

Next-to-Leading Order QCD Corrections to Three-Jet Cross Sections with Massive Quarks

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We calculate the cross section for e^+e^- annihilation into three jets for massive quarks at next-to-leading order in perturbative QCD, both on and off the Z resonance. Our computation allows the implementation of any jet clustering algorithm. We give results for the three-jet cross section involving b quarks for the JADE and Durham algorithms at c.m. energies $\sqrt{s} = m_Z$. We also discuss a three-jet observable that is sensitive to the mass of the b quark. [S0031-9007(97)03569-2]

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Jets of hadrons, which originate from the production and subsequent fragmentation of quarks and gluons in high energy electron positron annihilation have been among the key predictions [1,2] of quantum chromodynamics (QCD). For precision tests of QCD the e^+e^- experiments provide a particularly clean setup. There exist a number of jet observables that are well defined (i.e., infrared-finite) in QCD, and which can be calculated perturbatively as an expansion in the strong coupling α_s . The next-to-leading order (NLO) QCD corrections to three-jet production were computed more than a decade ago [3,4] for massless quarks, and subsequent implementations [5–9] of these results have been widely used for tests of QCD with jet physics.

To date, huge samples of jet events produced at the Z resonance have been collected both at LEP and SLC. From these data large numbers of jet events involving b quarks can be isolated with high purity using vertex detectors. For detailed investigations of b jets quark mass effects must be taken into account in the theoretical predictions [10]. Specifically, knowledge of the NLO three-jet fraction for nonzero quark mass opens the possibility to measure the mass of the b quark from b jet data at the Z peak [11]. Further applications include precision tests of the asymptotic freedom property of QCD by means of three-jet rates measured at various center-of-mass energies, also far below the Z resonance [12].

As far as massive quarks are concerned the three, four, and five jet rates are known to leading order (LO) in α_s only [13–15]. In this Letter we report the calculation of the e^+e^- annihilation cross section into three jets involving a massive quark antiquark pair at next-to-leading order QCD [16,17]. The determination of this cross section σ_{NLO}^3 to order α_s^2 consists of two parts: first, the computation of the amplitude of the partonic reaction $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow Q\bar{Q}g$ at leading and next-to-leading order in the QCD coupling. Here Q denotes a massive quark and g a gluon. We have calculated the complete decay amplitude and decay distribution structure for this reaction. This allows for predictions including oriented three-jet events. The differential cross section involves the so-called hadronic tensor which contains

five parity-conserving and four parity-violating Lorentz structures. Second, the leading order matrix elements of the four-parton production processes $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow ggQ\bar{Q}, Q\bar{Q}q\bar{q}, Q\bar{Q}Q\bar{Q}$ are needed. Here q denote light quarks which are taken to be massless.

The calculation of the $\mathcal{O}(\alpha_s^2)$ virtual corrections to the process $e^+e^- \rightarrow Q\bar{Q}g$ with massive quarks is straightforward albeit tedious. Non-neglecting of the quark mass leads to a considerable complication of the algebra. The infrared (IR) and ultraviolet (UV) singularities, which are encountered in the computation of the one-loop integrals, are treated within the framework of dimensional regularization in $D = 4 - 2\epsilon$ space-time dimensions. We remove the UV singularities by the standard $\overline{\text{MS}}$ renormalization. An essential aspect of our computation is to show that the IR singularities of the virtual corrections are canceled by the singularities resulting from phase space integration of the squared tree amplitudes for the production of four partons. Different methods to perform this cancellation have been developed (see [7–9], and references therein). We use the so-called phase space slicing method elaborated in [7]. The basic idea is to “slice” the phase space of the four parton final state by introducing an unphysical parton resolution parameter $s_{\text{min}} \ll s_{\text{ycut}}$, where s_{ycut} is the jet resolution parameter. The parameter s_{min} splits the phase space into a region where all four partons are “resolved” and a region where at least one parton remains unresolved. For massless partons, the resolved region may be conveniently defined by the requirement that all invariants $s_{ij} = (k_i + k_j)^2$ built from the parton momenta k_i are larger than the parameter s_{min} . We have modified this definition slightly to account for masses [16].

