# Parametric Interaction of Dipolar Spin Wave Solitons with Localized Electromagnetic Pumping

A. V. Bagada, G. A. Melkov, and A. A. Serga
*Department of Electrophysics, Kiev State University, Kiev, Ukraine*

A. N. Slavin
*Department of Physics, Oakland University, Rochester, Michigan 48309*
(Received 10 March 1997)

Parametric interaction of a propagating dipolar spin wave envelope soliton (carrier frequency $\omega_s$, duration $\tau_s$, and velocity $\nu_s$) with electromagnetic pumping (frequency $\omega_p = 2\omega_s$), localized in a spatial region of width $L$, has been observed in magnetic films for the first time. The necessary frequency bandwidth of interaction was guaranteed by choosing $L < |\nu_s|\tau_s$. The interaction results in soliton amplification if the soliton arrives at the pumping region not more than 1 $\mu$s after the pumping has been switched on. Otherwise, the soliton is scattered by short-exchange-dominated spin waves of frequency $\omega_s$ parametrically excited by the localized pumping. [S0031-9007(97)04025-8]

PACS numbers: 75.30.Ds, 76.50.+g, 85.70.Ge

Envelope solitons are stable nonlinear wave packets that preserve their shape when propagating in a nonlinear dispersive medium. When the medium is weakly dissipative the soliton profile broadens, and its amplitude decreases twice as fast as the amplitude of a sinusoidal signal. In fiber optic communication lines, where optical envelope solitons are used as information carriers, the adverse effects of dissipation are compensated by optical pumping of a higher frequency, and the optical pumping wave and the optical soliton are copropagating in the fiber [1].

The problem of dissipation compensation is even more important for dipolar spin wave (SW) envelope solitons at microwave frequencies propagating in yttrium-iron garnet (YIG) films, as dissipation per unit propagation length in YIG films is much higher than in fibers [2,3]. One can try to solve this problem using parametric interaction of a SW soliton (of carrier frequency $\omega_s$ and carrier wave number $k_s$) with an external electromagnetic (EM) pumping field of the frequency $\omega_p = 2\omega_s$ and wave number $k_p$ ($k_p \sim 2\pi/L$, where $L$ is the spatial size of the pumping localization region). This process satisfies the conservation laws

$$\omega_p = \omega_s + \omega_i, \qquad \mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i, \tag{1}$$

where $\omega_i$ and $k_i$ are the frequency and wave number of the "idle" spin wave. This parametric process is substantially different from the analogous parametric processes in nonlinear optics. First of all, the effective wave number of localized pumping is usually smaller than the wave number of the dipolar SW $k_p < k_s$, so that the signal and idle SW are propagating in opposite directions. Second, because of the multimode character of the SW spectrum in magnetic films the EM pumping will not only interact with the propagating SW soliton, but may also excite other SW modes having frequencies close to $\omega_s$. The interaction of the SW soliton with these parametrically excited SW modes can significantly affect the process of soliton propagation.

This Letter reports the first experimental demonstration of effective parametric interaction between propagating envelope soliton and a spatially localized monochromatic pumping, leading either to partial compensation of soliton's dissipative losses and soliton amplification, or to soliton attenuation. The pumping localization in a spatial region of the width

$$L < |\nu_s|\tau_s, \tag{2}$$

along the path of soliton propagation plays a critical role in this interaction, as it guarantees that the whole spectrum of the soliton pulse (full width $2\Delta\omega_s \sim 4\pi/\tau_s$) interacts effectively with the quasimonochromatic EM pumping field.

