Quantum Interference Effects in Spontaneous Emission from an Atom Embedded in a Photonic Band Gap Structure

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The spontaneous emission from a three-level atom embedded in a photonic band gap structure is studied. Interference between two transitions leads to quasiperiodic oscillations of population between the two upper levels with large amplitudes. The spontaneous emission of the atom is characterized by three components in the radiated field: a localized part, a traveling pulse, and a $(1/\sqrt{t})^3$ decaying part. An analytical expression for the localization distance of the localized field is obtained. The energy velocity for the traveling pulse could be close to zero. By selecting an appropriate initial superposition state, a large amount of population trapping can be achieved. [S0031-9007(97)03521-7]

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Quantum interference between different atomic transitions and atomic coherence can lead to various effects, such as change of spectra, population trapping, phase-sensitive amplification, and laser without inversion [1–3]. In the past, these phenomena were studied mostly in systems with a coherent driving field. In a four-level system the spontaneous emission from two neighboring levels (which emit photons of the same polarization) to the lower level can be totally suppressed when they are coupled to the fourth level by a driving field [2]. In this case, population trapping and oscillation of population in upper levels can be observed. In the absence of an external coherent field, oscillation of population in the upper levels also exists due to the emission and reabsorption of a single photon from the two upper levels as a result of the interference [3]. However, such an external field-free oscillation is very weak as the spontaneously emitted photon travels away from the atom in free space with the vacuum speed of light *c*.

It was shown that, in a photonic band gap structure (PBGS), the prohibition of light wave transmission can be achieved for some frequency ranges in all directions [4]. As a consequence the energy of the field can be localized in a space domain without propagating away. Recently, much effort has been concentrated on the study of photonic crystals in which a three-dimensional periodic dielectric structure is used to create one or several forbidden frequency bands [5].

An atom (impurity) embedded in such a structure will interact with field modes in the propagating frequency band as well as those in the forbidden band, localized field modes created by the atom [6,7]. Since an emitted photon can be trapped in the vicinity of the atom, the exchange of energy between the atom and field can be significant. It has been shown that a two-level atom embedded in a PBGS could retain some population in the upper level, even when the transition frequency was in the transmitting band [6,8]. The final state is a dressed state of the atom with a localized field mode, which lies in the forbidden band. A natural question arises: how to use the localized field to enhance quantum interference effect. Furthermore, one might ask what the properties of the field emitted by the atom are.

In this Letter we report the effect of quantum interference without a coherent driving field in the spontaneous emission of a three-level atom embedded in a photonic band structure. The atom has two upper levels and one lower level (which is different from the three-level atom treated in Ref. [8], where the atom has one upper level and two lower levels, and there is no interference between the two transitions [9,10]). The two upper levels are coupled to the lower one via the same field continuum. The interference between the two transitions leads to an exchange of population between the two upper levels, extended oscillations of energy distribution between the field and the atom, and a large population trapping in the upper levels. The field emitted by the atom is composed of three parts: a localized field, a traveling wave with very slow energy propagation velocity, and a decaying field.

Our model consists of a three-level atom with two upper levels $|a_1\rangle$, $|a_2\rangle$ and a lower level $|b\rangle$ (see Fig. 1). The dipole vectors between $|a_1\rangle$ and $|b\rangle$ and between $|a_2\rangle$ and $|b\rangle$ are parallel. The dispersion relationship of the band gap material near the band gap edge ω_c can be approximated by [6,8]

$$\omega_k = \omega_c + A(k - k_0)^2, \qquad A = \omega_c/k_0^2. \tag{1}$$

The Hamiltonian for the system, after carrying out the rotating wave approximation at ω_c, is

$$H = \sum_k \hbar(\omega_k - \omega_c)a_k^\dagger a_k + i\hbar\left[\sum_k (g_k^{(1)} a_k^\dagger |b\rangle\langle a_1| + g_k^{(2)} a_k^\dagger |b\rangle\langle a_2|) - \text{H.c.}\right]. \tag{2}$$