In the unresolved region soft and collinear divergences reside, which have to be isolated explicitly to cancel the singularities of the virtual corrections. This is considerably simplified due to collinear and soft factorizations of the matrix elements which hold in the limit $s_{\text{min}} \rightarrow 0$. In the presence of massive quarks, the structure of collinear and soft poles is completely different as compared to the massless case. As an example, we would like to discuss for $e^+e^- \rightarrow Q(k_1)\bar{Q}(k_2)g(k_3)g(k_4)$ the limit where one gluon, say $g(k_4)$, becomes soft. In this limit, the

squared matrix element can be written as a universal factor multiplying the squared Born matrix element for $e^+e^- \rightarrow Q\bar{Q}g$. The integration over the soft gluon mo-

mentum k_4 can then be carried out analytically in D dimensions. In the soft limit $k_4 \rightarrow 0$ the squared matrix element reads

$$|T_{fi}^{\text{soft}}(e^+e^- \rightarrow Q\bar{Q}gg)|^2 = \frac{g_s^2 N_C}{2} \left[\sum_{a=1,2} \left(\frac{4t_{a3}}{t_{a4}t_{34}} - \frac{4m^2}{t_{a4}^2} \right) - \frac{1}{N_C^2} \left(\frac{4t_{12}}{t_{14}t_{24}} - \frac{4m^2}{t_{14}^2} - \frac{4m^2}{t_{24}^2} \right) \right] |T_{fi}^{\text{Born}}(e^+e^- \rightarrow Q\bar{Q}g)|^2, \quad (1)$$

where g_s is the strong coupling constant, $N_C = 3$ is the number of colors, $t_{ij} = 2k_i k_j$, and m denotes the quark mass. Each of the three terms in (1) can now be integrated over the appropriate soft phase space volume which we define by the conditions $t_{a4} + t_{34} < 2s_{\min}$

(leading color terms), and $t_{14} + t_{24} < 2s_{\min}$ (subleading color term). The complete soft factor $S(k_1, k_2, k_3)$ multiplying the squared Born matrix element for $e^+e^- \rightarrow Q(k_1)\bar{Q}(k_2)g(k_3)$ which is obtained by this integration reads

$$S(k_1, k_2, k_3) = \frac{\alpha_s}{4\pi} N_C \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{s_{\min}} \right)^\epsilon \left[\left\{ \left(\frac{s_{\min}}{t_{13} + m^2} \right)^{-\epsilon} \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \left[\ln\left(1 + \frac{t_{13}}{m^2}\right) + 2\ln(2) - 1 \right] - \frac{\pi^2}{6} + 2\ln^2(2) - 2\ln(2) + \left[2\ln(2) + \frac{2m^2}{t_{13}} + 1 \right] \ln\left(1 + \frac{t_{13}}{m^2}\right) - \frac{1}{2}\ln^2\left(1 + \frac{t_{13}}{m^2}\right) - 2\text{Li}_2\left(\frac{t_{13}}{t_{13} + m^2}\right) + (t_{13} \leftrightarrow t_{23}) \right\} - \frac{1}{N_C^2} \left(\frac{s_{\min}}{t_{12} + 2m^2} \right)^{-\epsilon} \frac{1}{\beta} \left(\frac{1}{\epsilon} [2\beta + (1 + \beta^2)\ln(\omega)] - 4\beta\ln(2) - 2\ln(\omega) - 2\ln(2)(1 + \beta^2)\ln(\omega) - \frac{1 + \beta^2}{2} \ln^2(\omega) - 2(1 + \beta^2)\text{Li}_2(1 - \omega) \right) \right] + \mathcal{O}(\epsilon), \quad (2)$$

where $\beta = \sqrt{1 - 4m^2/(t_{12} + 2m^2)}$, $\omega = (1 - \beta)/(1 + \beta)$, and μ is an arbitrary scale introduced to keep α_s dimensionless in D dimensions. The poles in ϵ exhibited in (2) (and additional poles from the collinear region of phase space which we do not show explicitly) can now be canceled against the IR poles of the one-loop integrals entering the virtual corrections. One is then left with a completely regular differential three-parton cross section which depends on s_{\min} .