In our experiments we used a conventional delay-line structure [2,3] (Fig. 1) consisting of input and output transducers separated by $l = 8$ mm, to which we added a third (pumping) transducer placed in the middle. All three transducers were short-circuited copper wires of the width 50 $\mu$m, and were separated from the copper screen by a dielectric layer of the thickness $d = 50$ $\mu$m and dielectric permeability $\varepsilon \sim 6$. A tangentially magnetized

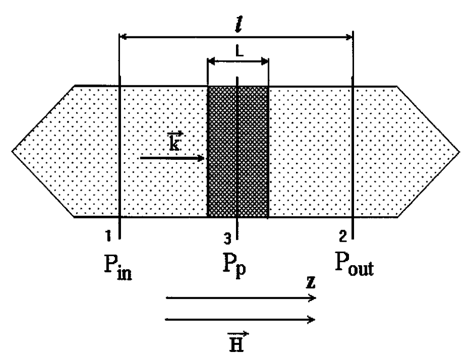


FIG. 1. Diagram of the experimental delay-line structure: 1 and 2 are the input and output transducers, 3 is the pumping transducer, and the dark region of the width $L$ is the region of pumping localization.

  

($H = 998$ Oe) YIG film (thickness 6.4 $\mu$m, saturation magnetization $4\pi M_0 = 1750$ Oe, cubic anisotropy field $H_a = -40$ Oe, ferromagnetic resonance linewidth measured at 5 GHz, $2\Delta H = 0.5$ Oe) was placed on top of the delay-line structure. The input rectangular pulse of duration $\tau_{\text{in}} = 30$ ns, carrier frequency $\omega_s/2\pi = 4630$ MHz, carrier wave number $k_s = 90$ cm$^{-1}$, and varying power $0 < P_{\text{in}} < 0.5$ W, was supplied to the input transducer. When the input power was approximately $P_{\text{in}} = 0.25$ W, this pulse created a well-developed single envelope soliton of dipole-dominated SW (backward volume magnetostatic waves or BVMSW [3]). The soliton propagated along the direction of the bias magnetic field $H$.

Under the conditions of our experiment the maximum frequency of BVMSW (corresponding to $k = 0$) was $\omega_0/2\pi = 4671$ MHz. The theoretical value of the BVMSW group velocity at the carrier frequency $\omega_s/2\pi = 4630$ MHz in the linear regime was $v_g \cong -2.9 \times 10^6$ cm/s, corresponding to a delay time between input and output transducers of $T_d = l/|v_g| = 275$ ns. The experimentally measured delay time in the linear regime was $T_d^{\text{lin}} = 290$ ns. This value is in a reasonable agreement with theory, taking into account slight inhomogeneity of the bias magnetic field $H$.

To create an active region under the middle transducer, a microwave pumping of the frequency $\omega_p/2\pi = 2\omega_s/2\pi = 9260$ MHz and power 0.3–10 W was supplied. The pumping was supplied in pulses of duration $\tau_p = 3$–$5$ $\mu$s with repetition frequency $f_R = 4$ kHz to avoid overheating the sample. The magnetic field of the pumping $\mathbf{h}(z)$ was nonhomogeneous, mostly parallel to the bias magnetic $\mathbf{H}$, and, according to our estimations, at $P_p = 10$ W the average pumping field of $h = 30$ Oe was localized in the region of the width $L = 150$ $\mu$m around the middle transducer.

The amplitude characteristics (i.e., the dependencies of the peak power of the output pulsed signal $P_{\text{out}}$ on the input power $P_{\text{in}}$) obtained with and without pumping are presented in Fig. 2. Both curves clearly show deviation of $P_{\text{out}}$ from the linear trend, which is a characteristic feature of soliton formation (see, e.g., [3,4]), and a small increase in the output power under the influence of pumping. These curves also show that the power threshold of soliton formation (i.e., the value of the input power at which the output power starts to deviate from the linear trend) decreases under the influence of pumping from $P_{\text{th}}^{\text{sol}} = 0.1$ W for $P_p = 0$ to $P_{\text{th}}^{\text{sol}} = 0.06$ W for $P_p = 7$ W.

