

## Unquenching the Scalar Glueball

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Computations in the quenched approximation on the lattice predict the lightest glueball to be a scalar in the 1.5–1.8 GeV region. Here we calculate the dynamical effect the coupling to two pseudoscalars has on the mass, width, and decay pattern of such a scalar glueball. Modeling this nonperturbative interaction reveals that, if the pure glue state has a width to two pseudoscalars of  $\sim 100$  MeV as predicted on the lattice, the resulting hadron has a width to these channels of only  $\sim 30$  MeV with a large  $\eta\eta$  component. Experimental results need to be reanalyzed in light of these predictions to decide if either the  $f_0(1500)$  or an  $f_0(1710)$  coincides with this dressed glueball. [S0031-9007(97)04076-3]

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Quantum chromodynamics without quarks, the gauge theory of gluon interactions, predicts a spectrum of hadrons quite unlike the world experiments so clearly reveal. The imprint of this pure gauge world would be a spectrum of glueballs, the lightest of which would be stable with scalar quantum numbers. Lattice calculations have now reached sufficient precision to predict that this scalar glueball has a mass of  $1740 \pm 71$  MeV [1] or  $1550 \pm 50$  MeV [2], depending on how the lattice data are analyzed. More recently, using improved lattice actions, Morningstar *et al.* [3] give results that allow a central mass of 1600 MeV to be deduced. Moreover, the IBM group [4] have calculated its coupling to two pseudoscalars which would give a width of 100 MeV for the scalar, if it could decay to these. How this state of the quarkless world would appear in the real world is what this Letter is about.

For more than thirty years we have understood that (most) hadrons are closely connected to the states of underlying quark multiplets: The vector and tensor mesons and the baryon octet and decuplet being the best known examples. The properties of these states are determined by the bound state dynamics of quarks. This fixes their masses and decays. Thus, it is natural that the  $f_2(1270)$  and  $f_2(1525)$  decay predominantly to  $\pi\pi$  and  $K\bar{K}$ , respectively. Indeed, it is the composition of the underlying state that determines its decays, while the masses of the open channels are incidental. However, as emphasized by Tornqvist [5,6] and by Geiger and Isgur [7], scalar mesons may be rather different. They couple far more strongly and are far more sensitive to the opening of thresholds, particularly those with  $S$ -wave interactions.

In the quark model we build  $0^{++}$  mesons by having a  $q\bar{q}$  system with spin one and a unit of orbital angular momentum. The lightest of these is expected in the 1–2 GeV region, in which the scalar glueball also occurs. The isospin zero  $q\bar{q}$  members are expected to mix with the glue state to give the hadrons we observe. While a simple quark model of mixing, using wholly perturbative methods, may be appropriate for other quantum numbers,

the properties of scalars make such calculations far too simplistic. Instead the mixing is highly nonperturbative and requires a more detailed discussion of the hadron propagators. The natural vehicle for modeling this is the appropriate Schwinger-Dyson equation. To calculate this, we first assume that the coupling to two pseudoscalar channels controls the dynamics and hence the mixing.

*Calculation.*—We begin with the quenched approximation, which delivers a bare state of mass  $m_0$  with point couplings  $g_i$  to each meson channel. Reflecting the spatial extent of hadrons, these couplings are multiplied by form factors  $F(k_i^2)$ , where  $k_i$  is the channel's c.m. three-momentum and

$$F(k_i^2) = \exp[-k_i^2/(2k_0^2)], \quad (1)$$

$k_0$  is related to the interaction radius, which is taken to be between 0.5 and 1 fm. In related earlier work [5,7,8], the production of mesons has been modeled by the  $q\bar{q}$  creation in a  $^3P_0$  state. While this gives a satisfactory picture in many cases, this model does not take into account the Goldstone boson nature of the light pseudoscalars, so essential here. Consequently, we do not adopt the  $^3P_0$  model, but rather build the couplings to two pseudoscalars to include the Adler zeros of chiral dynamics at  $s_{A,i}$ . Then the full coupling  $G_i$  is given by

