# Where is the Scissors Mode Strength in Odd-Mass Nuclei?

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It is demonstrated by a fluctuation analysis based on the assumption of a Wigner distribution for the nuclear level spacings and of a Porter-Thomas distribution for the transition strengths that significant parts of the dipole strength excited in photon scattering experiments in heavy, deformed odd-mass nuclei are hidden in the background of the experimental spectra. With this additional strength, the heretofore claimed severe reduction of the $B(M1)$ scissors mode strength in odd-mass nuclei compared to the one in neighboring even-even nuclei disappears. [S0031-9007(97)04060-X]

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Since the first experimental observation of a collective (on a single-particle scale) low-energy, orbital magnetic dipole excitation about a decade ago [1], the properties of this mode, nowadays commonly called scissors mode, still attract heavily the interest of both experimentalists and theorists [2]. The mode exhibits unusual structural features such as a proportionality of its strength to the square of the ground state deformation [3] and characteristic strength distributions which form a sensitive test for microscopic theories of nuclei (for some recent examples, see, e.g., [4–9] and references therein). Experimentally, the systematics of the mode has been established particularly in the rare-earth mass region by extensive investigations [10] with high-resolution nuclear resonance fluorescence (NRF). The variation of the total $B(M1)$ strengths over a mass region $A \approx 140-180$ has been successfully interpreted by a phenomenological sum rule [11] as well as within the interacting boson model [12]. Recently, first evidence for the scissors mode in a triaxially deformed ($\gamma$-soft) nucleus, $^{196}$Pt, was reported [13], and it could also be established [14] in another region of $\gamma$-softness around mass $A \approx 130$.

Current interest focuses on the question whether the scissors mode also exists in deformed, heavy odd-mass nuclei and what its properties are compared to the even-mass neighbors, where it is typically found in an excitation energy interval $E_x \approx 2.5-4$ MeV centered at about 3 MeV and with an average $B(M1)\uparrow$ strength of about $3\mu_N^2$ for large deformations. Intuitively, one would expect in odd-mass nuclei a larger degree of fragmentation of the strength because of the coupling to an unpaired nucleon and much higher level densities leading to an increased mixing of the scissors mode into the background mostly composed by dense quasiparticle states [15].

First experimental evidence for the scissors mode in an odd-mass nucleus was presented for $^{163}$Dy which showed a clustering of transitions around 3 MeV reminiscent of the even-mass cases, albeit with a total strength reduced by a factor of about 3 [16]. Subsequent studies of $^{155,157}$Gd, $^{159}$Tb, $^{161}$Dy, and $^{167}$Er revealed surprisingly distinct behaviors [17–19] which seem to contradict each other, but also the theoretical predictions. This is demonstrated in the upper part of Fig. 1, where the extracted dipole strength distributions are summarized. Note that rather than $B(M1)$, the reduced dipole strength $g\Gamma_0^{\rm red} = g\Gamma_0/E_x^3$ proportional to $B(M1)$ and $B(E1)$ is given [because the $(\gamma,\gamma')$ experiments did not distinguish the parities of the transitions]. Here, $\Gamma_0$ is the ground state decay width and $g = (2J_f + 1)/(2J_i + 1)$ a spin factor counting the initial and final substates. The figure also includes new