The coefficients $g_k^{(1)}$, $g_k^{(2)}$ are related to the decay coefficients of each upper level. We assume here, as in practical situations, $|\omega_{a_1} - \omega_{a_2}| \ll \omega_c$. For a special case $\omega_{a_1 b} - \omega_c = -(\omega_{a_2 b} - \omega_c) = \Delta$ with $g_k^{(1)} = g_k^{(2)}$, analytic results can be obtained for the evolution of an atom from an arbitrary initial excited state. The state vector at a time t is given by

$$|\psi(t)\rangle = [A^{(1)}(t)\,|a_1\rangle + A^{(2)}(t)\,|a_2\rangle]_a|0\rangle_f + \sum_k B_k(t)\,|b\rangle_a|1_k\rangle_f\,, \tag{3}$$

with $A^{(1)}(0)$, $A^{(2)}(0) \neq 0$, and $B_k(0) = 0$.

Following procedures similar to Ref. [8], we obtain the Laplace transform for the amplitudes $A^{(1)}(t)$, $A^{(2)}(t)$:

$$\begin{aligned} \tilde{A}^{(1)}(s) &= \frac{A^{(1)}(0)\,(s - i\Delta) - [A^{(1)}(0) - A^{(2)}(0)]\,(i\gamma)^{3/2}/\sqrt{s}}{s^2 + \Delta^2 - 2(i\gamma)^{3/2}\sqrt{s}}\,, \\ \tilde{A}^{(2)}(s) &= \frac{A^{(2)}(0)\,(s + i\Delta) - [A^{(2)}(0) - A^{(1)}(0)]\,(i\gamma)^{3/2}/\sqrt{s}}{s^2 + \Delta^2 - 2(i\gamma)^{3/2}\sqrt{s}}\,. \end{aligned} \tag{4}$$

Here $\gamma = [\omega_{ab}^{7/2} d_{ab}^2/6\pi\epsilon_0\hbar c^3]^{2/3}$. The right-hand side contains four poles, $s_i = x_i^2$, $i = 1,2,3,4$, where x_i are the roots of the equation, $x^4 + \Delta^2 - 2(i\gamma)^{3/2}x = 0$, and are located in the quadratures II, IV, III, and I, respectively. The inverse of Eq. (4) can be expressed as

$$A^{(1,2)}(t) = \sum_{i=1}^{4} a_i^{(1,2)}\Big((x_i + y_i)e^{x_i^2 t} + y_i[1 - \mathrm{erf}(\sqrt{x_i^2 t}\,)]e^{x_i^2 t}\Big)\,. \tag{5}$$

where $a_i^{(1,2)}$ is the expansion coefficient corresponding to the pole x_i, which depends on both Δ and $A^{(1,2)}(0)$, and $y_i = \sqrt{x_i^2}$. If x_i is on the right half plane, we have $y_i = x_i$, and if x_i is on the left half plane, we have $y_i = -x_i$, in order to keep the phase angle of x_2^2 within $-\pi$ to π [8]. Therefore, only two purely exponential terms due to x_4 (oscillatory without decay at a fixed space point) and x_2 (oscillatory with decay at a fixed space point) survive. The second half containing $\mathrm{erf}(\sqrt{x_i^2 t}\,)$ decays usually as $1/\sqrt{t}$, while similar terms in Ref. [8] decay at a faster pace as $(1/\sqrt{t}\,)^3$.

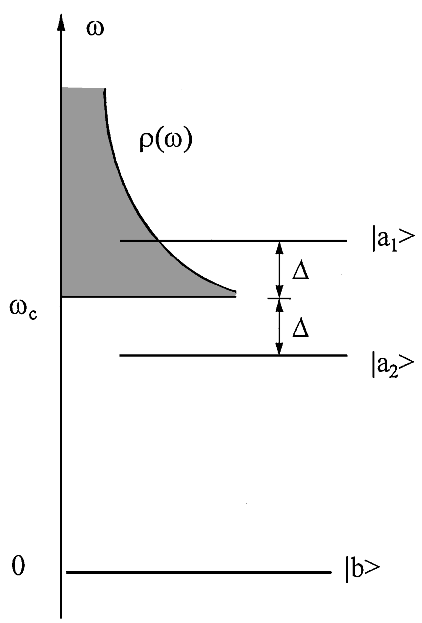

FIG. 1. A three-level atom in a photonic band gap structure. The two upper levels ($|a_1\rangle$ and $|a_2\rangle$) are symmetrically placed from the band gap edge by Δ.

