Resonant Decay of Cosmological Bose Condensates

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We present results of fully nonlinear calculations of decay of the inflaton interacting with another scalar field X. Combining numerical results for a cosmologically interesting range of the resonance parameter, $q \leq 10^6$, with analytical estimates, we extrapolate them to larger q. We find that scattering of X fluctuations off the Bose condensate is a very efficient mechanism limiting growth of X fluctuations. For a single-component X, the resulting variance, at large q, is much smaller than that obtained in the Hartree approximation. [S0031-9007(97)03914-8]

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In recent years, we have come to realize that the postinflationary universe had probably been a much livelier place than was previously thought. In many models of inflation, the decay of the inflaton field is not a slow perturbative process but a rapid, explosive one. At the initial stage of this rapid process, called preheating [1], fluctuations of Bose fields coupled to the inflaton grow exponentially fast, which can be thought of as "parametric resonance" [2], and achieve large occupation numbers. At the second stage, called semiclassical thermalization [3], the resonance smears out, and the fields reach a slowly evolving turbulent state with smooth power spectra [3,4].

The explosive growth of Bose fields leads to very large variances of these fields close to the end of the resonance stage. That could result in several important effects taking place shortly after the end of inflation. These include symmetry restoration, baryogenesis, and supersymmetry breaking [5]. To find out if these effects had indeed occurred, one needs a good estimate of the maximal size of Bose fluctuations. The semiclassical nature of processes involving states with large occupation numbers allows us to treat a resonant decay of the inflaton, and any Bose condensate in general, as a classical nonlinear problem with random initial conditions for fluctuations [3]. This classical problem can be analyzed numerically.

In this Letter we report results of fully nonlinear calculations for the most interesting case when the coupling of a massive inflaton ϕ to some other scalar field X is relatively large. That means, more precisely, that the system is in the regime of wide parametric resonance [1], characterized by a large value of the resonance parameter q, $q \gg 1$. We have studied both expanding and static universes, to cover both postinflationary dynamics and decays of possible other Bose condensates. Our objective was to obtain an estimate for the maximal size of X fluctuations, the importance of which we emphasized above.

Our results are as follows. We have found that scattering of X fluctuations off the Bose condensate of ϕ, which knocks inflaton quanta out of the condensate and into low-momentum modes, is very efficient in limiting the size of X fluctuations for large values of q, such as required [1,4] to produce particles much heavier than the inflaton. This scattering process involves the condensate of zero-momentum inflatons and, for that reason, is especially enhanced, cf. Refs. [3,6,7]. Fluctuations of X can reach larger values for smaller values of q. The suppression of the maximal size of X fluctuations for large q significantly restricts the possibility of grand unified theory (GUT) baryogenesis after inflation, as well as the types of phase transitions that could take place after preheating.

Our present results should be compared with those obtained in the Hartree approximation. We find that for a single-component field X in flat space-time, the Hartree approximation does not give an adequate estimate of the maximal size of X fluctuations for any $q \gg 1$. A similar conclusion was made in Ref. [4] for the conformally invariant case of massless inflaton interacting with a massless field X, based on our simulations of the fully nonlinear problem for that case. The Hartree approximation, with its characteristic positive feedback of X on the inflaton decay [1,4], may still apply when X has sufficiently many components; it remains to be seen if this can happen for realistic sizes of GUT multiplets.

In the model with a massive inflaton (model 1 of Ref. [4]), the full scalar potential is $V_1(\phi, X) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2X^2 + \frac{1}{2}M_X^2X^2$. For comparison, we will present results for the model with massless inflaton (model 2), in which the potential is $V_2(\phi, X) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2X^2$. Both fields ($\phi$ and X) have standard kinetic terms and are minimally coupled to gravity. We will often use rescaled variables: for model 1, $\tau = m\eta$, where η is the conformal time; $\boldsymbol{\xi} = m\mathbf{x}$; $\varphi = \phi a(\tau)/\phi(0)$; $\chi = Xa(\tau)/\phi(0)$. For model 2, one should replace m with $\sqrt{\lambda}\,\phi(0)$ in these rescalings. Here $a(\tau)$ is the scale factor of the Universe, defined so that $a(0) = 1$, and $\phi(0)$ is the value of the homogeneous inflaton field at the end of inflation, when $d\varphi/d\tau = 0$. The resonance parameter q is $q \equiv g^2\phi^2(0)/4m^2$ for model 1, and $q \equiv g^2/4\lambda$ for model 2.