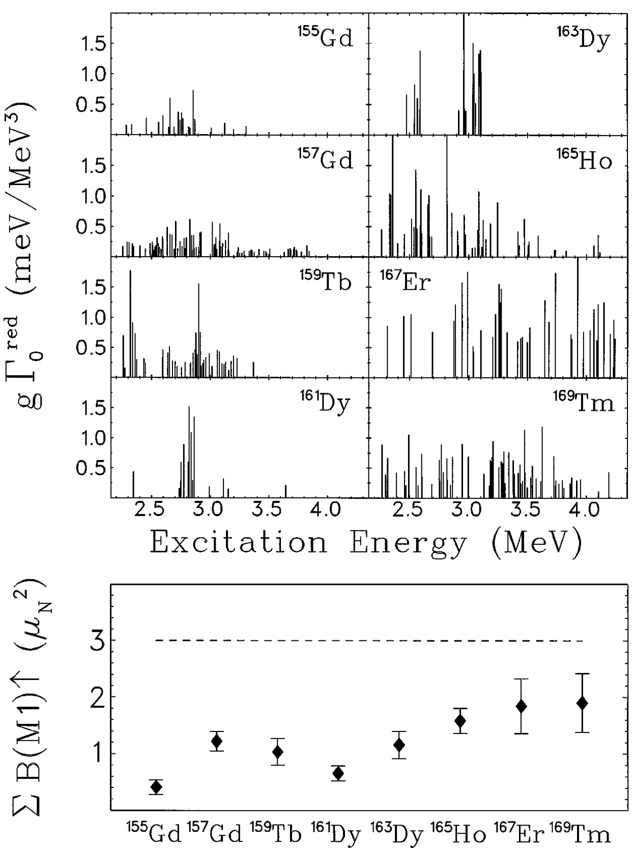


FIG. 1. Upper part: Dipole strength distributions of deformed odd-mass rare-earth nuclei in the energy region where the scissors mode is expected [16–20]. Lower part: Summed $B(M1)\uparrow$ strength of the scissors mode in the energy interval $E_x = 2.5-3.7$ MeV assuming pure $M1$ character of the transitions. The dashed line indicates the typical average $B(M1)\uparrow$ value of $3\mu_N^2$ in deformed even-mass nuclei.

 

results [20] for $^{165}$Ho and $^{169}$Tm obtained by a Darmstadt-Köln-Rossendorf collaboration with a EUROBALL cluster module which exhibits exceptional properties for NRF investigations [13,21–24]. Indeed, drastic differences are observed with respect to the fragmentation and the distribution of strength in the same energy interval. A particular pronounced example is the difference between $^{155,157}$Gd and $^{161,163}$Dy which is hard to understand since the scissors mode is little fragmented in the even-even Gd isotopes.

Another problem emerging from these results are the summed $B(M1)\uparrow$ strengths shown in the lower part of Fig. 1. To ensure compatibility of the different experiments, the further analysis is restricted to $E_x = 2.5$–$3.7$ MeV. Even with the assumption that the experimentally seen transitions have solely $M1$ character, all the results are a factor 2–3 below the value of about $3\mu_N^2$ (dotted line), typical for the even-even neighbors. Such a reduction would be in clear conflict with microscopic [9,25] or core-coupling [26,27] model calculations. Likewise, all studies within the interacting boson-fermion model [19,28–30] predict that the scissors mode strength should be nearly equal to that of the even-mass neighbors representing the boson core. Recently, this relation was explicitely derived by Ginocchio and Leviatan [31] in a sum-rule approach.

These puzzling differences raise the question whether they are due to unexpected nuclear structure effects or whether a significant part of the $M1$ strength escaped detection in the experiments performed so far. It is the purpose of this Letter to demonstrate by means of statistical methods that *the strength of the scissors mode in odd-mass nuclei is comparable to the even-even neighbors, but is so fragmented that a significant part lies below the experimental detection thresholds*. The approach furthermore explains (at least to a large extent) the vast differences discussed above.

Information about unresolved strength contributions buried in the background of dense photon scattering spectra can be obtained from a fluctuation analysis. The method is described in detail in [32] and has been successfully applied before to the analysis of $\beta$-delayed proton decay [33], giant resonance spectra from electron scattering [34,35], and recently also to features of $\gamma$-ray spectra above the yrast line [36]. It is valid in an energy region where the mean decay width $\Gamma$ of the levels is still smaller than their average spacing $\langle D \rangle$, but both are smaller than the experimental resolution $\Delta E$. With typical values $\Gamma \approx 10$–$100$ meV, $\langle D \rangle \approx 1$ keV, and $\Delta E \approx 3$–$4$ keV for the NRF spectra discussed here, this condition is well fulfilled. We use the $^{165}$Ho and $^{169}$Tm spectra measured with the EUROBALL cluster as a test case for the method, because significant suppression of Compton scattering edges which could be mistaken as true fluctuations was achieved by an active BGO shield.