$$G_i^2 = g_i^2(s - s_{A,i})F^2(k_i^2), \quad (2)$$

where the  $s_{A,i} = O(m_\pi^2)$ . The effect of unquenching is to dress the bare bound state propagators. This gives them imaginary parts that are the prerequisite for decay. The Dyson summation of Fig. 1 gives

$$m^2(s) = m_0^2 + \Pi(s), \quad (3)$$

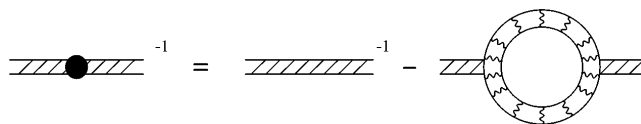


FIG. 1. The bare bound state propagator is dressed by hadronic interactions. The dot signifies the dressed propagator. The loop is of  $q\bar{q}$  pseudoscalars and the wiggly lines emphasize that these are also bound states.

where  $\Pi(s)$  is presently assumed to be dominated by the two pseudoscalar meson loops. Its imaginary part is simply related to the couplings by

$$\text{Im}\Pi_i(s) = -\rho_i(s)G_i^2(s)\theta(k_i^2), \quad (4)$$

where  $\rho_i(s) = k_i(s)/\sqrt{s}$  is the appropriate phase-space factor. The real part of  $\Pi(s)$  is now fixed by an unsubtracted dispersion relation—unsubtracted because hadrons have a spatial extent making only a finite number of hadronic channels relevant. Thus

$$\text{Re}\Pi(s) = \frac{1}{\pi} \mathcal{P} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}\Pi(s')}{s' - s}, \quad (5)$$

with  $\text{Im}\Pi(s) = \sum_i \text{Im}\Pi_i(s)$ , where we sum over all two-pseudoscalar modes, e.g.,  $K\pi, K\eta, K\eta'$  for the  $I = 1/2$  channel. The effect of loops is to produce a propagator,

$$\Delta = \frac{1}{m^2(s) - s} = \frac{1}{m_0^2 + \Pi(s) - s}, \quad (6)$$

where the pole is now in the complex energy plane (with reflections on each unphysical sheet generated by the thresholds to which the state couples). It is the position of this pole at  $s = m_{\text{pole}}^2 - im_{\text{pole}}\Gamma_{\text{pole}}$  that defines the mass,  $m_{\text{pole}}$ , and width,  $\Gamma_{\text{pole}}$ , of the hadron.

This is the formalism that applies in the  $I = 1/2$  and 1 scalar channels, where the ground state quark model gives one bare “seed.” However, in the  $I = 0$  channel, we have more bare states: two ground state  $q\bar{q}$ 's and the glueball. The philosophy here is that all mixing occurs through communicating hadronic final states. Thus the bare  $q\bar{q}$  nonet is assumed to be ideally mixed. This is in keeping with the notion that the  $\phi$ , for example, decays to  $\rho\pi$  through hadron interactions, e.g.,  $\phi \rightarrow K\bar{K} \rightarrow \rho\pi$ , and not through any nonhidden strangeness at the bare level. With three bare states ( $n\bar{n}, s\bar{s}, gg$ ) of the same quantum numbers, the propagator of Eq. (6) becomes a  $3 \times 3$  matrix. The hadrons are then the eigenstates of this matrix. To determine these requires a diagonalization. This diagonalization accounts for the mixing so crucial for scalar states [6,7].

*Input.*—The input for the bare seeds is as follows. There is an ideally mixed  $q\bar{q}$  multiplet. The mass of the nonstrange members is  $m_0(n\bar{n})$ , and the mass splitting is determined by the extra mass of the strange quark, where  $\Delta m_s \approx 100$  MeV from standard phenomenology. The couplings of these bare bound states to two pseudoscalars are assumed to be related by  $SU(3)_f$  symmetry, so that, in fact, there is effectively only one overall coupling for the nonet,  $\gamma$ . Similarly, the cutoff  $k_0$  in the form factor of Eq. (1) is assumed universal. The parameters  $m_0, \gamma$ , and  $k_0$  are to be determined.

