Exact Results for a Kondo Problem in a One-Dimensional t-J Model

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(Received 25 March 1997)

We propose an integrable Kondo problem in a one-dimensional t-J model. With the open boundary condition of the wave functions at the impurity sites, the model can be exactly solved via Bethe ansatz for a set of $J_{L,R}$ (Kondo coupling constants) and $V_{L,R}$ (impurity potentials) parametrized by a single parameter c. The integrable value of $J_{L,R}$ runs from negative infinity to positive infinity, which allows us to study both the ferromagnetic Kondo problem and the antiferromagnetic Kondo problem in a strongly correlated electron system. Generally, there is a residual entropy for the ground state, which indicates a typical non-Fermi liquid behavior. [S0031-9007(97)03860-X]

PACS numbers: 71.10.Pm, 72.10.Fk, 72.15.Qm, 75.20.Hr

Recently, considerable attention has been drawn by the theory of impurities in both the Fermi and the Luttinger liquids, and many new developments have been reported. This renewed interest in the quantum impurity problem was partially stimulated by the search for non-Fermi-liquid fixed points beyond the well known Luttinger liquid. A major progress is the extensive studies on the multichannel Kondo problem by the conformal field theory in the presence of boundary [1,2] and by the Bethe ansatz [3–5]. With these powerful methods, it becomes possible to obtain the low temperature thermodynamics near the critical point. At the same time, using bosonization and renormalization group techniques, Kane and Fisher have shown that a potential scatter center embedded in a Luttinger liquid is driven to a strong-coupling fixed point by the repulsive electron-electron interactions [6]. This shows for the first time that a single impurity in a Luttinger liquid behaves rather differently from its behavior in a Fermi liquid. This new finding stimulates the study on the problem of local perturbations in a Luttinger liquid and especially on the Kondo problem in a Luttinger liquid. The Luttinger-Kondo problem was first considered by Lee and Toner [7], who found a crossover of the Kondo temperature from power law dependence on the Kondo coupling constant to an exponential one. Subsequently, a poor man's scaling was performed by Furusaki and Nagaosa [8], who addressed the conjecture that ferromagnetic Kondo screening may occur in a Luttinger liquid. The boundary conformal field theory [9] yielded a classification of critical behaviors for a Luttinger liquid coupled to a magnetic impurity (without impurity potential). It turns out that there are only two possibilities, a local Fermi liquid with standard low-temperature thermodynamics, or a non-Fermi liquid as observed by Furusaki and Nagaosa [8]. It is now clear that the non-Fermi-liquid behavior is induced by the tunneling effect of conduction electrons through the impurity which depends only on the bulk properties but not on the detail of the impurity [10,11].

Despite such important progress, the problem of few impurities (potential, magnetic, especially both) embedded in a strongly correlated system is still not well understood. We remark that there is some progress related to this problem: the case of a spin $S > 1/2$ impurity in a spin $1/2$ Heisenberg chain solved many years ago by Andrei and Johannesson [12] and generalized to arbitrary spins by Lee and Schlottmann and by Schlottmann [13], and an integrable impurity in the supersymmetric t-J model [14] with a very complicated Hamiltonian solved by Bedürftig *et al.* We remark here that the above models are rather special for the absence of backward scattering off the impurity, and therefore cannot lead to a deep understanding of the Kondo problem in a Luttinger liquid, where the backward scattering is crucial to the fixed point of the system. In addition, Wang and Voit have proposed an integrable model of a single magnetic impurity in a δ-potential Fermi gas [11] with a special value of Kondo coupling constant.

Attempting to understand the effects of magnetic impurities in a strongly correlated electron system, we study the properties of the integrable t-J model coupled to two magnetic impurities sited at the ends of the system. Our starting point is the following Hamiltonian

$$
\begin{aligned}
H &= H_0 + H_i\,, \\
H_0 &= -\sum_{j=1,\sigma}^{N_a-1}(C_{j\sigma}^\dagger C_{j+1\sigma} + \text{H.c.}) \\
&\quad + 2\sum_{j=1}^{N_a-1}[\mathbf{S}_j\cdot\mathbf{S}_{j+1} + Vn_jn_{j+1}], \\
H_i &= J_L\mathbf{S}_1\cdot\mathbf{S}_L + V_Ln_1 + J_R\mathbf{S}_{N_a}\cdot\mathbf{S}_R + V_Rn_{N_a}\,,
\end{aligned}
\tag{1}
$$

where $C_{j\sigma}$ ($C_{j\sigma}^\dagger$) are annihilation (creation) operators of the conduction electrons; V, $J_{R,L}$, $V_{R,L}$ are the nearest neighbor interaction constant, the Kondo coupling constants, and the impurity potentials, respectively; $\mathbf{S}_j = \frac{1}{2}\sum_{\sigma,\sigma'}C_{j\sigma}^\dagger\sigma_{\sigma\sigma'}C_{j\sigma'}$ is the spin operator of the conduction electrons; $\mathbf{S}_{L,R}$ are the local moments with spin-$1/2$