The experimental profiles of the output soliton obtained with and without pumping for $P_{\text{in}} = 250$ mW are presented in Fig. 3. Two important facts can be seen in Fig. 3: (i) Even without pumping the soliton delay time $T_d^{\text{sol}} = l/|v_s| = 270$ ns is roughly 20 ns shorter than the delay time of a linear pulse $T_d^{\text{lin}} = l/|v_g| = 290$ ns, so that $|v_s| > |v_g|$. This is an additional proof of the soliton nature of the observed phenomena. (ii) Under the influence of pumping the soliton is amplified, so that its peak power increases 1.3 times, and delay time decreases by another 5 ns, while its shape and duration ($\tau_s = 22$ ns, defined as full width at half power) remain practically unchanged.

Our experiments have shown that the process of soliton interaction with localized pumping depends critically on the value of the time interval $\Delta t$ between the moment when the pumping is switched on, and the moment when the leading front of the soliton pulse arrives at the output transducer (see inset in Fig. 2). The data presented in Figs. 2 and 3 were obtained for $\Delta t = 260$ ns. Figure 4 shows the dependence of the soliton amplification coefficient $K_P = P_{\text{out}}^{\text{peak}}(P_p)/P_{\text{out}}^{\text{peak}}(0)$ (defined as the ratio of the output peak powers with and without pumping) on the magnitude of the time interval $\Delta t$. When $\Delta t$ is in the range 90 ns $< \Delta t < 1$ $\mu$s the propagating soliton is *amplified* through the parametric interaction with localized EM pumping, and the maximum amplification is observed for $\Delta t = 260$ ns. When $\Delta t > 1$ $\mu$s the soliton amplification

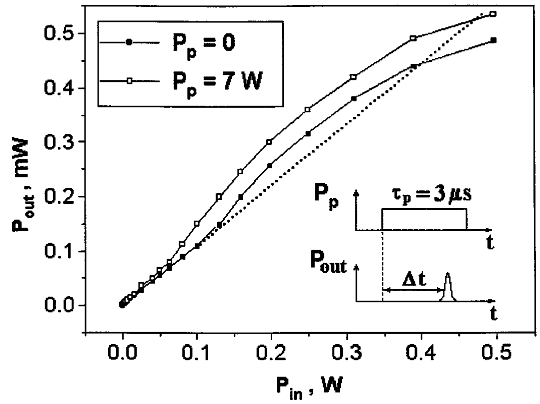


FIG. 2. Amplitude characteristics of the experimental delay line obtained with and without pumping. Dotted line shows amplitude characteristic in the linear regime. The inset shows relative positions in time of the pumping pulse and the output SW envelope soliton ($\Delta t = 260$ ns for this experiment).

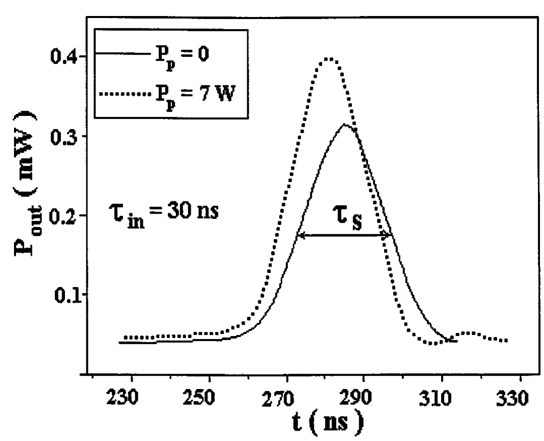


FIG. 3. Profiles of output pulses (power vs time) obtained with and without pumping for $P_{\text{in}} = 0.25$ W, and $\Delta t = 260$ ns.