In order for resonance to fully develop in the expanding universe, the resonance parameter q should exceed a certain minimum value, $q_{\min}$, which depends on the mass M_X of X [4]. In model 1, for $M_X = 0$, $q_{\min} \sim 10^4$; for $M_X = 10m$, $q_{\min} \sim 10^8$. We have simulated this model for q up to $q = 10^4$ in flat space-time and for a few q ranging from 10^4 to 10^6 in the expanding universe. We have developed an analytical approach, and we use analytical estimates to extrapolate our results to larger q, needed to produce heavier particles.

The full nonlinear equations of motion for φ and χ, which follow from the action described above, are solved directly in configuration space with initial conditions corresponding to conformal vacuum at the end of inflation for all modes with nonzero momenta. The initial conditions for χ are given in Ref. [4]; those for $\delta\varphi$ are obtained similarly, along the lines of Ref. [3]. Classical fluctuations evolve from quantum ones, and, in cases we consider, their typical initial sizes are much smaller than the scale of nonlinearity. Hence, initial evolution is linear with respect to fluctuations. During the initial linear stage, the equation of motion for Fourier components of χ can be approximated as

$$\ddot{\chi}_{\mathbf{k}} + \omega_k^2(\tau)\chi_{\mathbf{k}} = 0\,, \tag{1}$$

$$\omega_k^2(\tau) = m_\chi^2 a^2 + k^2 - \ddot{a}/a + 4q\varphi_0^2(\tau)\,, \tag{2}$$

$m_\chi \equiv M_X/m$, and φ_0 is the zero-momentum mode of φ; $\dot{\varphi}_0(0) = 0$. By virtue of our rescaling, $\varphi_0(0) = 1$.

The computations were done on 128^3 lattices, for a single-component X. For the case of the expanding universe, we used $m^2 = 10^{-12}M_{\rm Pl}^2$. Energy nonconservation in flat space-time typically was less than 10^{-3}; in the expanding universe in linear regime our calculations closely reproduced calculations in the Hartree approximation, which have much better accuracy.

Let us first consider the case without expansion of the Universe. In the formulas above, one substitutes $a(\tau) = 1$. The variances of the fields χ and φ in model 1 at $q = 2000$, $m_\chi = 0$ as functions of time are shown in Fig. 1. The angular brackets denote averaging over random initial conditions; for space-independent quantities, like variances, this is equivalent to averaging over space. The exponential growth of the variance $\langle\chi^2\rangle$ at early times is a parametric resonance. The linear stage in the present case ends at $\tau \approx 40$.

At large q, fluctuations of X are produced by the resonance stage only during short intervals of time near moments when φ_0 passes through zero [1,8]. These are intervals in which the adiabatic (WKB) condition $\dot{\omega}_k/\omega_k^2 \gg 1$ is broken for some k. Notice a series of spikes in $\langle\chi^2\rangle$ at the same moments of time. They are due to modulation of ω_k by the oscillating φ_0. Indeed, introduce analogs of occupation numbers n_k via $\langle\chi^2\rangle = \int d^3k\, P_\chi(k) \propto \int d^3k\, n_k(\tau)/\omega_k(\tau)$. Even at the resonance stage, change in n_k during one oscillation is much smaller than variation of ω_k: when instead of $\langle\chi^2(\tau)\rangle$ we plot $\int d^3k\, n_k(\tau)$, the spikes are replaced by comparatively small steps at times when $\varphi_0 = 0$. We will distinguish between $\langle\chi^2\rangle$ in spikes and in "valleys" between them by a subscript: "s" or "v."