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sited at the left and the right ends of the system, respectively; $n_j = C_{j\uparrow}^\dagger C_{j\uparrow} + C_{j\downarrow}^\dagger C_{j\downarrow}$ is the number operator of conduction electrons; N_a is the length or site number of the system. Notice that the single occupation condition $n_j \leq 1$ is assumed for the Hamiltonian (1). We remark that the model is very reasonable for the absence of physically questionable terms in the Hamiltonian.

It is well known that H_0 is exactly solvable for $V = -1/4$ and $3/4$ [15,16]. Here we study only the $V = 3/4$ case while both the charge and spin sectors can be described by a Luttinger liquid. The $V = -1/4$ (supersymmetric t-J model) case can then be treated similarly without any difficulty. By including the impurities, any electron impinging on the boundaries will be completely reflected and suffer a reflection matrix $R_{j,L}$ or $R_{j,R}$ [11]. The waves are therefore reflected at either end as

$$\begin{aligned} e^{ik_jx} &\rightarrow R_{j,L}(k_j)e^{-ik_jx}, \qquad x \sim 1\,, \\ e^{ik_jx} &\rightarrow R_{j,R}^{-1}(k_j)e^{-ik_jx-2ik_jN_a}, \qquad x \sim N_a\,. \end{aligned} \tag{2}$$

The reflecting Yang-Baxter equation [17,18],

$$\begin{aligned} S_{jl}(q_j - q_l)R_{j,a}(q_j)S_{jl}(q_j + q_l)R_{l,a}(q_l) \\ = R_{l,a}(q_l)S_{jl}(q_j + q_l)R_{j,a}(q_j)S_{jl}(q_j - q_l)\,, \\ a = R, L\,, \end{aligned} \tag{3}$$

constrains the integrability of the present model. Here S_{jl} is the electron-electron scattering matrix in the bulk. By evaluating Eq. (3), we find that the present model is exactly solvable with the following parametrized J_a and V_a ($a = L, R$): $J_a = -8/(2c_a + 3)(2c_a - 1)$, $V_a = (4c_a^2 - 7)/(2c_a + 3)(2c_a - 1)$, where $c_{L,R}$ are two arbitrary real constants. Generally, J_L and J_R may take different values. Here we consider only the $J_R = J_L$, $V_R = V_L$, i.e., $c_L = c_R = c$, case. The solution of our model in the integrable line is quite similar to those of other integrable models with open boundaries [17,18]. Notice that the reflection matrix $R_{L,R}$ ($K_\pm$ in Ref. [17]) is an operator matrix rather than a c-number matrix. The spectrum of the Hamiltonian (1) is uniquely determined by the following Bethe ansatz equations:

$$\begin{aligned} \left(\frac{q_j - \frac{i}{2}}{q_j + \frac{i}{2}}\right)^{2N_a} &= \left[\frac{q_j + i(c-1)}{q_j - i(c-1)}\right]^2 \prod_{r=\pm1}\left\{\prod_{l\neq j}^{N}\frac{q_j - rq_l - i}{q_j - rq_l + i}\prod_{\alpha=1}^{M}\frac{q_j - r\lambda_\alpha + \frac{i}{2}}{q_j - r\lambda_\alpha - \frac{i}{2}}\right\} \\ \left\{\frac{\lambda_\alpha - i(c-\frac{1}{2})\lambda_\alpha + i(c+\frac{1}{2})}{\lambda_\alpha + i(c-\frac{1}{2})\lambda_\alpha - i(c+\frac{1}{2})}\right\}^2 \prod_{r=\pm1}\prod_{j=1}^{N}\frac{\lambda_\alpha - rq_j + \frac{i}{2}}{\lambda_\alpha - rq_j - \frac{i}{2}} &= \prod_{r=\pm1}\prod_{\beta\neq\alpha}^{M}\frac{\lambda_\alpha - r\lambda_\beta + i}{\lambda_\alpha - r\lambda_\beta - i}\,, \end{aligned} \tag{4}$$

with the eigenvalue of (1) given by $E = 2N - \sum_{j=1}^{N} 4/(4q_j^2 + 1)$. Here N is the number of conduction electrons, $q_j = 1/2\tan(k_j/2)$, and k_j and λ_α are the rapidities of charge and spin, respectively. M is the number of spin-down electrons.