The quenched glue state has mass  $m_0(gg)$  specified by lattice calculations. The coupling  $\gamma'$  is arranged to give a width of the unmixed glue state, to the sum of the two pseudoscalar channels we consider, of  $(108 \pm 29)$  MeV as computed by the IBM group [4]. The coupling of the bare state to individual channels is assumed to be according

to one of two schemes, either (a) as a  $SU(3)_f$  singlet, or (b) as found by the IBM group [4] on the lattice.

The size of the effect of turning on hadron loops can be understood qualitatively. For  $S$ -wave decays, each threshold produces a jump, Eq. (4), in the imaginary part of the mass function directly related to the strength of the coupling to the opening threshold with a corresponding cusp in the real part. If the coupling strength gives a width of 100 MeV, for example, then this may shift the mass of any bare state that is within 100 MeV or so of this threshold by roughly 100 MeV, and generally towards this threshold. This naturally has the effect of making the scalar mesons appear close to strongly coupled thresholds [9], potentially attracting the  $a_0(980)$  and  $f_0(980)$  to  $K\bar{K}$  threshold, if the parameters are suitable.

$I = 1/2$  and 1 sectors.—Although all the light scalars feature in our discussion, the major experimental ingredient in determining  $m_0, \gamma, k_0$  is the well established  $K_0^*(1430)$ , which from the Particle Data Tables [10] has

$$m = (1429 \pm 6) \text{ MeV}, \quad \Gamma = (287 \pm 23) \text{ MeV}. \quad (7)$$

That this  $I = 1/2$  state is a member of the lightest  $q\bar{q}$  nonet is noncontroversial. In contrast, whether the  $a_0(980)$ , or a possible  $a_0(1430)$ , belongs to this same multiplet is *a priori* not so clear.

In a closely related analysis [6], Tornqvist has determined the parameters  $m_0, \gamma, k_0$  with great precision from the  $K\pi$  phase shift that reflects the presence of the  $K_0^*(1430)$ . To do this, he assumes the  $K\pi$  scattering amplitude is pole dominated. This not only requires the denominator of any pseudoscalar scattering amplitude to be given by the propagator of Eq. (6) but assumes the numerator has a most specific form. This is a very special unitarization that is far from the most general; in particular, it lacks crossing symmetry. We do not enter here into the controversy [11] about what effect Tornqvist's strong assumptions have on the existence of a light  $\sigma$  resonance, suffice it to say that the treatment by Tornqvist and Roos [12] neglects the cross-channel dynamics that would appear in the numerator of a  $N/D$  analysis. Here it is only the  $D$  function that enters our discussion.

Since we consider just the denominators of the scalar amplitudes, we find a range of values of the parameters  $m_0, \gamma, k_0$  to be consistent with the strange state's mass and width, Eq. (7)—a range that includes Tornqvist's numbers. With central values, we show in Fig. 2 the real and imaginary parts of the  $I = 1/2$  mass function  $m^2(s)$ , together with that for  $I = 1$ , the parameters of which are also implicitly constrained by Eq. (7). This fixes the bare  $n\bar{n}$  state to be  $\sim 1420$  MeV. In contrast, the dressed  $I = 1$  bound state containing just 20%  $n\bar{n}$ , but with roughly 70%  $K\bar{K}$  and 10%  $\pi\eta'$ , has a Breit-Wigner mass and width [6] of 984 MeV and 130 MeV, respectively, as phenomenology requires [10]. However, it is only the pole in a propagator that truly fixes the parameters of a state. Consequently, we have to continue Eqs. (5) and (6) into the