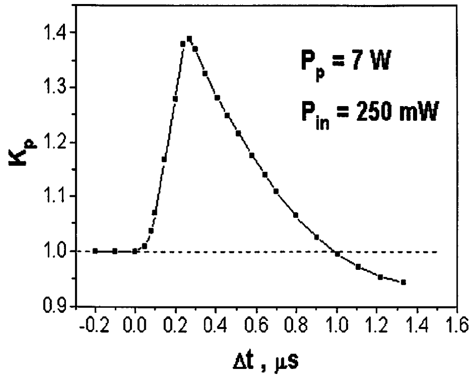


FIG. 4. Soliton amplification coefficient $K_p$ vs time delay $\Delta t$ between pumping pulse and the output soliton (see inset in Fig. 2) obtained for $P_{\mathrm{in}} = 0.25$ W, $P_p = 7$ W, and $T_p = 3\ \mu$s.

coefficient becomes $K_P < 1$, and the soliton interaction with localized pumping leads to *attenuation*.

This attenuation of the propagating soliton $\Delta t > 1\ \mu$s, in our opinion, is caused from the scattering of the dipolar SW soliton by short exchange-dominated spin waves ($k = k_{\mathrm{ex}} \sim 10^5\ \mathrm{cm}^{-1}$), parametrically excited by the pumping in the process

$$\omega_p = \omega_{k_1}^{\mathrm{ex}} + \omega_{k_2}^{\mathrm{ex}} = 2\omega_k^{\mathrm{ex}}, \qquad \mathbf{k}_1 \cong -\mathbf{k}_2 = \mathbf{k}. \quad (3)$$

For the process (3), involving short SW with $k \gg k_p$, pumping can be considered quasihomogeneous. In a dissipative medium this process has the threshold [5,6]

$$h_{\mathrm{th}}^{\mathrm{ex}} = \frac{\gamma_k}{V_k}\sqrt{1 + \left(\frac{\omega_k^{\mathrm{ex}} - \omega_p/2}{\gamma_k}\right)^2}, \quad (4)$$

where $V_k$ is the coupling coefficient of SW with pumping

$$V_k = g[(\omega_p/2)^2 - \omega_H^2]/2\omega_p\omega_H \quad (5)$$

(in our experiment $V_k/2\pi = 0.67$ MHz/Oe), $\gamma_\kappa = g\Delta H_k$ is the dissipation parameter for the exchange SW, $\Delta H_k$ is the half-width of the resonance curve for the exchange SW (in our experiment $\Delta H_k \cong \Delta H$), $g$ is the gyromagnetic ratio ($g/2\pi = 2.8$ MHz/Oe), and $\omega_H = gH$.

It is well known [5,6], that exchange-dominated spin waves at half pumping frequency, propagating in the direction perpendicular to the bias field **H**, have the lowest threshold, and will be excited by parallel pumping. The power threshold of the process (3) was measured in our experiments to give the value of $P_{\mathrm{th}}^{\mathrm{ex}} = 10$ mW. Thus, for all the values of pumping power in our soliton amplification experiments ($P_p = 0.3$–$10$ W) this threshold was substantially exceeded.

When strong pumping ($hV_k \gg \gamma_k$) is switched on, a SW packet of the spectral width $\Delta\omega_k \sim hV_k$ is excited, and the amplitudes of these parametrically excited exchange SW start to grow exponentially from the thermal level. A finite time $\tau_g$ is necessary for them to grow to the magnitudes at which they can significantly affect the propagation of the dipolar SW soliton. This growth time depends on the pumping amplitude $h$, and can range from several microseconds to several hundreds of nanoseconds [7].

If the soliton enters the region of pumping localization later than $\tau_g$ after the pumping has been switched on, the soliton suffers intensive scattering on the parametrically excited exchange SW propagating along the axis of pumping localization. This conclusion is supported by Fig. 5 where the soliton amplification coefficient is shown as a function of the pumping power $P_p$ for two values of $\Delta t : \Delta t_1 = 260$ ns and $\Delta t_2 = 3\ \mu$s. It is clear from Fig. 5 that for $\Delta t = \Delta t_1 < \tau_g$ the soliton is amplified, and the amplification grows with $P_p$, while for $\Delta t = \Delta t_2 > \tau_g$ the soliton is scattered, and its attenuation increases with the increase of $P_p$.