Below we discuss the ground state properties for different regions of parameter c.

(i) $c \geq 1$.—The system falls into the ferromagnetic Kondo coupling regime and no bound state can exist at low energy scales. The ground state is thus described by two sets of real parameters $\{q_j\}$ and $\{\lambda_\alpha\}$. Define the quantities

$$\begin{aligned} Z_{N_a}^c(q) &= \frac{1}{\pi}\left\{-\theta_1(q) + \frac{1}{2N_a}\left[\phi_c(q) - \sum_{\alpha=-M}^{M}\theta_1(q-\lambda_\alpha) + \sum_{j=-N}^{N}\theta_2(q-q_j)\right]\right\}, \\ Z_{N_a}^s(\lambda) &= \frac{1}{2N_a\pi}\left\{\phi_s(\lambda) - \sum_{j=-N}^{N}\theta_1(\lambda-q_j) + \sum_{\alpha=-M}^{M}\theta_2(\lambda-\lambda_\alpha)\right\}, \end{aligned} \tag{5}$$

with the phase shifts $\phi_c(q) = -\theta_2(q) + 4\tan^{-1}[q/(c-1)]$, $\phi_s(\lambda) = -\theta_2(\lambda) + 4\tan^{-1}[\lambda/(c+1/2)] - 4\tan^{-1}\times[\lambda/(c-1/2)]$ induced by the impurities in the charge and spin sectors, respectively, and $\theta_n(x) = -2\tan^{-1}(2x/n)$. Note above we have used the reflection symmetry of the Bethe ansatz equations to include solutions with $q_{-j} = -q_j$ and $\lambda_{-\alpha} = -\lambda_\alpha$. The Bethe ansatz equations are solved by $Z_{N_a}^c(q_j) = I_j/N_a$ and $Z_{N_a}^s(\lambda) = J_\alpha/N_a$, where I_j and J_α are nonzero integers. In the ground state, I_j and J_α must be consecutive numbers around zero symmetrically to minimize the energy. The roots q_j and λ_α become dense in the thermodynamic limit, and we define their densities as

$$\rho_{N_a}^c(q) = \frac{dZ_{N_a}^c(q)}{dq}, \qquad \rho_{N_a}^s(\lambda) = \frac{dZ_{N_a}^s(\lambda)}{d\lambda}. \tag{6}$$

The cutoffs of q and λ in the ground state are $\pm Q$ and $\pm\Lambda$, respectively, which correspond to $Z_{N_a}^c(\pm Q) = \pm(N + 1/2)/N_a$, $Z_{N_a}^s(\pm\Lambda) = \pm(M + 1/2)/N_a$. In the thermodynamic limit $N_a \rightarrow \infty$, $N \rightarrow \infty$, and $N/N_a \rightarrow$ finite, we find that the energy is minimized at $\Lambda \rightarrow \infty$, which gives a result of $N = 2M$ by integrating Eq. (6). The magnetization is given by $1/2(N + 2 - 2M)$. This indicates a spin triplet ground state which is in contradiction to the Furusaki-Nagaosa conjecture [8].

(ii) $1/2 < c < 1$.—The system falls also into the ferromagnetic Kondo coupling regime. However, unlike case (i), two bound states of electrons can be formed around the impurities in the ground state, which correspond to two imaginary q modes at $q = \pm i(1-c)$. The Bethe ansatz equations for the real modes are thus reduced to

$$\left(\frac{q_j - \frac{i}{2}}{q_j + \frac{i}{2}}\right)^{2N_a} = \left[\frac{q_j + i(c-1)}{q_j - i(c-1)}\frac{q_j - ic}{q_j + ic}\frac{q_j + i(c-2)}{q_j - i(c-2)}\right]^2 \prod_{r=\pm1}\left\{\prod_{l\neq j}^{N-2}\frac{q_j - rq_l - i}{q_j - rq_l + i}\prod_{\alpha=1}^{M}\frac{q_j - r\lambda_\alpha + \frac{i}{2}}{q_j - r\lambda_\alpha - \frac{i}{2}}\right\}$$
$$\left\{\frac{\lambda_\alpha - i(c-\frac{3}{2})}{\lambda_\alpha + i(c-\frac{3}{2})}\frac{\lambda_\alpha + i(c+\frac{1}{2})}{\lambda_\alpha - i(c+\frac{1}{2})}\right\}^2 \prod_{r=\pm1}\prod_{j=1}^{N-2}\frac{\lambda_\alpha - rq_j + \frac{i}{2}}{\lambda_\alpha - rq_j - \frac{i}{2}} = \prod_{r=\pm1}\prod_{\beta\neq\alpha}^{M}\frac{\lambda_\alpha - r\lambda_\beta + i}{\lambda_\alpha - r\lambda_\beta - i}. \tag{7}$$