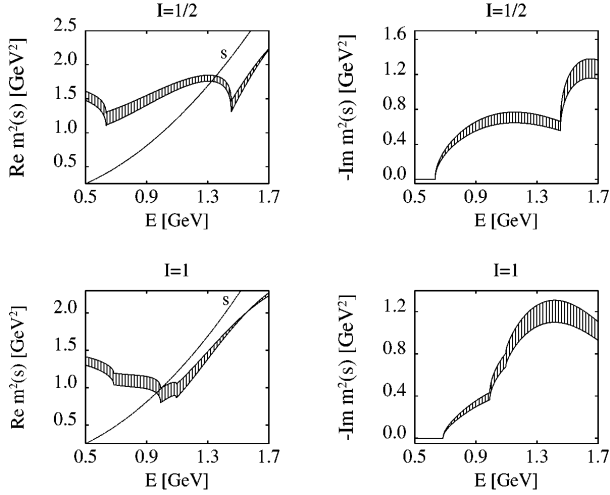


FIG. 2. The real and imaginary parts of  $m^2(s)$  as functions of  $E = \sqrt{s}$  for the  $I = 1/2$  and 1 propagators. Here  $m_0 = 1420$  MeV, and  $\gamma$  is taken to be between 1.1 and 1.2. The upper end of each band has  $\gamma = 1.1$ , and the lower end has  $\gamma = 1.2$ .

complex  $s$  plane onto the appropriate unphysical sheets. The scalars being generally broad and the effect of thresholds marked, the mass functions vary strongly as one moves into the  $s$  plane. The sheets are specified by the signs of the  $\text{Im}k_i$ , as there is a bifurcation at each threshold  $i$ . In Table I we give the mass and width  $m_{\text{pole}}, \Gamma_{\text{pole}}$  on the nearest unphysical sheet.

$I = 0$  sector. —Turning to the  $I = 0$  sector, the masses and couplings of the bare  $q\bar{q}$  nonet members are now fixed. The bare glueball mass  $m_0(gg)$  is taken to be either 1600 MeV from Morningstar *et al.* [3] or 1740 MeV from the IBM group [1]. Unquenching by first turning on the pseudoscalar loops of Fig. 1 gives the complex mass function shown in Fig. 3.  $I_0, II_0$  correspond to the pure glueball having, respectively, either  $SU(3)_f$  singlet couplings or the pattern computed by the IBM group that favors the higher mass channels with the couplings to  $\pi\pi : K\bar{K} : \eta\eta$  in the ratio  $(0.3 \pm 0.2) : 1 : (1.2 \pm 0.2)$ . It is by making the imaginary part of this mass function correspond to a width of  $\sim 100$  MeV at the glue state mass that fixes the coupling  $\gamma'$ . There are, of course, analogous mass functions for the  $n\bar{n}$  and  $s\bar{s}$  quark model scalars. To find the physical hadrons, we have to diagonalize the  $3 \times 3$  mass matrix formed from the  $n\bar{n}, s\bar{s}$ , and  $gg$  states. In physical terms, this allows the quark and glue

TABLE I. Masses and widths of the scalar nonet members in MeV, with  $\gamma = 1.15$ . The sheets are defined by the signs of  $\text{Im}k_i$  for the thresholds given.

| Resonance     | $m_{\text{pole}}$ | $\Gamma_{\text{pole}}$ | Thresholds                    | Sheet     |
|---------------|-------------------|------------------------|-------------------------------|-----------|
| $K_0^*(1430)$ | 1445              | 334                    | $\pi K, \eta K, \eta' K$      | $-, -, +$ |
| $a_0(980)$    | 1082              | 309                    | $\pi\eta, K\bar{K}, \pi\eta'$ | $-, +, +$ |
| $f_0(980)$    | 1006              | 54                     | $\pi\pi, K\bar{K}, \eta\eta$  | $-, +, +$ |
| $f_0(1300)$   | 1203              | 361                    | $\pi\pi, K\bar{K}, \eta\eta$  | $-, -, -$ |