The contribution to σ_{NLO}^3 of the resolved part of the four-parton cross section is finite and may be evaluated in $D = 4$ dimensions, which is of great practical importance. It also depends on s_{\min} and is most conveniently obtained by a numerical integration. Since the parameter s_{\min} is completely arbitrary, the sum of all contributions to $\sigma_{\text{NLO}}^3(y_{\text{cut}})$ must not depend on s_{\min} . In the soft and collinear approximations one neglects terms which vanish as $s_{\min} \rightarrow 0$. This limit can be carried out numerically. Since the individual contributions depend logarithmically on s_{\min} , it is a nontrivial test of the calculation to demonstrate that σ_{NLO}^3 becomes independent of s_{\min} for small values of this parameter. Moreover, in order to avoid large numerical cancellations, one should determine the largest value of s_{\min} which has this property.

The three-jet cross section depends on the experimental jet definition. We consider here the JADE [18] and Durham [19] clustering algorithms, although other schemes [20] can also be easily implemented. We have checked that we recover the result of [6] in the massless limit.

We now present our results for the cross section $\sigma_{\text{NLO}}^{3,b}$ for b quarks in the $\overline{\text{MS}}$ scheme. We require that at least two of the jets that remain after the clustering procedure contain a b or \bar{b} quark [21]. We use the b quark mass parameter $m_b(\mu)$ defined in the $\overline{\text{MS}}$ scheme at a scale μ . The asymptotic freedom property of QCD predicts that this mass parameter decreases when being evaluated at a higher scale. With $m_b(m_b) = 4.36$ GeV [22] and $\alpha_s(m_Z) = 0.118$ [23] as an input and employing the standard renormalization group evolution of the coupling and the quark masses, we use the value $m_b(\mu = m_Z) = 3$ GeV.

Figures 1(a) and 1(b) show the three-jet cross section $\sigma_{\text{NLO}}^{3,b}$ at $\sqrt{s} = \mu = m_Z$ with b quarks of mass $m_b = 3$ GeV as a function of $y_{\min} = s_{\min}/(s_{\text{cut}})$ for the JADE and Durham algorithms at a value of the jet resolution parameter $y_{\text{cut}} = 0.03$. It can be clearly seen that the cross section reaches a plateau for small values of the parameter y_{\min} . The error of the numerical integration becomes bigger as $y_{\min} \rightarrow 0$. In order to keep this error as small as possible without introducing a systematic error from using the soft and collinear approximations, we take $y_{\min} = 10^{-2}$ for the JADE algorithm and $y_{\min} = 5 \times 10^{-3}$ for the Durham algorithm. These values are used in Figs. 1(c) and 1(d), where we plot $\sigma_{\text{NLO}}^{3,b}$ as a function of y_{cut} together with the LO result. The QCD corrections to the LO result are quite sizable, as known also in the massless case. The renormalization scale dependence which is also shown in Figs. 1(c) and 1(d) is modest in the whole y_{cut} range exhibited for the Durham and above $y_{\text{cut}} \sim 0.01$ for the

