Do the Quark Masses Run? Extracting $\overline{m}_b(m_Z)$ from CERN LEP Data

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We present the first results of next-to-leading order QCD corrections to three-jet heavy quark production at the CERN e^+e^- collider LEP including mass effects. Among other applications, this calculation can be used to extract the bottom-quark mass from LEP data and, therefore, to test the running of masses as predicted by QCD. [S0031-9007(97)03560-6]

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The decay width of the Z gauge boson into three jets has already been calculated at the leading order (LO) including complete quark mass corrections [1,2]. There it has been shown that for the b quark the effects of the mass could be as large as 1% to 6%, depending on its value and the jet-resolution parameter y_c. In fact, these effects had already been seen in the experimental tests of the flavor independence of the strong coupling constant [3–5].

In view of that we proposed [2], together with the DELPHI Collaboration [6], the possibility of using the ratio [2–4]

$$R_3^{bd} \equiv \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^d(y_c)/\Gamma^d} \tag{1}$$

as a means to extract the bottom-quark mass from LEP data. In this equation, $\Gamma_{3j}^q(y_c)/\Gamma^q$ is the three-jet fraction, and q denotes the quark flavor. Since the measurement of R_3^{bd} is done far away from the threshold of b-quark production, it will allow one, for the first time, to test the running of a quark mass as predicted by QCD. However, we discussed in [2] that the leading order calculation does not distinguish among the different definitions of the quark mass: perturbative pole mass M_b, running mass at M_b scale, or running mass at m_Z scale. The numerical difference is significant when the different definitions of masses are used in LO calculations. Therefore, in order to correctly take into account mass effects, it is necessary to perform a complete next-to-leading order (NLO) calculation of three-jet ratios including quark masses [7,8]. In this Letter we sketch the main points of this calculation, leaving the details for other publications [9,10], and we present the results that have been used by the DELPHI Collaboration to measure the running mass of the bottom quark at $\mu = m_Z$ [11].

To define the jets we use the jet-clustering algorithms (see, e.g., [12]) because they lead to IR finite observables and have small hadronization corrections at LEP.

The decay width of the Z boson into three jets with a heavy quark can be written as follows [2]:

$$\Gamma_{3j}^b(y_c) = \frac{m_Z g^2 \alpha_s(m_Z)}{c_W^2 64\pi^2}[g_V^2 H_V(y_c, r_b) + g_A^2 H_A(y_c, r_b)], \tag{2}$$

where g is the SU(2) gauge coupling constant, c_W and s_W are the cosine and the sine of the weak mixing angle, $g_V = -1 + 4/3 s_W^2$, and $g_A = 1$ are the vector and axial-vector coupling of the Z boson to the bottom quark. Functions $H_{V(A)}(y_c, r_b)$ contain all the dependences on y_c and the quark mass $r_b = (M_b/m_Z)^2$ for the different algorithms. These functions can be written as an expansion in α_s

$$H_{V(A)}(y_c, r_b) = A^{(0)}(y_c) + \frac{\alpha_s}{\pi} A^{(1)}(y_c) + r_b\left[B_{V(A)}^{(0)}(y_c, r_b) + \frac{\alpha_s}{\pi} B_{V(A)}^{(1)}(y_c, r_b)\right] + \ldots . \tag{3}$$

Here, $A^{(0)}$ is the tree-level contribution in the massless limit. It is the same function for the vector and the axial-vector parts, and it is known for the different algorithms in analytic form. The function $A^{(1)}$ gives the QCD NLO correction for massless quarks and, to a good approximation, it is also the same for the vector and the axial-vector parts. [There are triangle diagrams contributing to the axial part even for $m_q = 0$; however, they are small [13] and we are not going to consider them here.] This function is also known for the different algorithms [12]. Functions $B_{V(A)}$ take into account residual mass effects, once the leading dependence in r_b has been factorized. The tree-level contributions $B_{V(A)}^{(0)}$ were calculated numerically in [2] for the different algorithms, and results were presented in the form of simple fits to the numerical results. Finally, the functions $B_{V(A)}^{(1)}$, which were completely unknown, contain the NLO corrections depending on the quark mass and are the main result of our work.

Note that the way we write $H_{V(A)}$ in Eq. (3) is not an expansion for small r_b. We keep the exact dependence on r_b in the functions $B_{V(A)}$. Factoring out r_b makes it easier to analyze the massless limit and the dependence of the results on r_b in the region of interest. This means that our results can also be adapted, by including the photon exchange, to compute the e^+e^- cross section into three jets out of the Z peak at lower energies or at higher energies for top quark production.