configurations to mix through their common communicating channels:  $\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$ , and  $\eta'\eta'$ . Importantly, as noted earlier by Tornqvist [6], with the parameters of the quark multiplet fixed largely by the  $K_0^*(1430)$ , the ground state isoscalars are naturally the  $f_0(980)$  and  $f_0(1300)$ . A ground state  $s\bar{s}$  scalar up at 1700 MeV [13] is alien to the nonperturbative mixing computed here. The resulting  $I = 0$  pole positions are given in Table II, again for central values of the parameters. The presence of a glue state, with the mass and width as predicted on the lattice, has little effect on these predominantly  $q\bar{q}$  states. However, as seen in Fig. 3 from curves I, II, the mixing has an appreciable effect on the glue state. Its coupling to the  $n\bar{n}$  and  $s\bar{s}$  states dramatically reduces the width of the unquenched hadron: by how much depends on its mass and coupling pattern. For an underlying flavor singlet, the width is down from 100 MeV to  $\sim 30$  MeV if  $m_0(gg) = 1740$  MeV and to only a few MeV if  $m_0(gg) = 1600$  MeV. With this suppression of the couplings to the decay channels, the real part of the mass function, labeled I, II in Fig. 3, becomes almost independent of  $s$ . The suppression is most

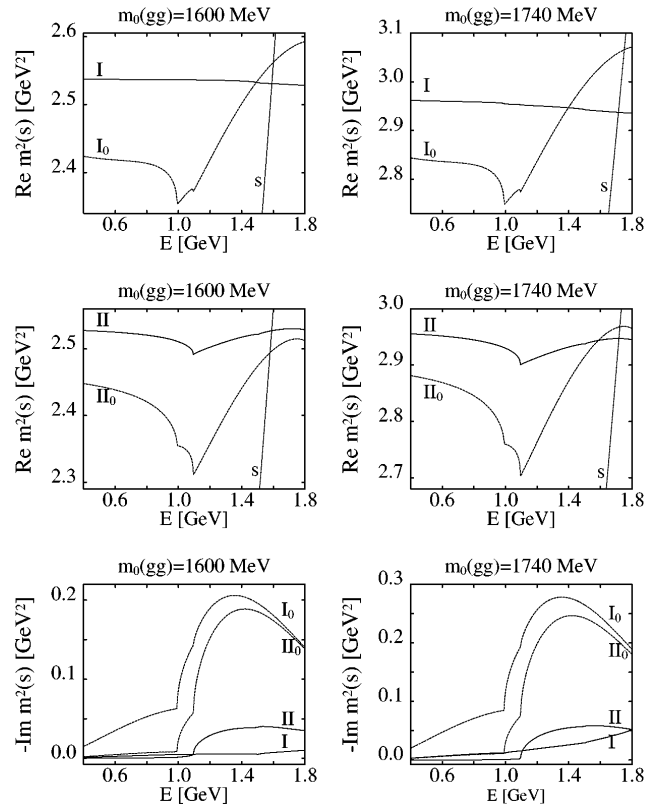


FIG. 3. The real and imaginary parts of the glue state  $m^2(s)$  as functions of  $E = \sqrt{s}$  for two values of bare mass  $m_0(gg) = 1600, 1740$  MeV, suggested by lattice calculations. The curves ( $I_0, I$ ) are obtained using  $SU(3)_f$  singlet couplings. ( $I_0$ ) corresponds to the mass function for the unmixed glueball, while ( $I$ ) shows the effect of its mixing with the  $q\bar{q}$  states. The curves ( $II_0, II$ ) are the analogs obtained using the pattern of couplings found by the IBM group. Here there is negligible difference between the curves with  $\gamma = 1.1, 1.2$ .

TABLE II. Mass and width of the glue state in MeV corresponding to various choices of bare mass and couplings. The sheets are defined as in Table I.