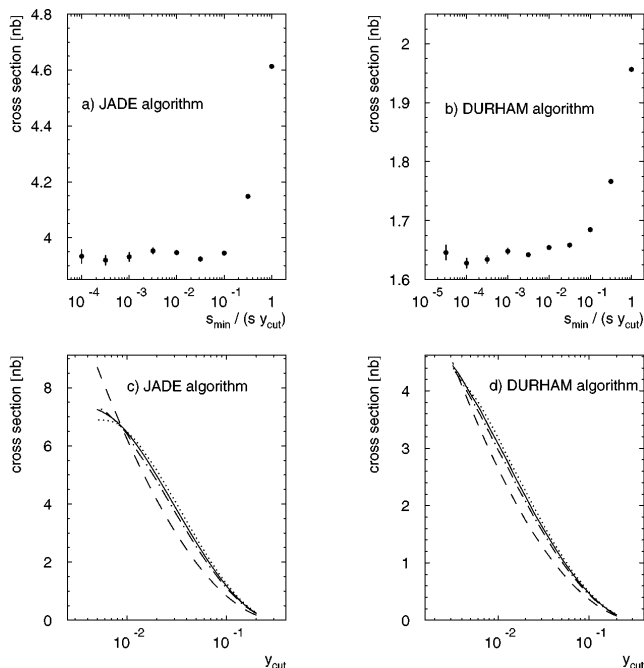


FIG. 1. (a) and (b) show $\sigma_{\text{NLO}}^{3,b}$ as defined in the text at $\sqrt{s} = \mu = m_Z$ as a function of $y_{\text{min}} = s_{\text{min}}/(s y_{\text{cut}})$ for the JADE and Durham algorithms at a value of the jet resolution parameter $y_{\text{cut}} = 0.03$ with $m_b(\mu = m_Z) = 3$ GeV and $\alpha_s(\mu = m_Z) = 0.118$. (c) and (d) show $\sigma^{3,b}$ as a function of y_{cut} for the JADE and Durham algorithms, respectively. The dashed line is the LO result. The NLO results are for $\mu = m_Z$ (solid line), $\mu = m_Z/2$ (dotted line), and $\mu = 2m_Z$ (dash-dotted line). Initial-state photon radiation is not included in the cross sections.

JADE algorithm. Below this value, perturbation theory is not applicable in the JADE scheme.

The effects of the b quark mass at the Z peak may be exhibited with the following ratio [24]:

$$\mathcal{B}(y_{\text{cut}}) = \frac{R_3^b(y_{\text{cut}})}{R_3^{udsc}(y_{\text{cut}})}. \quad (3)$$

Here we define $R_3^b = \sigma^{3,b}/\sigma(e^+e^- \rightarrow b\bar{b})$, and R_3^{udsc} is the three-jet fraction for the four light quarks with no flavor tagging. (Note that $\mathcal{B} \neq 1$ in the limit $m_b \rightarrow 0$ due to the different definitions of R_3^b and R_3^{udsc} .) The LO and NLO results for the observable \mathcal{B} are shown in Figs. 2(a) and 2(b) for the JADE and the Durham algorithms. We took the massless $\mathcal{O}(\alpha_s^2)$ results from [6,20]. As both the LO and NLO terms in R_3^b depend on m_b it is clear that comparison of our result with measured values of $\mathcal{B}(y_{\text{cut}})$ would allow for an unambiguous determination of m_b within a given renormalization scheme. In view that the b mass effect in \mathcal{B} —and in other observables, for instance the differential two-jet distribution [16]—is only of the order of a few percent this constitutes an experimental challenge. Moreover, further work is needed to assess in detail the theoretical uncertain-

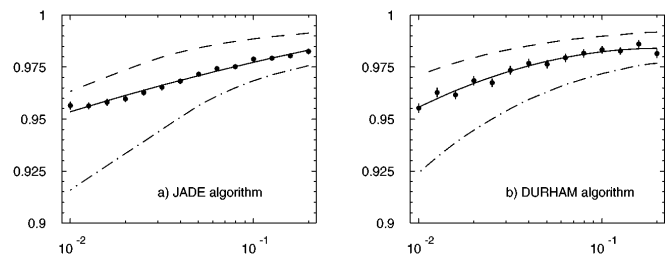


FIG. 2. The ratio \mathcal{B} of Eq. (3) as a function of y_{cut} for (a) the JADE and (b) the Durham algorithm, using values for m_b and α_s , as in Fig. 1. The dashed line is the LO result. The points and the solid line are the NLO result for $\mu = m_Z$. For illustrative purposes, the dash-dotted line shows the LO result for $m_b = 5$ GeV.

ties involved [25]. Yet such an analysis would be worth the effort: It would be the first determination of the b quark mass at a high scale, and it might also experimentally establish the “running” of a quark mass as predicted by QCD.

Summarizing, we have computed the NLO QCD corrections for $e^+e^- \rightarrow 3$ jets for massive quarks. Our results, which we shall report on in detail in future work, should find applications to a number of precision tests of QCD involving b and c quarks at various c.m. energies, and to theoretical investigations of top quark production at very high-energetic electron positron collisions.

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Note added.—A ratio R_3^{bd} similar to the ratio \mathcal{B} defined in Eq. (3) was calculated at NLO in α_s in [17] and by Rodrigo, Santamaria, and Bilenky [26]. When adopting the definition of R_3^{bd} , we find agreement with the results of these authors. We wish to thank A. Santamaria for detailed correspondence on this matter.

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