At the NLO we have contributions to the three-jet cross section from three- and four-parton final states.

One-loop three-parton amplitudes are both infrared (IR) and ultraviolet (UV) divergent. Therefore, some regularization procedure is needed. We use dimensional regularization for both IR and UV divergences, because it preserves the QCD Ward identities. The three-parton transition amplitudes can be expressed in terms of a few scalar one-loop integrals [9]. The result contains poles in $\epsilon = (4 - D)/2$, where D is the number of space-time dimensions. Some of the poles come from UV divergences and the others come from IR divergences. The UV divergences, however, are removed after the renormalization of the parameters of the QCD Lagrangian. After that we obtain analytical expressions, which contain terms proportional to the IR poles and finite contributions.

The four-parton transition probabilities for $Z \to b\overline{b}gg(b\overline{b}q\overline{q})$ are split in two parts. The first part contains the terms which are divergent when one gluon is soft or two gluons (or light quarks) are collinear. These terms are integrated analytically in arbitrary D dimensions in the soft and collinear regions of phase space. This way we obtain the IR singular contributions of four partons in the three-jet region and show that they are canceled exactly by the corresponding IR poles coming from the virtual corrections according to [14]. The second part, corresponding to the radiation of hard gluons, gives rise to finite contributions and can be calculated in $D = 4$ dimensions. The three-jet Z width is obtained by integrating both renormalized three-parton contribution and four-parton transition probabilities in the three-jet phase-space region defined by the different jet-clustering algorithms. This quantity is infrared finite and well defined.

Following Ellis, Ross, and Terrano [15] (ERT) we have classified both three-parton and four-parton transition probabilities according to their color factors. It is clear that the cancellation of IR divergences between three-parton and four-parton processes can occur only inside groups of diagrams with the same color factor. The cancellation of IR divergences can be seen more clearly by representing the different amplitudes as the different cuts one can perform in the three-loop bubble diagrams contributing to the Z-boson self-energy. After summing up the three-parton and four-parton contributions to the three-jet decay width of the Z boson we obtain the functions $H_{V(A)}$ in Eq. (2) at order α_s. Since a large part of the calculation has been done numerically, it is important to have some checks of it. We have performed the following tests: (i) We have checked our four-parton probabilities in the massless limit against the amplitudes presented in Ref. [15]. The three-parton amplitudes for massive quarks cannot be compared directly with the corresponding massless result, as they have different structure of IR singularities. (ii) The four-parton transition amplitudes have also been checked in the case of massive quarks by comparing their contribution to four-jet processes with the known results [1]. (iii) To check the performance of the numerical programs we have applied our method to the massless amplitudes of ERT and obtained the known results for the functions $A^{(1)}$. (iv) We have checked, independently for each of the groups of diagrams with different color factors, that the final result obtained with massive quarks reduces to the massless result in the limit of very small masses.

The last test is the main check of our calculation. We have calculated the functions $H_{V(A)}$ for several values of r_b, in the range $M_b \sim 1 - 5$ GeV, and then we have extrapolated the results for $r_b \to 0$. In that limit we reproduce the values for the function $A^{(1)}$ in the different algorithms considered and the different groups of diagrams. This check is not trivial at all since the structure of IR divergences for massive quarks is quite different from the case of massless quarks: For massive quarks collinear divergences are regulated by the quark mass, and therefore some of the poles in ϵ that appear in the massless case are softened by $\log r_b$.

Combining Eqs. (1)–(3) and using the known expression for Γ^b [2,16] we write R_3^{bd} as the following expansion in α_s:

$$R_3^{bd} = 1 + r_b\left(b_0 + \frac{\alpha_s}{\pi}\, b_1\right), \tag{4}$$

where the functions b_0 and b_1 are an average of the vector and axial-vector parts, weighted by $c_V = g_V^2/(g_V^2 + g_A^2)$ and $c_A = g_A^2/(g_V^2 + g_A^2)$, respectively. They can be written in terms of the different functions introduced before, Eq. (3) [2,8], and also depend on y_c and r_b.

It is important to note that because of the particular normalization we have used in the definition of R_3^{bd}, which is manifested in the final dependence on c_V and c_A, most of the electroweak corrections cancel. Those are about 1% [17] in total rates, while in R_3^{bd} they are below 0.05%. Therefore, for our estimates it is enough to consider tree-level values of g_V and g_A. The same argument applies for the passage from decay widths to cross sections. Contributions from photon exchange are small at LEP and can be absorbed in a redefinition of g_V^2 and g_A^2 [18]. They will add a small correction to our observable.