Following the same procedure discussed above, we obtain again a spin triplet ground state. This can be understood in the following picture: the bounded electrons and the local moments form two spin-1 local composites due to the effective attraction and the ferromagnetic Kondo coupling. However, the itinerant electrons impinging on these composites will screen their moments partially due to the indirect Kondo coupling induced by the electron-electron correlation. In such a sense, we recover Furusaki-Nagaosa's conjecture [8], though the local moments are not completely screened.

(iii) $-1/2 < c < 1/2$.—The system falls into the antiferromagnetic Kondo coupling regime. No bound state appears in the ground state. By integrating Eq. (6) in the thermodynamic limit, we have $N + 2 = 2M$. This indicates a spin singlet ground state, a similar result to that of the Kondo problem in a Fermi liquid.

(iv) $-1 \leq c \leq -1/2$.—No bound state exists in the ground state. It seems that the ground state should be a spin triplet. We note that both J_a and V_a take positive values in this region. The repulsive boundary potential dominates over the Kondo coupling, and bars the conduction electrons from forming singlet with the local moments. In such a sense, no Kondo screening occurs, which strongly indicates that both the boundary potential and the electron-electron correlation in the bulk have significant effects to the Kondo problem in a Luttinger liquid.

(v) $-3/2 < c < -1$.—For this case, the system is still in the regime of antiferromagnetic Kondo coupling but with a weaker or attractive boundary potential. Two imaginary λ modes at $\lambda = \pm i(c + 1/2)$ and two imaginary q modes at $q = \pm i(c + 1)$ appear in the ground state. These modes correspond to the formation of bound singlet pairs of two conduction electrons with the local moments. By integrating the densities of the real modes in the thermodynamic limit, we still arrive at a spin singlet ground state.

(vi) $c < -3/2$.—The Kondo coupling is ferromagnetic and the boundary potential is repulsive. There is no bound state in the ground state. Following the same procedure discussed in (i), we obtain again $N = 2M$ for the ground state. Therefore, there is no Kondo screening in this region, which contradicts the Furusaki-Nagaosa conjecture.

The thermodynamics of the present model can be calculated in a closed form based on the Bethe ansatz equations (4). This allows us to obtain the temperature and magnetic field dependence of the free energy which contains three parts of contributions, i.e., the bulk term, the boundary term, and the Kondo effect term. Here we omit the details of calculation which follow the standard procedure, and can be found in some excellent works [3,19–21]. The Kondo effect induced free energy is the most interesting term which takes the following form at low temperatures (hereafter we assume c as an integer or half integer):

$$F_k = -T\,\mathrm{sgn}(n_1)\int \frac{\ln[1 + \zeta_{|n_1|}(\lambda)]\,d\lambda}{2\cosh(\pi\lambda)} - T\,\mathrm{sgn}(n_2)\int \frac{\ln[1 + \zeta_{|n_2|}(\lambda)]\,d\lambda}{2\cosh(\pi\lambda)}, \tag{8}$$

where n_1 and n_2 are two c-dependent integers. $\zeta_n(\lambda)$ are elements of the following coupled integral equations:

$$\begin{aligned}
\ln\eta(\lambda) &= \frac{\epsilon_0(\lambda) - \mu}{T} - ([1]G + [2])\ln[1 + \eta^{-1}(\lambda)] - G\ln[1 + \zeta_1(\lambda)],\\
\ln\zeta_n(\lambda) &= G\{\ln[1 + \zeta_{n+1}(\lambda)] + \ln[1 + \zeta_{n-1}(\lambda)]\}, \qquad n > 1,\\
\ln\zeta_1(\lambda) &= -G\ln[1 + \eta^{-1}(\lambda)] + G\ln[1 + \zeta_2(\lambda)],\\
\lim_{n\to\infty}&\{[n]\ln[1 + \zeta_{n+1}(\lambda)] - [n+1]\ln[1 + \zeta_n(\lambda)]\} = \frac{2H}{T},
\end{aligned} \tag{9}$$

where μ is the chemical potential, $\epsilon_0(\lambda) = 2 - 4\lambda^2/(4\lambda^2 + 1)$, $[n]$ and G are integral operators with the kernels $a_n(\lambda) = (n/2\pi)/[\lambda^2 + (n/2)^2]$ and $1/[2\cosh(\pi\lambda)]$, respectively, and H is the external magnetic field. Notice that we have omitted the excitations breaking the bound states in deriving (9). Their contributions to the free energy are exponentially small at low enough temperatures.

