSSM with an exact  $R$  parity. Namely, when  $\Omega_c$  gets a VEV, the couplings  $\alpha$  in Eq. (6) lead to the splitting of the bidoublets

into two light  $SU(2) \times U(1)$  doublets and two heavy ones (with masses proportional to  $\langle \omega_c \rangle$ ). Of course, the light doublets are light only with the usual fine-tuning between the  $\mu$  and the  $\alpha(\Omega_c)$  terms in the effective potential.

In this case  $\langle L \rangle \neq 0$  [11] would imply the existence of the Majoron [12], which corresponds to the spontaneous breaking of the global  $B - L$  symmetry. Such a Majoron can be ruled out due to its couplings to the  $Z$  boson.

Next, for finite  $M_R$ , it is a simple exercise to show that the Majoron becomes massive and, as expected on general grounds, one finds

$$m_J^2 \simeq \frac{M_S^3}{M_R}, \quad (16)$$

where  $M_J$  is the Majoron's mass. This follows from soft terms in the potential of the type

$$\Delta V_{\text{soft}} = \dots + M_S L^T \tau_2 \Phi_i \tau_2 L_c + \dots \quad (17)$$

Clearly for  $M_R \gg M_S$  there is no possibility that  $M_J > M_Z$ , and the bounds on the doublet Majoron from the  $Z$  width in fact rule out the possibility that  $\langle \tilde{\nu} \rangle \neq 0$ .

Thus we have a remarkable prediction: *the low-energy effective theory of the MSLRM is the MSSM with unbroken  $R$  parity, and the lightest supersymmetric partner (LSP) is stable.* This has profound phenomenological and specially cosmological consequences. In particular it allows the LSP to be a dark matter candidate.

*See-saw mechanism.*—Maybe the nicest feature of the theory is the implementation of the see-saw mechanism. As is well known, in the ordinary L-R symmetric theories, the left-handed triplet  $\Delta$  necessarily gets a nonvanishing vacuum-expectation value [4]

$$\langle \Delta \rangle = \alpha \frac{M_W^2}{M_{BL}}, \quad (18)$$

where  $\alpha$  is the ratio of some unknown couplings in the Higgs potential. This, while preserving the see-saw effect, unfortunately introduces additional unknown parameters and spoils the canonical form we cited in the introduction. However, in the supersymmetric version, as we have seen,  $\Delta$  has no VEV due to the absence of tadpole terms in the effective Higgs potential. Thus the see-saw mechanism is “clean,” since it depends only on the neutrino Dirac mass terms, i.e.,

$$m_\nu \simeq \frac{m_D^2}{M_{BL}} \quad (19)$$

This is especially important when one studies the  $SO(10)$  extensions of these theories, where the Dirac neutrino masses became related to the up quark masses, and the see-saw mechanism becomes potentially predictive once the intermediate mass scale  $M_{BL}$  is determined.

In summary, supersymmetry and left-right symmetry have grown with time into the central extension of the standard model, and L-R symmetry seems to play an important role in providing a gauge rationale for  $R$  parity. However, a construction of the SUSY L-R theory is by no means trivial. As we know from the work of Kuchimanchi

and Mohapatra [5], and as we have reviewed here, in the minimal version of the theory the symmetry cannot be spontaneously broken, whereas when this is cured by the addition of a parity-odd singlet one ends up breaking also electromagnetic charge invariance.

The minimal price to be paid at the renormalizable level is then to accept an intermediate  $B - L$  scale. Phenomenologically this of course is a blessing, for it leads to a whole plethora of new Higgs particles, potentially accessible to experiment. Of particular interest is the triplet  $\Omega$ , whose mass is of order  $\max[M_{BL}^2/M_R; M_S]$ .

We are deeply grateful to Alejandra Melfo and Anđrija Rašin for many useful discussions, for their invaluable help in checking most of our computations, and for the careful reading of our manuscript. We also acknowledge valuable discussions with Kaladi Babu and Anjan Josphura. C. S. A. thanks ICTP High Energy Group for hospitality during the initial stages of this work. The work of K. B. is supported by DOE Grant No. DE-FGO3-95ER40917, and that of G. S. by EEC under TMR Contract No. ERBFMRX-CT960090.

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