| Couplings                  | $m_0(gg)$ | $m_{\text{pole}}$ | $\Gamma_{\text{pole}}$ | Sheet      |
|----------------------------|-----------|-------------------|------------------------|------------|
| SU(3) <sub>f</sub> singlet | 1600      | 1591              | 3                      | -, -, +    |
| SU(3) <sub>f</sub> singlet | 1740      | 1715              | 32                     | -, -, +    |
| IBM                        | 1600      | 1589              | 22                     | -, -, -    |
| IBM                        | 1740      | 1715              | 28                     | -, -, -    |
| IBM + 4 $\pi$ channel      | 1600      | 1564              | 108                    | -, +, -, - |
| IBM + 4 $\pi$ channel      | 1740      | 1706              | 127                    | -, -, -, - |

appreciable for a lighter glueball with SU(3)<sub>f</sub> couplings. This is because the mixing with quark states occurs most through  $\pi\pi$  and  $K\bar{K}$  intermediate states. The IBM pattern of couplings favors the heavier pseudoscalars and the width suppression is consequently less. With the bare  $n\bar{n}$  state being at 1420 MeV and the  $s\bar{s}$  at 1620 MeV, a glue state at 1600 MeV interacts quite differently from one with a bare mass of 1740 MeV. In Table III we give the corresponding widths of the mainly gluish hadron to each two-pseudoscalar channel. Notice that there is no large width to  $K\bar{K}$ . Indeed, it is the  $\eta\eta$  decay mode that provides the largest width. These are the predictions that experiment has to check.

Experiment, in fact, delivers two potential candidates for the unquenched glueball:  $f_0(1500)$  with a width of  $(120 \pm 19)$  MeV seen by the Crystal Barrel experiment in  $p\bar{p}$  annihilation in several different channels [14], and the  $f_J(1710)$  with even spin and width of  $(175 \pm 9)$  MeV, first identified in  $J/\psi$  radiative decays [15]. Our results show that there is *prima facie* difficulty in identifying either of these with the glueball. Notice that the coupling pattern computed by the IBM group [4] for a quenched (nondecaying) glueball does not survive strong mixing with the other scalars and their decay channels. The resulting hadron, rather than coinciding with the  $f_0(1710)$  of Lindenbaum *et al.* [16], has a large  $\eta\eta$  decay width much more like the  $G(1590)$  state from the GAMS collaboration [17]. The latest Particle Data Tables [10] identify this with the WA91  $f_0(1450)$  [18] and the Crystal Barrel  $f_0(1500)$  [19], despite largely contradictory decay information—agreed with relatively small statistics from GAMS.

There is little doubt that the opening of multipion channels in the 1400 MeV region, which we have so far neglected, can have a marked effect on the glue state, while

TABLE III. Widths of the glue state to each single two-pseudoscalar channel, for various choices of bare mass and couplings.  $\Gamma_{\text{PS}}$  is the total width to two pseudoscalars.

| Couplings                  | $m_0(gg)$ | $\Gamma_{\text{PS}}$ | $\Gamma_{\pi\pi}$ | $\Gamma_{K\bar{K}}$ | $\Gamma_{\eta\eta}$ | $\Gamma_{\eta\eta'}$ |
|----------------------------|-----------|----------------------|-------------------|---------------------|---------------------|----------------------|
| SU(3) <sub>f</sub> singlet | 1600      | 4                    | 2                 | 0.2                 | 0.5                 | 1.5                  |
| SU(3) <sub>f</sub> singlet | 1740      | 26                   | 5                 | 6                   | 9                   | 6                    |
| IBM                        | 1600      | 25                   | 0.5               | 1.5                 | 21                  | 2                    |
| IBM                        | 1740      | 32                   | 0.1               | 1                   | 26                  | 5                    |

changing the ground state quark mesons rather little. Since we have no predictions from the lattice of the importance of these decay modes, we mimic these by arranging for just the 4 $\pi$  channel [19] to enhance the dressed glue state to  $\sim 120$  MeV total width. This leaves its partial widths to two pseudoscalars as in Table III. However, the larger couplings shift the mass of the “unquenched” glueball downwards and this becomes much more like the  $f_0(1500)$  with poles as in Table II.

In conclusion, the present work yields definite predictions for the decay pattern of the dressed glueball to be compared with experiment. The analysis of experiment is however not without its ambiguities. Consequently, the challenge is to perform a consistent analysis of data on all peripheral and central production,  $J/\psi$  radiative decays, and  $p\bar{p}$  annihilation [20] and show which of the predictions of Table III is in best agreement with experiment. Only then can one claim to have discovered the lightest glueball.

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