For $H = 0$ and $T \to 0$, the driving term $G\ln[1 + \eta^{-1}(\lambda)]$ in (9) is nothing but $T^{-1}\epsilon_s(\lambda)$. $\epsilon_s(\lambda)$ is the dressed energy of the spin waves and has the asymptotic form $\epsilon_s(\lambda) \to 2\pi e_0 \exp(-\pi|\lambda|)$ for $|\lambda| \to \infty$, where e_0 is the energy density of the ground state. This allows us to formulate the low-temperature expansion of (9). The asymptotic solution of (9) is given by functions $\zeta_n(x)$, with

$x = \ln[(\pi e_0)/T] + \pi|\lambda|$. $\zeta_n(x)$ decrease monotonically with x for all n, and tend to finite limits ζ_{n+} as $x \rightarrow \infty$. The limits are given by $\zeta_{n+} = \sinh^2(nH/T)/\sinh^2(H/T) - 1$. No anomaly appears in the free energy of the bulk and the open boundary. They contain only constant terms and T^2 terms up to the order $o(T^2)$. An interesting feature is that with different c values, the system may show either Fermi- or non-Fermi-liquid behavior. To show this, we study the entropy of the ground state for a variety of c regions. For $c > 1$ or $c < -3/2$, there is no bound state in the ground state. n_1 and n_2 take the values $2|c| + 1$ and $1 - 2|c|$, respectively. Therefore, the two local spins are renormalized to two effective moments with the strengths $|c| + 1/2$ and $|c| - 1/2$, respectively. However, it seems that the larger "moment" couples with the conduction electron antiferromagnetically, while the smaller moment couples with the conduction electrons ferromagnetically. The residual entropy of the ground state can be easily calculated from Eq. (8) as $S_g = \ln[(2|c| + 1)/(2|c| - 1)]$. For $c = 0$, the local moments are completely screened, and both n_1 and n_2 are equal to unity. Thus the entropy of the ground state is zero and the system flows to a Fermi-liquid fixed point. For $c = -1/2$, $n_1 = 2$, and $n_2 = 0$, the residual entropy takes a value of $\ln 2$. While for $c = -1$, $n_1 = -1/2$, $n_2 = 3/2$, $S_g = \ln 3$. Notice that for $c = -1/2$ and $c = -1$, the system falls into the regime of antiferromagnetic Kondo coupling but with a nonzero residual entropy. We remark that $c = 1/2$ and $c = -3/2$ are two critical points, since at these points, both J_a and V_a are divergent. The residual entropy has different limits for $c \rightarrow 1/2 + 0^+(-3/2 + 0^+)$ and $c \rightarrow 1/2 + 0^-(-3/2 + 0^-)$. From the above discussion we conclude that the system generally has a c-dependent residual entropy which strongly indicates a non-Fermi-liquid behavior. The system can flow to a Fermi-liquid fixed point only in a narrow parameter region of $c \sim 0$. Such a fascinating effect can be understood in the following picture: The charge-spin coupling induced by the backward scattering off the impurity introduces an effective boundary field to the local moment, which means the charge degrees of freedom join the Kondo effect. However, unlike a real magnetic field, the "effective field" does not split the degeneracy of different orientation of the local moment. In fact, when the conduction electrons are impinging and leaving the impurity, the incident waves and the reflection waves feel different strengths of the impurity spin. One is $1/2 - c$ and the other is $1/2 + c$ or vice versa. The finite residual entropy is a result of the cooperative effect of the charge and spin sectors.

In summary, we introduce an integrable model of the Kondo problem in a 1D strongly correlated electron system. With different values of the parameter c, the system can show either a Fermi-liquid behavior or a non-Fermi-liquid behavior beyond that obtained by Furusaki and Nagaosa [8]. A nonsinglet ground state in an antiferromagnetic Kondo coupling region is obtained. It is found that the residual entropy depends not only on the self-magnetization of the ground state, but also on the interaction parameter c, which we interpret as a "cooperative effect" of the Kondo coupling and the impurity scattering. It would be instructive to apply renormalization group analysis and conformal field theory to reveal a more complete picture for such an interesting problem.

One of the authors (Y. W.) is supported in part by the National Natural Science Foundation of China and the Natural Science Foundation for Young Scientists, Chinese Academy of Sciences.

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