The radiated field amplitude at a particular space point, r, is [9]

$$E(r,t) = \sum_k \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}\, e^{-i(\omega_k t - kr)}B_k(t)\,, \tag{6}$$

where the one-photon state amplitude

$$B_k(t) = -\int_0^t dt'\,[g_k^{(1)}A^{(1)}(t') + g_k^{(2)}A^{(2)}(t')]e^{-i(\omega_c - \omega_k)t'}. \tag{7}$$

We find that the right-hand side of Eq. (6) can be expressed as the sum of three contributions, I_1, I_2, and I_3, for large times ($t \longrightarrow \infty$).

The first part, I_1, comes from the x_4 term in Eq. (5) and does not decay in time. By neglecting $(kr)^{-2}$, and higher order terms, we found its amplitude drops exponentially as e^{-r/l_c}, and its frequency $(\omega_c - |x_4|^2)$ is within the forbidden band [9,10]. It represents a localized field. The size of the localized photon mode is $l_c = \sqrt{A/|x_4|^2}$. The amplitude of the localized photon mode is proportional to $\sin\theta(1 - i\sqrt{\omega_c/|x_4|^2}\,)/kr$, which goes to zero as Δ increases. Here θ is the angle between the atomic dipole vector and the r vector.

The I_2 part comes from the x_2 term. Spatially, it is an exponential pulse with the phase velocity v_p and energy velocity v_e,

$$\begin{aligned} v_p^{-1} &= [1 + \mathrm{Re}(\sqrt{ix_2^2/\omega_c}\,)]k_0/\alpha\,, \\ v_e^{-1} &= -\mathrm{Im}(\sqrt{ix_2^2/\omega_c}\,)k_0/\beta\,. \end{aligned} \tag{8}$$

The phase and amplitude propagation is proportional to $\sin\theta e^{-i\alpha(t - r/v_p)}e^{-\beta(t - r/v_e)}$ with $\beta = -\mathrm{Re}(x_2^2)$. Its frequency $\alpha = \omega_c - \mathrm{Im}(x_2^2)$ is in the transmitting band. The energy velocity v_e is considerably smaller than c (could be close to zero).

For the third part, I_3, the exact result cannot be found. An approximate expression was obtained for large γt. The

I_3 term decays to zero, but, unlike I_2, it does not have the form of an exponential of $e^{\eta(t-r/v)}$ representing a wave packet traveling away from the atom. It contains only a phase propagating factor $e^{-i(\omega_c t - k_0 r)}$. At any fixed time, the amplitude of the third part decreases to zero exponentially as the distance from the atom increases. At any space point, the amplitude decays to zero as time goes to infinity. Therefore, it represents a decaying field. The amplitude decay is proportional to $(\frac{1}{\sqrt{t}})^3$.

We examined the evolution behaviors and the final state with various initial superposition states to analyze the roles played by the coupling of the decaying field to the traveling wave and the localized wave, and by the interference due to the two interaction channels. A picture of how the population transfers between levels, and how the energy is transferred from the atom to the traveling wave and localized field, is thus obtained.

Interference leads to the transfer of population from $|a_1\rangle$ to $|a_2\rangle$ or vice versa, as witnessed by the oscillations in Fig. 2 (initially the atom in $|a_2\rangle$). The population trapping in the two upper levels is due to the photonic band gap with a nondecaying component to form the final dressed state. In Fig. 2, the dominating part decays at a rate $(\sqrt{\gamma t})^{-1}$ due to the interference. The populations oscillate many cycles ($\sim 10^2$) before eventually decaying to their final values (for other initial states, the oscillations are similar).