Let us now consider the process of soliton amplification by localized parallel pumping for $\Delta t < \tau_g$ when the influence of the exchange-dominated SW is negligible. The soliton is propagating perpendicular to the axis of the pumping localization region. Therefore, pumping localization plays an important role in the process of interaction with the soliton.

Parametric excitation of SW, propagating perpendicular to the axis of pumping localization, has been studied in [8]. The threshold of this process for the case of a constant-amplitude pumping field, localized in the region of the width $L$, is given by [8]

$$h_{\mathrm{th}}^{\mathrm{dip}} = \frac{\gamma}{V_k}\sqrt{1 + \left(\alpha\pi\frac{\lambda^{\mathrm{dip}}}{L}\right)^2}, \quad (6)$$

where $\gamma = g\Delta H$ is the dissipation parameter for dipolar SW, and $\alpha$ is the dimensionless coefficient which depends on the ratio of $L$ and the mean free path $\lambda^{\mathrm{dip}} = |v_g|/\gamma$ of a dipolar SW. In our experiment $\lambda^{\mathrm{dip}} \gg L$, and $\alpha = 1/2$. Using (6) it is easy to estimate that even for

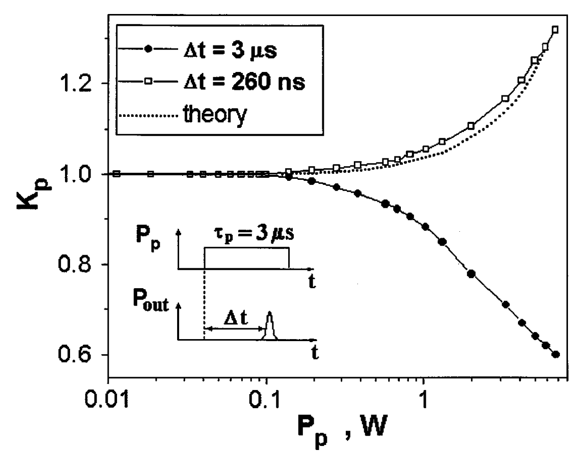


FIG. 5. Soliton amplification coefficient $K_P$ vs pumping power $P_p$ measured for $P_{\mathrm{in}} = 0.25$ W and two values of $\Delta t : \Delta t_1 = 260$ ns and $\Delta t_2 = 3\ \mu$s. Dotted line shows the theoretical curve calculated from Eq. (12).

the highest value of the pumping field in our experiment ($h = 30$ Oe, corresponding to the pumping power of $P_p = 10$ W) this threshold was not exceeded. Thus, the soliton amplification takes place below the threshold for parametric excitation of dipole-dominated SW (BVMSW).

Another important feature of our experiment is that the time $T_{\mathrm{int}} = L/|v_s|$, during which the localized quasi-monochromatic pumping interacts with each part of the propagating soliton profile, is much smaller than the durations of both the soliton and the pumping pulse

$$T_{\mathrm{int}} \ll \tau_s \ll \tau_p \,. \tag{7}$$

Thus, inside the region of pumping localization both the EM pumping and the amplified signal (soliton) can be considered quasistationary, and we can use the theory of stationary parametric amplification of SW (see, e.g., [9]) to calculate the soliton amplification coefficient $K_p$.