The basic steps of the application are demonstrated in Fig. 2 for the example of $^{165}$Ho. Part 2(a) shows a background subtracted original $(\gamma, \gamma')$ spectrum in the energy range 2.5–4 MeV. It is first smoothed using a Gaussian function with a width of 12 keV to eliminate contributions of the fluctuations from counting statistics [thin line in Fig. 2(b)] and then folded with a Gaussian of the width $\Delta E_{\mathrm{fit}} = 50$ keV, which results in the thick line in Fig. 2(b). This line defines the mean about which the original points fluctuate. In order to remove the variations of the mean, i.e., the long-range correlations in the spectrum, the ratio of the original and the smoothed spectrum is calculated. The resulting spectrum called stationary in Fig. 2(c) now fluctuates around unity. It is a direct measure of the local fluctuations which can be expressed in terms of an autocorrelation function

$$C(\epsilon) = \langle S(E_x) S(E_x + \epsilon) \rangle , \qquad (1)$$

where $S(E_x)$ denotes a data point in the stationary spectrum at excitation energy $E_x$ and $S(E_x + \epsilon)$ is one shifted by an energy increment $\epsilon$. The brackets indicate averaging over the energy interval where the scissors mode is expected. Application of Eq. (1) to the experimental data results in

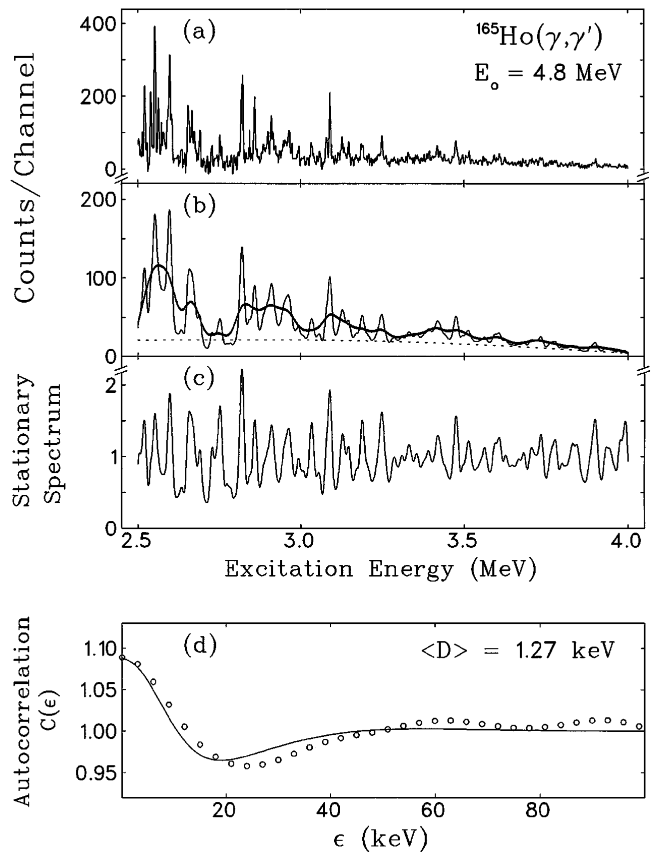


FIG. 2. Fluctuation analysis of a $^{165}$Ho$(\gamma, \gamma')$ spectrum measured at an endpoint energy of 4.8 MeV: (a) background subtracted spectrum, (b) spectrum from (a) smoothed with a Gaussian of a width of 12 keV (thin line) and then folded with one of $\Delta E_{\mathrm{fit}} = 50$ keV to obtain the mean (thick line), (c) stationary fluctuating spectrum obtained by dividing the spectrum of (b) by its mean, and (d) autocorrelation function from experiment (open circles) and from Eq. (2) with $C(0)$ fixed by the prediction of the mean level spacing taken from Table I (solid line). The physical background derived by this method is indicated in (b) by the short-dashed line.

the open circles displayed in part (d) of Fig. 2. The experimental autocorrelation function can be well approximated [32,33] by the analytical expression

$$C(\epsilon) = 1 + \frac{\alpha \langle D \rangle}{4\sqrt{\pi 2 \ln 2}\ \Delta E} \times \left[ \exp - \left( \frac{\epsilon}{4\sqrt{2 \ln 2}\ \Delta E} \right)^2 + f(\epsilon, \Delta E, \Delta E_{\mathrm{fit}}) \right]. \tag{2}$$