This quasioscillation has quite a large amplitude of the order of 0.5, a feature significantly different from the two-level case. In the current situation, the interference between the two transitions is enhanced by the localized field. Consequently, we have a larger oscillation amplitude compared to either a two-level atom in a PBGS or a three-level atom in vacuum. If the atom were continuously pumped to one of the upper levels, the emitted field would show a strong beat signal [10,11]. From the pattern of decay we determined that the strong quasioscillation is due mainly to the energy exchange between the atom and the field. The energy in the decaying localized field (given by the atom initially) will transfer back to the atom and then become the energy of the traveling wave and the localized field. This explains the small value of v_e.

It can be proven analytically that the amplitude of the oscillations can be minimized by choosing a special initial state $A^{(1)}(0) = A^{(2)}(0)$ (minimizing the interference), and with this initial state the amplitude of the population oscillation decays as $(1/\sqrt{t})^3$, as shown in Fig. 3. [Note, decay as $(1/\sqrt{t})^3$ is the situation for a two-level atom.]

The amount of population trapped in the upper levels depends on the initial condition. The population in the upper level within the band gap ($|a_2\rangle$) could be transferred to the upper level in the transmitting band ($|a_1\rangle$) from which it could emit the traveling wave. The final state contains an upper level part (trapped excited state population) and a lower level part with one photon in the localized mode. The phase difference between the two upper levels at a final state is always zero. The ratio of the populations in the two upper levels, $A^{(1)}(\infty)/A^{(2)}(\infty)$, is independent of the initial state, which has been proven numerically. This ratio, as well as the portion in the lower level, depends on Δ/γ.

Since the true trapped final state is a dressed state with a lower level component and a localized field (together with the upper level components), no superposition of the two upper levels can evade decay completely. This

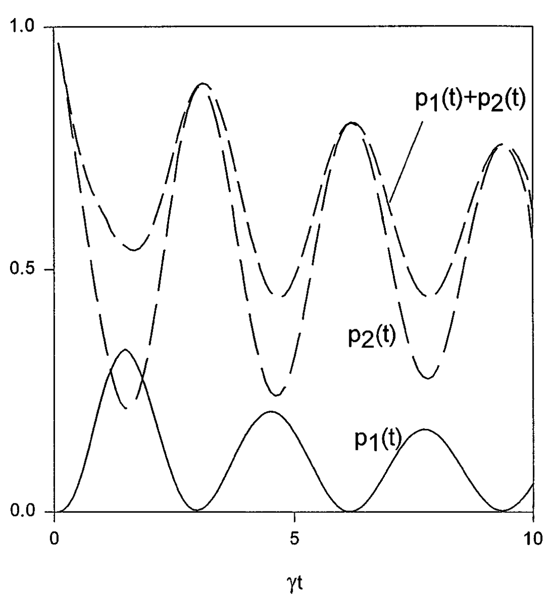

FIG. 2. Upper state population evolution for the initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle - |a_2\rangle)$. $\Delta = \gamma = 1$. p_1, p_2 are the populations in levels $|a_1\rangle$ and $|a_2\rangle$, respectively.