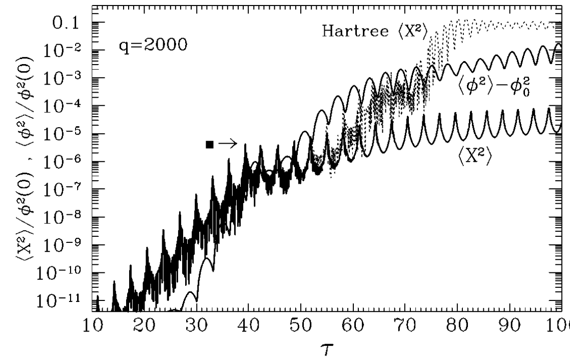

FIG. 1. Variances of the fields X and ϕ in model 1 in flat space-time ($m_\chi = 0$). The filled square marks the spike value of $\langle X^2\rangle$ at the end of the linear stage. The dotted curve is $\langle X^2\rangle$ in the Hartree approximation.

We have monitored the power spectra, $P(k)$, of φ and χ in all our integrations. The strongest resonant momentum of χ is typically of order $q^{1/4}$; for some q, though, it can be close to zero. Development of resonance peaks for χ is followed by appearance of peaks for φ due to rescattering, cf. Ref. [3]. Later, rescattering leads to a turbulent state, characterized by smooth power spectra.

The linear stage is followed by a plateau in $\langle\chi^2(\tau)\rangle$ (unless we consider an exceptional q, for which the resonance peak was close to zero). There, the variances of fluctuations do not grow, but an important restructuring of the power spectrum of χ takes place. The power spectrum of χ changes from being dominated by a resonance peak at some nonzero momentum to being dominated by a peak near zero. The width of this new peak at $k \approx 0$ is of order one. When this peak becomes strong enough, the growth of variances resumes. The resumed growth (at $\tau \gtrsim 50$ in Fig. 1) is quite rapid (compared to the subsequent slow evolution) and is strongly affected by rescattering. This stage can be called semiclassical thermalization [3], or chaotization stage. During it, the power spectra smoothen out; both power spectra are now dominated by momenta of order one.

An important effect seen in Fig. 1 is the rapid growth of fluctuations of the field φ. Indeed, at late times, they are much larger than fluctuations of χ. Fluctuations of φ are produced by the scattering process in which χ fluctuations knock φ out of the zero mode. For some time, though, the fluctuation $\delta\varphi = \varphi - \varphi_0$ can be neglected. Then, in addition to Eq. (1) we obtain the equation

$$\ddot{\varphi}_0 - (\ddot{a}/a)\varphi_0 + a^2(\tau)\varphi_0 + 4q\langle\chi^2\rangle\varphi_0 = 0\,. \tag{3}$$

Equations (1) and (3) comprise the Hartree approximation in the present model. The classical average in (3) approximates the corresponding quantum average (used, e.g., in the second paper by Boyanovsky *et al.* in Ref. [2]) with accuracy $O(1/n_{\rm amp})$, where $n_{\rm amp}$ is a typical occupation number in amplified modes.

The end of the linear stage is (at large q) a Hartree effect: it is due to the shift of the frequency of φ_0 caused by $\langle\chi^2\rangle$; see Eq. (3). This is confirmed by Fig. 1 and the data in Fig. 2, where we compare spike values, $\langle\chi^2\rangle_{\rm s}$, at the end of the linear stage in the Hartree approximation and in the full problem; they agree well. Using analytical results of Ref. [8], we find that the width of the resonance peak scales as $q^{1/4}$, up to a power of $\ln q$. We then estimate that the linear stage ends when $\langle\chi^2\rangle_{\rm s} \sim q^{-3/2}$. The data of Fig. 2 at $q \gtrsim 100$ are well fitted by the $q^{-3/2}$ dependence.