The function $f(\epsilon, \Delta E, \Delta E_{\mathrm{fit}})$ depends only on the experimental resolution $\Delta E$ and the width $\Delta E_{\mathrm{fit}}$ introduced above. One sees immediately that the autocorrelation function at $\epsilon = 0$ is proportional to 1 plus a term rising linearly with the mean level spacing $\langle D \rangle$, and it is seen indeed in Fig. 2(c) that the strength of the fluctuations decreases with increasing energy (i.e., level density). In turn, with the average spacing fixed by level density predictions, the physical background in the experimental spectrum can be determined by adjusting the experimental autocorrelation function to reproduce the prediction of Eq. (2). The normalized variance $\alpha$ is fixed assuming Porter-Thomas intensity and Wigner-type spacing distributions weighted by the fraction of $B(M1)$ and $B(E1)$ strengths and the corresponding excited levels of the even-mass neighbors, respectively [35]. Such a treatment is supported by a statistical analysis of the complete data set in Fig. 1. The nearest-neighbor spacing statistics obtained after proper unfolding for each nucleus is well described by a superposition of two Wigner distributions [20].

The theoretical value of $C(\epsilon = 0)$ in Fig. 2(d) is determined using an average of the so-called backshifted Fermi-gas model and the constant temperature model level density predictions from [37] leading to $\langle D \rangle = 1.27(13)$ for the present example. In order to reproduce the resulting value of $C(\epsilon = 0)$, the physical background indicated by the short-dashed line in Fig. 2(b) must be added to the experimental spectrum. It should be noted that its distribution as a function of energy, while initially assumed to be constant, can be determined by performing the same type of analysis for smaller energy intervals of the spectrum.

Since we are interested in the scissors mode content in the spectra, the $E1$ contribution to the total reduced dipole widths has to be estimated. We took the average of the measured $\sum B(E1)$strengths in the even-mass neighbors [38]. With this assumption we find $\sum B(M1)\uparrow$ values for $^{165}$Ho and $^{169}$Tm of $3.5(1.2)\mu_N^2$ and $3.4(1.4)\mu_N^2$, respectively. The estimated errors include the model dependence of the level densities, uncertainties of the background subtraction and the assumption of a smooth variation of the $E1$ strength with mass number. The total $B(M1)\uparrow$ strengths now fit well into the systematics observed for the even-even neighbors.

One may ask whether this approach can be generalized and to what extent it offers explanations for the differences seen in Fig. 1. For this purpose we attempt to reconstruct the characteristic properties of the observed dipole strength distributions with Monte Carlo methods using the statistical model assumptions introduced above. The input quantities derived by averaging over the even neighbors are summarized in Table I.

Finally, it is investigated whether the number of levels ($N_{\mathrm{lev}}$) visible above the experimental thresholds and their summed reduced dipole strengths ($\sum g\Gamma_0^{\mathrm{red}}$) can be reproduced. For a meaningful comparison to the data, the energy dependence of the detection thresholds must be considered. In case of the measurements with a EUROBALL cluster module these were determined from the original spectra. The same energy dependence was adopted for $^{167}$Er. Otherwise, it was taken from Fig. 4 of [18] which should reasonably approximate all experiments described in [16–18]. An overall normalization factor must be taken into account because of the different definition of the threshold in the experiment and in the model. However, this does not add additional uncertainty to the description, since the factor is uniquely determined by the condition of an optimum *simultaneous* reproduction of $N_{\mathrm{lev}}$ and $\sum g\Gamma_0^{\mathrm{red}}$.

The results are summarized in Fig. 3. Within error bars the experimentally observed $\sum g\Gamma_0^{\mathrm{red}}$, showing large differences of a factor of about three, can be sucessfully reproduced. This is also true for most nuclei with respect to

TABLE I. Input quantities for the statistical model simulation of dipole strength distributions in heavy odd-mass nuclei obtained from averaging over the even-mass neighbors: ground state spin and paritiy $J_0^\pi$, total reduced dipole strength $\sum g\Gamma_0^{\mathrm{red}}$, ratio of dipole strengths, and number of levels excited by $M1$ and $E1$ transitions. The average level spacing $\langle D \rangle$ is taken from [37].