Although intermediate calculations have been performed using the pole mass, we can also re-express our results in terms of the running quark mass by using the known perturbative expression $M_b^2 = \overline{m}_b^2(\mu)[1 + 2\alpha_s(\mu)/\pi(4/3 - \log[m_b^2/\mu^2])]$. The connection between pole and running masses is known up to order α_s^2; however, consistency of our pure perturbative calculation requires we use only the expression above. We obtain

$$R_3^{bd} = 1 + \overline{r}_b(\mu)\left[b_0 + \frac{\alpha_s(\mu)}{\pi}\left(\overline{b}_1 - 2b_0 \log\frac{m_Z^2}{\mu^2}\right)\right], \tag{5}$$

where $\overline{r}_b(\mu) = \overline{m}_b^2(\mu)/m_Z^2$ and

$$\overline{b}_1 = b_1 + b_0[8/3 - 2\log(r_b)]. \tag{6}$$

$\overline{r}_b(\mu)$ can be expressed in terms of the running mass of the b quark at $\mu = m_Z$ by using the renormalization group. At the order we are working $\overline{r}_b(\mu) = \overline{r}_b(m_Z)[\alpha_s(m_Z)/\alpha_s(\mu)]^{-4\gamma_0/\beta_0}$ with $\alpha_s(\mu) = \alpha_s(m_Z)/[1 + \alpha_s(m_Z)\beta_0 t]$ and $t = \log(\mu^2/m_Z^2)/(4\pi)$, $\beta_0 = 11 - 2N_f/3$, $N_f = 5$, and $\gamma_0 = 2$.

At the perturbative level, Eqs. (4) and (5) are equivalent. However, they neglect different higher order terms and lead to different answers. Since the experiment is performed at high energies (the relevant scales are m_Z and $m_Z\sqrt{y_c}$) one would think that the expression in terms of the running mass is more appropriate because the running mass is a true short distance parameter, while the pole mass contains in it all the complicated physics at scales $\mu \sim M_b$. Moreover, by using the expression in terms of the running mass we can vary the scale in order to estimate the error due to the neglect of higher order corrections. In any case, if one would use in Eq. (5) scales as low as $\mu = 5$ GeV, one would get something closer to the pole mass result.

Although we have studied the observable Eq. (1) for the four jet-clustering algorithms discussed in [2,8,12], here we will present results only for the DURHAM algorithm [19], which is the one that gives smaller radiative corrections, and we postpone the presentation of results for the different algorithms for another publication [10].

The function b_0 gives the mass corrections at leading order. As shown in [2] it depends very mildly on the quark mass in the region of interest ($M_b \sim 3 - 5$ GeV). Therefore it is appropriate to present our results for b_0 as a fit in only y_c: $b_0 = \sum_{n=0}^{2} k_0^{(n)} \log^n y_c$. For the DURHAM algorithm, in the range $0.01 < y_c < 0.10$ and 3 GeV $< M_b <$ 5 GeV, using $s_W^2 = 0.2315$, we obtain $k_0^{(0)} = -10.521$, $k_0^{(1)} = -4.4352$, $k_0^{(2)} = -1.6629$.

The function $\overline{b}_1$ is the main result of this paper. It gives the NLO massive corrections to R_3^{bd}. It is important to note that $\overline{b}_1$ contains significant logarithmic corrections depending on the quark mass. We take them into account by using the form $\overline{b}_1 = k_1^{(0)} + k_1^{(1)}\log(y_c) + k_m^{(0)}\log(r_b)$ in the fit. The coefficients we obtain, for the DURHAM scheme and ranges for y_c and r_b mentioned above, are $k_1^{(0)} = 297.92$, $k_1^{(1)} = 59.358$, $k_m^{(0)} = 46.238$.

In Fig. 1 we present R_3^{bd} for $\mu = m_Z$ (dashed), $\mu = 30$ GeV (dash-dotted), and $\mu = 10$ GeV (dotted) for $\overline{m}_b(m_Z) = 3$ GeV and $\alpha_s(m_Z) = 0.118$. For comparison