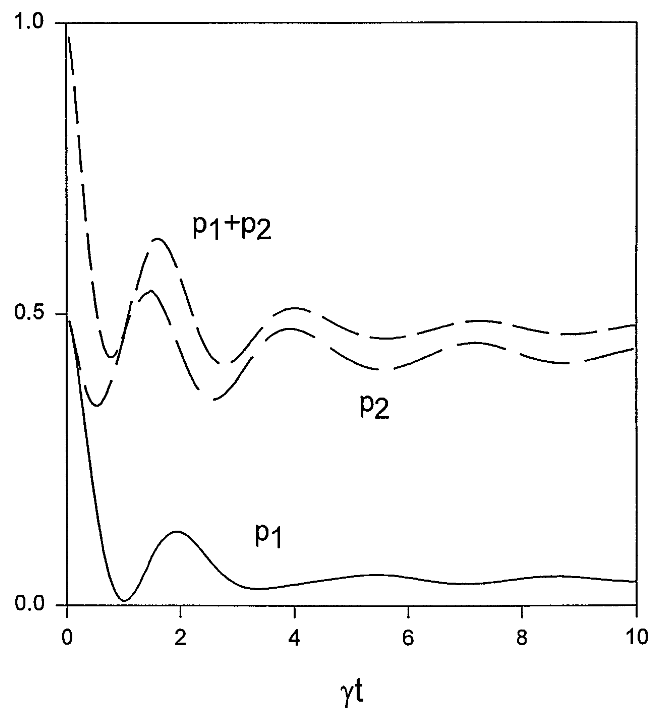

FIG. 3. Upper state population evolution for the initial state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|a_1\rangle + |a_2\rangle)$. $\Delta = \gamma = 1$. Notice the significantly different decay time scale compared with Fig. 2.

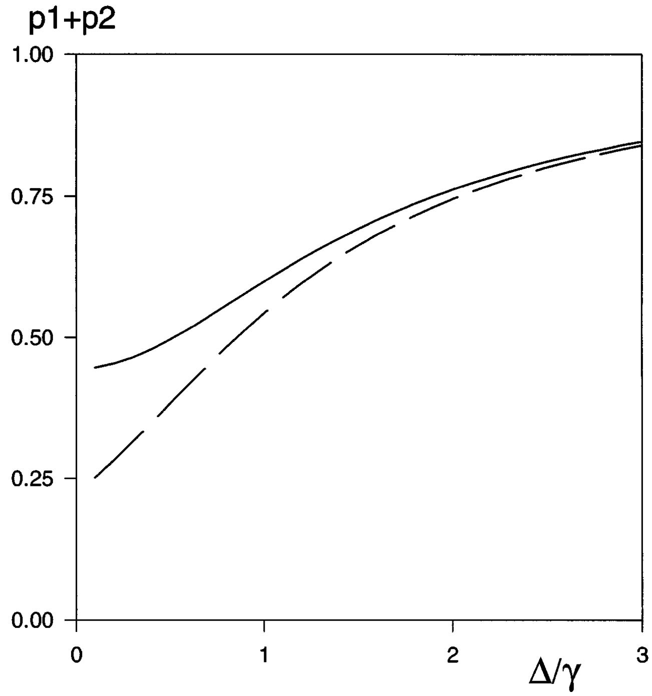

FIG. 4. Trapped population in the two upper levels as a function of Δ for the initial state which is a renormalized state of the final state projected onto the manifold of the two upper levels and started from any initial state (solid line). For comparison, the trapped population for the initial state $|\psi(0)\rangle = |a_2\rangle$ is also plotted (dashed line).

is in contrast with the dark state of a driven four-level system. However, if the atom is prepared in a state, which is a renormalized state of the final state projected onto the manifold of the two upper levels starting from any initial state (i.e., by making $B_k(t = \infty) = 0$ in Eq. (3) and renormalizing the resulting state), we can minimize the energy emitted into the traveling wave and could have more population trapped in the upper levels (even more than the case that the atom is initially in the level in the gap, i.e., $|a_2\rangle$). This final state population trapped in the two upper levels is plotted in Fig. 4 (where for comparison we also plotted the final population trapped in the two upper levels starting from the atom initially in the upper level in the band gap). We have established that the trapped population for each value of Δ is higher than that from any other initial states (see Fig. 4). This is similar to the situation of a driven four-level system [2]. If the four-level system is prepared in a nondecaying dressed state, the four-level atom will never decay (no traveling wave). In the current case, the atom gives energy mainly to the localized field to form the required component of the lower level with one photon. For the same Δ, more population trapped in the upper level means more energy in the localized field.

In conclusion, we found that, in a photonic band gap structure, the interference of spontaneous emission from a three-level atom with the two upper levels coupled to the same continuum can be significantly enhanced without the help of a driving field, which is essential in free space. The reabsorption and reemission of photons in the three-level system embedded in PBGS are more pronounced. The atom releases its energy during the spontaneous emission process in three forms, a localized field with a localization distance l_c, a traveling wave (an exponential pulse with an energy velocity much slower than c), and a slowly decaying field. Since there is one level in the forbidden band and one level in the transmitting band, it could also serve as a medium to study the coupling of the external field and the local trapped field.

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