To go beyond the Hartree approximation, we develop the perturbation expansion in $\delta\varphi$ near it. This assumes that $|\delta\varphi| \ll \bar{\varphi}$, where $\bar{\varphi}$ is the amplitude of φ_0. Solving the equations obtained in the first order in $\delta\varphi$ with the help of the Green functions, we get, for Fourier components of $\delta\varphi$ and χ,

$$\varphi_{\mathbf{p}}(\tau) \approx -\frac{4q}{\Omega_p}\int_0^\tau d\tau' \sin[\Omega_p(\tau-\tau')]\varphi_0(\tau') \times \int d^3k\, \chi^*_{\mathbf{k}}\chi_{\mathbf{k}+\mathbf{p}}(\tau')\,, \tag{4}$$

$$\chi_{\mathbf{k}}(\tau) = \chi_{\mathbf{k}}^{(0)} + 8q\int_0^\tau d\tau'\, F_k(\tau,\tau')\varphi_0 \int d^3p\, \varphi^*_{\mathbf{p}}\chi_{\mathbf{k}+\mathbf{p}}\,. \tag{5}$$

Here $\Omega_p = (p^2+1)^{1/2}$; $\chi_{\mathbf{k}}^{(0)}(\tau)$ solves Eq. (1) in the Hartree approximation, and $F_k(\tau,\tau')$ is the retarded Green function for it. At times when χ grows on average, the main contribution to the time integral in (5) comes from τ' near τ. Using the random phase approximation, we then obtain $\langle|\varphi_{\mathbf{p}}|^2\rangle \sim q^2\bar{\varphi}^2V^{-1}\int d^3k\,\langle|\chi_{\mathbf{k}}|^2\rangle\langle|\chi_{\mathbf{k}+\mathbf{p}}|^2\rangle$, where V is the total spatial volume.

The Hartree approximation breaks down when the second term in the right-hand side of (5) becomes of the order of the first. We find that that happens at time $\tau_{\rm sc}$, when, in terms of the physical fields,

$$\langle X^2\rangle_{\rm v} \sim \frac{\phi^3(0)}{q^{3/2}\bar{\phi}(\tau_{\rm sc})}\,; \qquad \langle(\delta\phi)^2\rangle \sim \phi^2(0)/q\,, \tag{6}$$

where $\bar{\phi}$ is the amplitude of ϕ_0. At that time the system becomes dominated by rescattering. In Fig. 1, that time is $\tau_{\rm sc} \sim 60$. When variances of fields reach values (6), the analogs of occupation numbers, introduced as before, reach values of order $1/g^2$ for *both* X and ϕ.

At $\tau > \tau_{\rm sc}$, the maxima and minima of $\langle\chi^2\rangle$ evolve slowly. The variance of φ continues to grow rapidly for a while; see Fig. 1. Indeed, according to Eq. (4), a periodic (or close to periodic) χ can still drive the growth of $\delta\varphi$, via a force-driven (as opposed to parametric) resonance. This growth of $\delta\varphi$ will stop only when the approximation leading to (4) breaks down, that is, $\delta\phi \sim \bar{\phi}$.

Let us now turn to model 1 in the expanding universe, see Figs. 3 and 4. We used $\phi(0) = 0.28M_{\rm Pl}$ [4]. For massless X, the evolution of the scale factor was determined self-consistently, including the influence of produced fluctuations in the Einstein equations; for massive X, the Universe was assumed matter dominated.

In the expanding universe, particle creation acquires a qualitatively new feature [4]: because of the time dependence of $\bar{\varphi}$, the resonance peak scans the entire instability band; see Fig. 4. As our numerical integrations confirm (see also [4]), in order for production of fluctuations to be efficient, variation of the frequency of χ, Eq. (2), need not be periodic. What is required is that every once in a while the adiabatic condition breaks down. In this situation, "nonadiabatic amplification" seems to be a better term for stimulated particle creation than "parametric resonance."

Time dependence of variances is shown in Fig. 3 for the case $q = 10^6$ and $m_\chi = 2$. The variances of fields at the time when rescattering begins to dominate ($\tau_{\rm sc} \approx 8.5$ in the figure) can be estimated in the same way as before, and we obtain again Eq. (6). Note that now $\bar{\phi}(\tau_{\rm sc}) \ll \phi(0)$, due to the redshift of the field.