The system of equations describing the process (1) of interaction of localized parallel EM pumping with the signal and idle SW has been obtained in [8] [see Eq. (6.5.10) in [8]]. In the stationary case ($\partial/\partial t \equiv 0$) when $\omega_s \cong \omega_i \cong \omega_p/2$ this system has the form [9]

$$\gamma_s A_k + v_s \partial A_k/\partial z + ih(z)V_k A^*_{-k} = 0\,, \tag{8a}$$

$$\gamma_s A^*_{-k} - v_s \partial A^*_{-k}/\partial z - ih(z)V^*_{-k} A_k = 0\,, \tag{8b}$$

where $A_k$ and $A^*_{-k}$ are the amplitudes of the signal SW (BVMSW soliton) and the idle SW correspondingly, $\gamma_s = 2\gamma$ is the dissipation parameter for the soliton (which dissipates twice as fast as the linear signal in the same medium), $v_s$ is the soliton velocity, and $h(z)$ is the pumping amplitude which is constant $[h(z) = h]$ in the region of pumping localization $0 < z < L$, and is vanishing everywhere else. For the problem of soliton amplification, when the propagating soliton arrives at the point $z = 0$ having amplitude $A_0$, the boundary conditions are

$$A_k(0) = A_0\,, \qquad A^*_{-k}(L) = 0\,. \tag{9}$$

The coefficient of signal power amplification by localized pumping, defined as $K_p = [A_k(L)/A_k(0)]^2$, can be found using (8) and (9) in the form [9]

$$K_p = \{v_s\chi/[\gamma_s \sinh(\chi L) + v_s\chi\cosh(\chi L)]\}^2, \tag{10}$$

where

$$\chi = \sqrt{\gamma_s^2 - (hV_k)^2}/v_s\,. \tag{11}$$

Under the conditions of our experiment $hV_k \gg \gamma_s$, and the expression (10) can be simplified to

$$K_p = [\cos(hV_k L/v_s)]^{-2}. \tag{12}$$

Assuming that the squared amplitude of magnetic field of the parallel pumping $h^2$ is proportional to the pumping power $P_p(h^2 = CP_p)$ with coefficient $C = 90$ Oe$^2$/W, and using Eq. (12), we calculated the theoretical dependence of the soliton amplification coefficient $K_p$ on the pumping power $P_p$. This theoretical curve is shown as a dotted line in Fig. 5. It is clear from Fig. 5 that Eq. (12) gives a good quantitative explanation for the observed values of soliton amplification by localized parallel EM pumping.

In conclusion, we have shown experimentally that EM pumping localized in a spatial region of width $L$ interacts effectively with a propagating dipolar SW soliton if $L < |v_s|\tau_s$. The character of this interaction depends on the time delay $\Delta t$ between the moment when the pumping is switched on and the moment when soliton arrives at the output transducer. We have also shown that the power threshold of soliton formation is reduced under the influence of localized pumping.

The authors are grateful to Dr. C. E. Patton and Dr. B. A. Kalinikos for many helpful discussions. This work was supported by the National Science Foundation, Grant No. DMR-9701640, Ukrainian Fund for Fundamental Research, Grant No. 2.4/707, International Science Foundation, Grant No. SPU-062034, Research Corporation, Grant No. CC-3401, and by the Oakland University Foundation.

[1] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, Boston, 1995).

[2] A. N. Slavin, B. A. Kalinikos, and N. G. Kovshikov, in *Nonlinear Phenomena and Chaos in Magnetic Materials*, edited by P. E. Wigen (World Scientific, Singapore, 1994), Chap. 9, p. 209.

[3] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. B **49**, 12 773 (1994).

[4] J. M. Nash, C. E. Patton, and P. Kabos, Phys. Rev. B **51**, 15 709 (1995).

[5] E. Schloemann, J. Green, and V. Milano, J. Appl. Phys. **31**, 3865 (1960).

[6] A. G. Gurevich and G. A. Melkov, *Magnetization Oscillations and Waves* (CRC Press, New York, 1996), p. 265.

[7] R. I. Joseph and E. Schloemann, J. Appl. Phys. **38**, 1915 (1967).

[8] V. S. L'vov, *Wave Turbulence Under Parametric Excitation* (Springer, Berlin, Heidelberg, 1994), p. 157.

[9] G. A. Melkov and S. V. Sholom, Sov. Phys. Tech. Phys. **35**, 943 (1990).