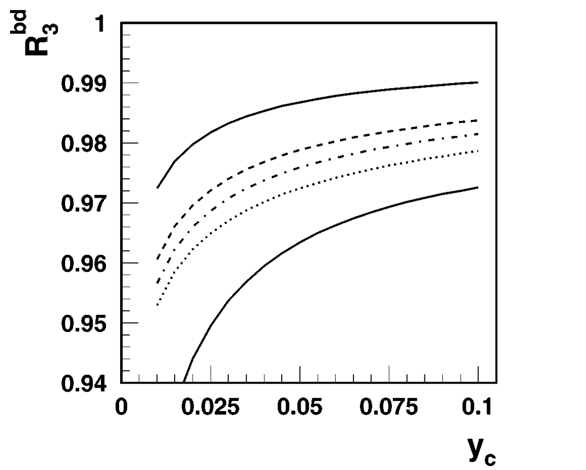

FIG. 1. NLO results for R_3^{bd} (DURHAM) for $\mu = m_Z$ (dashed), $\mu = 30$ GeV (dash-dotted) and $\mu = 10$ GeV (dotted) for $\overline{m}_b(m_Z) = 3$ GeV and $\alpha_s(m_Z) = 0.118$. For comparison we also plot the LO results for $M_b = 5$ GeV (lower solid line) and $\overline{m}_b(m_Z) = 3$ GeV (upper solid line).

we also present the LO results for the quark mass equal to 5 GeV (lower solid line) which is, roughly, the value of the pole mass obtained at low energies and 3 GeV (upper solid line) which is, roughly, the value one obtains for the running mass at the m_Z scale by using the renormalization group. Note that choosing a low value for μ makes the result closer to the LO result written in terms of the pole mass, while choosing a large μ makes the result approach to the LO result written in terms of the running mass at the m_Z scale.

If R_3^{bd} is measured to good accuracy one could use Eq. (5) and the relationship between $\overline{m}_b(\mu)$ and $\overline{m}_b(m_Z)$ to extract $\overline{m}_b(m_Z)$. However, the extracted result will depend on the scale μ. For illustration, in Fig. 2 we represent, as a function of μ, the value one would obtain for $\overline{m}_b(m_Z)$ if $R_{3\,\mathrm{exp}}^{bd} = 0.96$ for $y_c = 0.02$. What is the best scale one should choose in three-jet quantities is a long standing discussion. One would think that if the energy is equally distributed among the three jets one should choose $\mu \sim m_Z/3$. However, a more conservative approach is to vary the scale in an appropriate range and take the spread of the result as an estimate of the error due to higher order corrections. From Fig. 2 we see that if we take μ in the range $m_Z/10 - m_Z$ the error due to the scale in the determination of $\overline{m}_b(m_Z)$ would be of about 0.20 GeV. If scales as low as $\mu = 5$ GeV are accepted, the error increases to 0.23 GeV. Whether this error can be reduced by a clever choice of the scale or resummation of leading logs in y_c or r_b remains to be seen.

The calculation presented in this paper has already been used by the DELPHI Collaboration [11] to extract the value of $\overline{m}_b(m_Z)$ from R_3^{bd}. The preliminary result [we would like to thank the DELPHI Collaboration for allowing us to use these numbers here] they

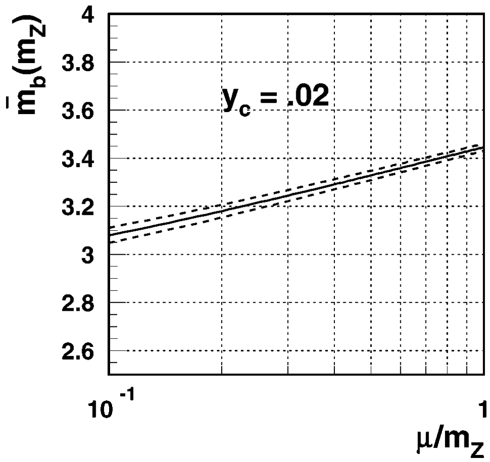

FIG. 2. Extracted value of $\overline{m}_b(m_Z)$ if $R_{3\,\mathrm{exp}}^{bd} = 0.96$ as a function of the scale μ. We take $\alpha_s(m_Z) = 0.118$ (solid) and $\Delta\alpha_s = 0.003$ (dashed).

obtain is

$$\overline{m}_b(m_Z) = 2.85 \pm 0.22(\mathrm{stat}) \pm 0.20(\mathrm{theo}) \pm 0.36(\mathrm{frag})\ \mathrm{GeV}, \qquad (7)$$

which has to be compared with low energy determinations of the bottom quark mass. The last analysis of the Υ system using QCD sum rules [20] gives $\overline{m}_b(\overline{m}_b) = 4.13 \pm 0.06$ GeV, which translates into $\overline{m}_b(m_Z) = 2.83 \pm 0.10$ GeV if one uses three-loop renormalization group running and $\alpha_s(m_Z) = 0.118 \pm 0.003$. A previous analysis [21] using only order α_s expressions, as we use, gives $\overline{m}_b(M_b) = 4.23 \pm 0.04 \pm 0.02$ GeV. On the other hand, the last lattice result is [22] $\overline{m}_b(\overline{m}_b) = 4.15 \pm 0.20$ GeV and $\overline{m}_b(m_Z) = 2.84 \pm 0.21$ GeV. Given the errors it is clear that central values agree so well just by chance.