| Nucl. | $J_0^\pi$ | $\sum g\Gamma_{0,\mathrm{tot}}^{\mathrm{red}}$ (meV/MeV$^3$) | $\sum g\Gamma_0^{\mathrm{red}}$ $M1/E1$ (%) | $N_{\mathrm{lev}}$ $M1/E1$ (%) | $\langle D \rangle$ (keV) |
|---|---|---|---|---|---|
| $^{155}$Gd | $3/2^-$ | 41.6(84) | 76/24 | 73/27 | 2.98 |
| $^{157}$Gd | $3/2^-$ | 48.2(93) | 77/23 | 74/26 | 2.27 |
| $^{159}$Tb | $3/2^+$ | 42.8(72) | 86/14 | 75/25 | 2.29 |
| $^{161}$Dy | $5/2^+$ | 36.0(36) | 86/14 | 75/25 | 2.03 |
| $^{163}$Dy | $5/2^-$ | 56.8(52) | 73/27 | 60/40 | 1.30 |
| $^{165}$Ho | $7/2^-$ | 63.1(98) | 62/38 | 56/44 | 1.27 |
| $^{167}$Er | $7/2^+$ | 65.2(119) | 61/39 | 56/44 | 1.14 |
| $^{169}$Tm | $1/2^+$ | 76.2(115) | 63/37 | 56/44 | 1.60 |

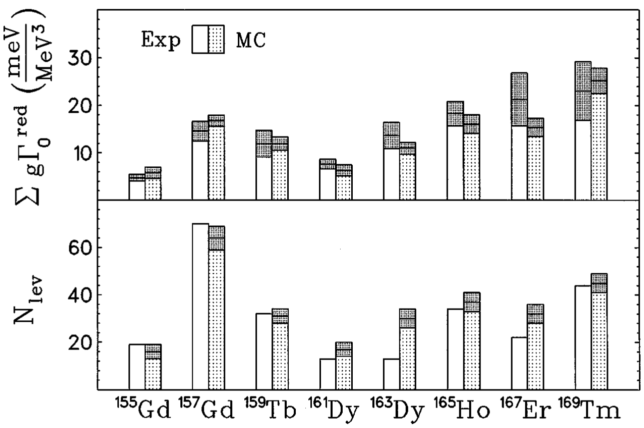


FIG. 3. Number of levels and their summed reduced dipole strengths $\sum g\Gamma_0^{\rm red}$ from the experimental data in Fig. 1 (open bars) and from Monte Carlo generated strength distributions based on a statistical model approach (dotted bars). The uncertainties of the statistical model results represent the $1\sigma$ width from 1000 repetitions of the calculations.

the number of observed levels which again display rather strong variations from about 15 to 70. There is some tendency to overestimate the fragmentation for $^{161}$Dy and in particular $^{163}$Dy, and also $^{167}$Er. Indeed the distributions for the Dy isotopes shown in Fig. 1 indicate that the assumptions of the statistical approach might not fully apply. The experimental results for $^{167}$Er show the unexpected feature of a large fraction of strength at energies above the region considered here [19]. If this strength is included, the number of levels would resonably agree, but the spectroscopic strength would be larger than in all the other cases. Nevertheless, it seems justified to state an overall very satisfactory description of the experimental quantities when assuming a $B(M1)$ scissors mode strength comparable to the even-even nuclei. The remaining question is, how sensitive are these results to variations of the total reduced dipole strength $\sum g\Gamma_{0,\rm tot}^{\rm red}$ used as input? The dependence of $N_{\rm lev}$ and the spectroscopic strength $\sum g\Gamma_0^{\rm red}$ is rather strong. An increase (reduction) of $\sum \Gamma_{0,\rm tot}^{\rm red}$ by, e.g., 25% leads to a corresponding increase (reduction) of a factor of 2 for both.

In summary, the available experimental information on the scissors mode in heavy odd-mass nuclei has been analyzed by a statistical model approach. A fluctuation analysis of high-quality $^{165}$Ho$(\gamma,\gamma')$ and $^{169}$Tm$(\gamma,\gamma')$ spectra obtained with a EUROBALL cluster module reveals that a significant part of the dipole strength is hidden in the background. Assuming a smooth variation of the experimentally indistinguishable $E1$ contributions with mass number the total $B(M1)$ strengths agree well with the findings for the scissors mode in the neighboring even-mass nuclei. Monte Carlo generated dipole strength distributions based on these assumptions and including detection thresholds from the experiments are able to quantitatively reproduce the strong variations of characteristic quantities such as the number of observed levels and their summed transition strengths. We thus conclude that the scissors mode is present in deformed, heavy odd-mass nuclei with the strength expected from systematics, but a significant part—which can vary depending on the experimental conditions and the different level densities—escapes detection in the NRF experiments because of the strong fragmentation.

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