Appropriate Null Hypothesis for Cosmological Birefringence

A recent Letter [1] claims to have discovered evidence for birefrigence in the propagation of radio waves across cosmological distances. Unfortunately, this claim is based on a flawed statistical analysis.

To search for birefringence, the authors look for correlations between the direction and distance to a galaxy and the angle β between the polarization direction and the galaxy's major axis. Plotting the data as shown in Fig. 1(d) of their paper, here reproduced as Fig. 1, they use the correlation coefficient R_{xy} as their statistic.

To estimate the significance of their result, the authors use mock data samples constructed by randomly picking the angle β from a uniform distribution of allowed angles. This is not the proper null hypothesis for testing the dependence of β on the direction and distance to the galaxies. Rather, for the null hypothesis, one should draw the angles from the observed distribution. The high-redshift subsample, upon which the primary conclusions were based, is clearly not drawn from a uniform distribution. From Fig. 1, one can see by eye that the polarization in these galaxies tends to align with the galaxy's minor axis, i.e., $\beta^{\pm}$ prefers $\pm\pi/2$ and avoids 0 or $\pm\pi$. For example, $\sim 3/4$ of the points have $\pi/4 \leq |\beta| \leq 3\pi/4$.

That this matters can easily be seen from the following example. Consider the region in the x-y coordinate plane spanning -1 to 1 in both directions. If we uniformly fill the first and third quadrants, the correlation coefficient R_{xy} will be 0.75. If, however, we fix $y = 0.5\ \mathrm{sgn}(x)$ while allowing x to span -1 to 1 uniformly as before, then $R_{xy} = \sqrt{3}/2 \approx 0.867$. Collapsing the y direction in this way allows more of the scatter to be explained by the best-fit line.

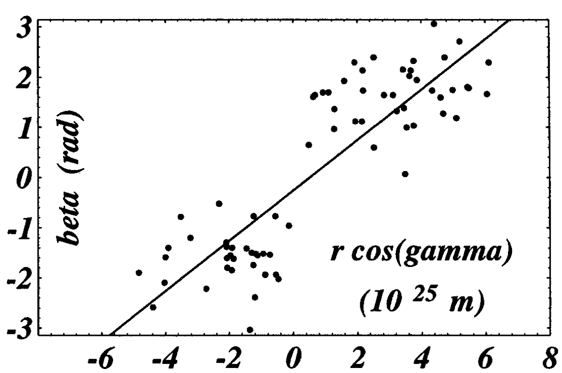

FIG. 1. Figure 1(d) reproduced from [1]. The 71 galaxies with redshifts above 0.3 are shown. γ is the angle from the proposed birefringence direction, r is the distance to the galaxy, and β is the angle between the galactic major axis and the polarization direction. See [1] for more details. Our claim is that the β are not uniformly distributed between 0 and $\pm\pi$ but rather are clumped toward $\pm\pi/2$.

Hence, we should expect that the tendency of the angle β to prefer $\pm\pi/2$ will cause R_{xy} to be higher than it would be if β were uniformly distributed between 0 and $\pm\pi$. By using the latter as their null hypothesis, the authors find a spuriously high statistical significance for their result. Indeed, if the underlying galaxy population truly had a uniform intrinsic distribution of β, it would be impossible to measure the proposed birefringence at all; one could not detect a rotation of such a distribution.

Stated another way and estimating by eye, in Fig. 1 the data *are* more tightly correlated than they would be if the β values were randomly and uniformly distributed between 0 and $\pm\pi$. However, they are not significantly more correlated than they would be if the β values in a quadrant were shuffled among themselves while the best-fit line was adjusted accordingly. Hence, the claimed correlation of the angle β with the position and distance of the galaxy is not statistically significant.

Taking the null hypothesis that the birefringence does not exist and that the angles between the polarization directions and galaxies' major axes are distributed as the data indicate, one is left to explain why the particular direction in the sky turned out to yield a higher R_{xy} than other directions. This most likely results from combining the inhomogeneous sky coverage—the sample is mostly from the northern sky and avoids low galactic latitudes—and the propensity of the chosen statistic R_{xy} to prefer directions that place many galaxies near the center of the spread in $r\cos\gamma$ where the tendence of β to prefer the center of its range can best reduce the scatter. It is not surprising that such a direction could exist.

A second error relating to the choice of statistic and null hypothesis is the authors' use of the slope of the best-fit line in Fig. 1 as a measure of the inverse birefringence scale Λ_s^{-1}. Because the null hypothesis (either the one they used or the one proposed here) produces a nonzero slope in the absence of birefringence, this is clearly a highly biased and inappropriate estimator.

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[1] B. Nodland and J. P. Ralston, Phys. Rev. Lett. **78**, 3043 (1997).