This preliminary measurement is in full compatibility with low energy data and it is accurate enough to test the running of the bottom quark mass from $\mu = M_b$ to $\mu = m_Z$ since the result for $\overline{m}_b(m_Z)$ in Eq. (7) and the previous values for $\overline{m}_b(\overline{m}_b)$ differ by more than 2.5 standard deviations. We believe that these results can be substantially improved with more experimental and theoretical work.

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Note added.—The results in this paper have been previously presented at several conferences [7,11] and seminars. While preparing the manuscript a preprint dealing with the same problem has appeared [23].

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[1] A. Ballestrero, E. Maina, and S. Moretti, Phys. Lett. B **294**, 425 (1992); Nucl. Phys. **B415**, 265 (1994).
[2] M. Bilenky, G. Rodrigo, and A. Santamaria, Nucl. Phys. **B439**, 505 (1995).
[3] J. Chrin, in *Proceedings of the 28th Rencontre de Moriond, Les Arcs, Savoie, France, 1993,* edited by J. T. T. Van (Editiones Frontieres, Gif-sur-Yvette, France, 1993), p. 313; J. Valls, Ph.D. thesis, Universitat de València, 1994; J. Fuster, in *Proceedings of the XXII International Meeting on Fundamental Physics, Jaca, Spain, 1994* (Editiones Frontieres, Gif-sur-Yvette, France, 1995).
[4] DELPHI Collaboration, P. Abreu *et al.,* Phys. Lett. B **307**, 221 (1993).
[5] ALEPH Collaboration, D. Buskulic *et al.,* Phys. Lett. B **355**, 381 (1995).
[6] J. Fuster (private communication).
[7] G. Rodrigo, Nucl. Phys. (Proc. Suppl.) **56A**, 60 (1997).
[8] G. Rodrigo, Ph.D. thesis, Universitat de València [Report No. hep-ph/9703359, ISBN: 84-370-2989-9, 1996].
[9] G. Rodrigo, A. Santamaria, and M. Bilenky, Report No. hep-ph/9703360, 1997.
[10] G. Rodrigo, A. Santamaria, and M. Bilenky (to be published).
[11] J. Fuster, S. Cabrera, and S. Martí:, Nucl. Phys. (Proc. Suppl.) **54A**, 39 (1997); J. Fuster, in Proceedings of the XXV International Meeting on Fundamental Physics, Formigal-Huesca, Spain, 1997.
[12] S. Bethke, Z. Kunszt, D. E. Soper, and W. J. Stirling, Nucl. Phys. **B370**, 310 (1992); Z. Kunszt, P. Nason, G. Marchesini, and B. R. Webber, in Proceedings of the 1989 LEP Physics Workshop, Geneva, Switzerland, 1989.
[13] K. Hagiwara, T. Kuruma, and Y. Yamada, Nucl. Phys. **B358**, 80 (1991).
[14] F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937); T. Kinoshita, J. Math. Phys. **3**, 650 (1962); T. D. Lee and M. Nauenberg, Phys. Rev. **133**, B1549 (1964).
[15] R. K. Ellis, D. A. Ross, and A. E. Terrano, Nucl. Phys. **B178**, 421 (1981).
[16] A. Djouadi, J. H. Kühn, and,P. M. Zerwas, Z. Phys. C **46**, 411 (1990); K. G. Chetyrkin and J. H. Kühn, Phys. Lett. B **248**, 359 (1990).
[17] J. Bernabéu, A. Pich, and A. Santamaria, Nucl. Phys. **B363**, 326 (1991).
[18] J. Jersák, E. Laermann, and P. M. Zerwas, Phys. Rev. D **25**, 1218 (1982); Phys. Rev. D **36**, 310(E) (1987).
[19] N. Brown and W. Stirling, Z. Phys. C **53**, 629 (1992); S. Catani, Y. L. Dokshitser, M. Olsson, G. Turnock, and B. R. Webber, Phys. Lett. B **269**, 432 (1991).
[20] M. Jamin and A. Pich, Report No. hep-ph/9702276, 1997.
[21] S. Narison, Phys. Lett. B **341**, 73 (1994).
[22] V. Giménez, G. Martinelli, and C. T. Sachrajda, Nucl. Phys. (Proc. Suppl.) **53**, 365 (1997); Phys. Lett. B **393**, 124 (1997).
[23] W. Bernreuther, A. Brandenburg, and P. Uwer, Phys. Rev. Lett. **79**, 189 (1997).