We may assume that, because of the fast scattering processes, the system at $\tau > \tau_{\rm sc}$ loses memory of the

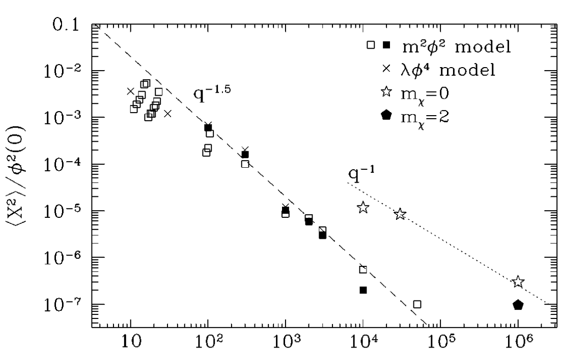

FIG. 2. Filled squares and crosses are the spike values, $\langle X^2\rangle_{\rm s}$, at the end of the linear stage, obtained in fully nonlinear simulations of models 1 and 2 in flat space-time; empty boxes are the spike values at the first plateau in the Hartree approximation. Stars and pentagon correspond to $\langle X^2\rangle$ at the moment when the zero-momentum mode decays in model 1 in the expanding universe.

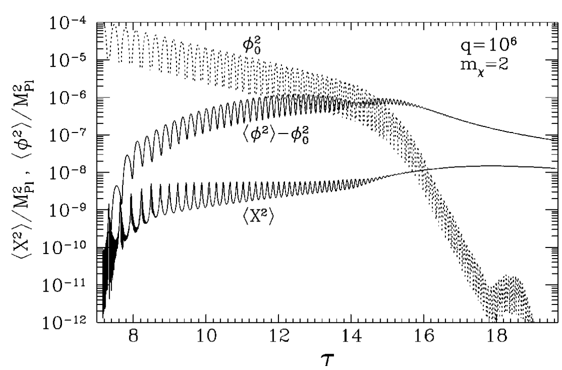

FIG. 3. Variances of fields X and ϕ, together with the inflaton zero-momentum mode, in model 1 in the expanding universe.

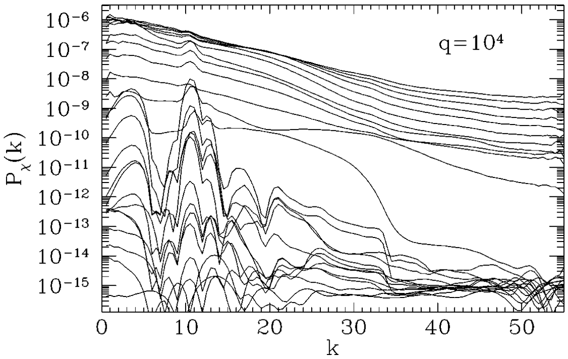

FIG. 4. Power spectrum of the field X in model 1, $m_\chi = 0$, in the expanding universe, output every period at maxima of $\varphi_0(\tau)$. The result can be trusted for k smaller than the Nyquist momentum k_{N_y}; in this particular case, $k_{N_y} \approx 32$ (in units of the inflaton mass). After rescattering, $P_\chi(k)$ decreases exponentially in a range of k below k_{N_y}.

initial conditions, so that $\langle X^2\rangle_{\mathrm{v,s}}$ depend only on the current value of $\bar\phi$ and the effective, slowly changing value of the resonance parameter, $q_{\mathrm{eff}}(\tau) \equiv q\bar\phi^2(\tau)/\phi^2(0)$. Then, on dimensional grounds, $\langle X^2\rangle_{\mathrm{v,s}}(\tau) = \bar\phi^2(\tau) f_{\mathrm{v,s}}(q_{\mathrm{eff}})$. This relates the time dependence of $\langle X^2\rangle_{\mathrm{v,s}}$ (or their scaling with $\bar\phi$) to their scaling with q.

According to Eq. (6), $f_{\mathrm{v}}(q_{\mathrm{eff}}) \sim q_{\mathrm{eff}}^{-3/2}$, which gives $\langle X^2\rangle_{\mathrm{v}}(\tau) \propto \bar\phi^{-1}$. We can construct an analog of particle density, n, as $n/m = [4q(\phi_0^2 + (\delta\phi)^2)/\phi^2(0) + m_\chi^2]^{1/2}\langle X^2\rangle$. As long as the term with ϕ_0 is the leading term in the square brackets, $n \propto \langle X^2\rangle_{\mathrm{v}}\bar\phi$, so that n, at $\tau > \tau_{\mathrm{sc}}$, should be time independent. Our data confirm that. Thus, X fluctuations are being produced at the same rate as they are diluted by the expansion. This can be called negative feedback amplification.

A striking feature of the data of Fig. 3 is that, at $\tau > \tau_{\mathrm{sc}}$, $\langle X^2\rangle$ in spikes becomes essentially time independent. According to our scaling argument, this means that $f_{\mathrm{s}} \propto q_{\mathrm{eff}}^{-1}$, so that $\langle X^2\rangle_{\mathrm{s}}$ scales as $1/q$. In Fig. 2, we show by stars the values of $\langle X^2\rangle$ in the expanding universe, taken at times when ϕ_0 decays (more precisely, when $\bar\phi^2 \approx \langle\delta\phi^2\rangle$), and the spike and the valley values of $\langle X^2\rangle$ coalesce. The scaling of these values with q is well fitted by $1/q$.

The values of $\langle X^2\rangle_{\mathrm{s}}$ at the time when ϕ_0 decays are within a factor of a few from the maximal value, $\langle X^2\rangle_{\max}$, that $\langle X^2\rangle$ achieves in the expanding universe. The exact value of that factor depends on the parameters; for $q = 10^6$ it was the largest, equal to 2.6, at $M_X = 0$. Our final estimate for $\langle X^2\rangle_{\max}$, obtained using the $1/q$ scaling to extrapolate the data, is $\langle X^2\rangle_{\max} \sim 0.1\phi^2(0)/q$. For example, for $q = 10^8$, required to produce scalar leptoquarks with $M_X = 10m$, we get $\langle X^2\rangle_{\max} \sim 10^{-10}M_{\mathrm{Pl}}^2$. In comparison, in the Hartree approximation, the maximal value of $\langle X^2\rangle$ is $\langle X^2\rangle_{\max}^{\mathrm{H}} \sim q^{-1/2}\bar\phi\phi(0)$ [1,4]; for $q = 10^8$ and $M_X = 10m$ this gives $\langle X^2\rangle_{\max}^{\mathrm{H}} \sim 10^{-7}M_{\mathrm{Pl}}^2$. In general, $\langle X^2\rangle_{\max}^{\mathrm{H}} \gg \langle X^2\rangle_{\max}$ whenever $q_{\mathrm{eff}}(\tau_{\mathrm{sc}}) \gg 1$.

If instead of a single-component X we consider X with N real components, our estimate for $\langle X^2\rangle_v$ at $\tau = \tau_{\mathrm{sc}}$ becomes larger than in (6) by a factor of $\sqrt{N}$. For realistic GUT values of N, this can increase $\langle X^2\rangle_{\max}$ by an order of magnitude.

Because the number density n stays constant for a while, despite the expansion, the time-integrated conversion of inflatons into X fluctuations in this two-field model is, in fact, quite efficient. For definite predictions for baryon asymmetry generated in decays of leptoquarks, however, one has to include other fields, which can have smaller q and thus provide faster alternative channels of inflaton decay.

The relatively quick complete exponential decay of ϕ_0, see Fig. 3, is a distinctive feature of model 1, as opposed to the conformally invariant model 2. In the latter case, we find that at the time when rescattering starts to dominate, $\langle\delta\phi^2\rangle \sim \bar\phi^2/q$, $\langle X^2\rangle_{\mathrm{v}} \sim \bar\phi^2/q^{3/2}$, and in the subsequent evolution $\langle X^2\rangle$ redshifts together with $\bar\phi^2$.

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Note added.—As the writing of this paper was nearing completion, we received a preprint, Ref. [9], in which the method of Ref. [3] was also applied to two-field models. The authors of Ref. [9] consider only flat space-time and conformally invariant cases; their results for these cases partially overlap with ours.

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