Only Fermi Liquids Are Metals

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Any singular deviation from Landau Fermi-liquid theory appears to lead, for an arbitrarily small concentration of impurities coupling to a nonconserved quantity, to a vanishing density of states at the chemical potential and infinite resistivity as temperature approaches zero. Applications to copper-oxide metals including the temperature dependence of the anisotropy in resistivity, and to other cases of non Fermi-liquids are discussed. [S0031-9007(97)03911-2]

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The resistivity in the c direction, $\rho_c(T)$, in the normal phase of most copper-oxide (CuO) compounds increases with decreasing temperature while the in-plane resistivity $\rho_{a,b}(T)$ has the opposite behavior, which for compositions near those for the highest T_c is proportional to T down to $T \simeq T_c$ [1]. One may be led to suppose that, if superconductivity were not to intervene, $\rho_c \to \infty$ while $\rho_{a,b} \to$ finite value as $T \to 0$. For any finite quantum-mechanical transfer matrix element $t_\perp$ between adjacent planes, the asymptotic low temperature dependence in different directions must be identical for $T \ll T_{xa}$, where

$$\tau_{\rm in}^{-1}(T_{xa}) \simeq t_\perp . \tag{1}$$

Here $\tau_{\rm in}^{-1}(T)$ is the inelastic scattering rate. Anderson and Zhou [2] conjectured that the renormalized matrix elements $t_\perp(T) \to 0$ in CuO compounds at low temperatures due to orthogonality effects. An alternative conjecture [3] (with a supporting calculation whose conditions of validity were not clear) put forward to resolve the issue is that $\rho_{a,b}$ also $\to \infty$ as $T \to 0$ due to impurity scattering in a non-Fermi-liquid. Recent experiments [4,5] measuring the resistivity at low temperatures by suppressing T_c in a large magnetic field support this conjecture and find $\rho_{a,b}(T) \sim \rho_c \sim \ln T$ at low temperature. Similar behavior is also found (without applying a magnetic field) in samples of the single layer Bi compound [6]. Here theoretical support for the conjecture that the resistivity of a non-Fermi-liquid is infinity for $T \to 0$ for any finite concentration of impurities as well as the logarithm temperature dependence is obtained.

A Landau Fermi liquid has the property that the real part of the single particle self-energy

$$\text{Re } \Sigma(\omega, T, k_{\rm F}) \sim x^\alpha, \quad \text{with } \alpha = 1 . \tag{2}$$

Here $x = \omega$ for $|\omega| \gg T$ and $= \pi T$ for $T \gg |\omega|$. For $\alpha < 1$, the pole in the single particle Green's function, the quasiparticle, vanishes at the chemical potential and is replaced by a branch cut. $\alpha < 1$ may be used to characterize a non-Fermi-liquid. The gentlest departure from a Landau Fermi liquid with

$$\text{Re } \Sigma(\omega, T, k_{\rm F}) \sim \omega \left| \ln \frac{\omega_c}{x} \right| \tag{3}$$

has been termed a marginal Fermi liquid [7].

The observed linear temperature dependence of the resistivity and corresponding behavior of the frequency dependent conductivity implies directly that the momentum scattering rate, $\tau_{\rm mom}^{-1}$, in Cu-O compounds is proportional to $\max(|\omega|, T)$. The imaginary part of the single particle self-energy $\text{Im } \Sigma(\omega, T, k_{\rm F})$ cannot have a higher power dependence on $(|\omega|, T)$ than the momentum scattering rate. Through Kramers-Kronig transformation, this implies $\alpha < 1$. The marginal case, Eq. (3), is consistent with the measured tunneling conductance [8] as well as the deduced electronic heat capacity [9], as is the low temperature resistivity derived here.

I discuss below the (ω, T) dependence of the scattering rate from *s-wave* scattering off impurities for the non-Fermi-liquids characterized in the pure limit by the marginal case as well as other $\alpha < 1$. This can be done to a considerable extent without reference to any microscopic theory of the non-Fermi-liquid. Also, for a given α, the dimensionality d will not be important as long as $d > 1$. Of course α may depend on d.

The problem of impurities in a one-dimensional interacting system has already been discussed [10]. The strong electronic localization at low energies due to impurities in the noninteracting problem becomes even stronger with interactions unless the interactions are attractive and above a critical value depending on disorder. Then the ground state has long-range superconductive correlations. The situation I discuss here is different. As seen below the leading effect of impurities in a non-Fermi-liquid in $d = 2$ or higher is not a localization of states but a renormalization of one-particle density of states to zero at the chemical potential.

Consider impurities which in the absence of electron-electron interactions have finite s-wave scattering amplitude [11]. The vertex correction due to the interactions will in general introduce scattering into higher partial waves. But there is no reason why the s-wave scattering should vanish. Consider only this part for which the forward scattering limit characterizes the properties. In this limit, a Ward identity provides the vertex including the effect of electron-electron interactions.

Let the bare impurity potential be

$$V_{\rm imp}(r) = \sum_i v\,\delta(r - r_i)\,, \tag{4}$$

where $\{r_i\}$ are the position of the impurities assumed randomly distributed. We will assume also that $v/\epsilon_{\rm F} \ll 1$ and the concentration of impurities c is low enough so that the mean-free path ℓ_0 calculated without electronic renormalizations satisfies $(k_{\rm F}\ell_0)^{-1} \ll 1$. The s-wave part of the renormalized scattering from a given impurity is

$$\tilde{v} = v \lim_{q\to 0} \Lambda_{k_{\rm F},\omega}(q,0)\,, \tag{5}$$

where $\Lambda_{k_{\rm F},\omega}(q,0)$ is the irreducible vertex due to electron-electron interactions for elastic scattering with momentum transfer q.

If an impurity couples to a nonconserved quantity (for instance, for Cu-O compounds, it can change the equilibrium charge difference between Cu and O in a unit cell or alter the local kinetic energy in a Cu-O bond), a Ward identity gives that [12–14] (at $T = 0$),

$$\lim_{q\to 0} \Lambda_{k_{\rm F},\omega}(q,0) = z^{-1}(\omega)\,, \tag{6}$$

where

$$z^{-1}(\omega) = \left(1 - \frac{\partial}{\partial\omega}{\rm Re}\;\Sigma(k_{\rm F},\omega)\right). \tag{7}$$

The self-energy due to impurities from such a vertex for a given impurity configuration illustrated in Fig. 1(a) is

$$\Sigma_{\rm imp}(\omega) = \frac{\tilde{V}_{\rm imp}(\omega)}{1 - \tilde{V}_{\rm imp}(\omega)\sum_k G(\mathbf{k},\omega)}\,, \tag{8}$$

where $\tilde{V}_{\rm imp}(\omega) = \sum_i \tilde{v}(\omega)\delta(r - R_i)$ and $G(\mathbf{k},\omega)$ is the exact single particle Green's function including the effect of impurities.

In Eq. (8), vertex renormalization at a given impurity alone has been considered. Vertex renormalization which spans across impurities, such as in Fig. 1(b), are proportional to $(\omega/\epsilon_{\rm F})$ so that (8) may be considered asymptotically exact.

The strategy adopted here is to first evaluate a subset of the processes in the expression (8), which are expected to be the most singular. The result so obtained is used to calculate corrections which show that the expectation is borne out. The most singular processes for the impurity averaged self-energy are expected to be given by the self-consistent Born approximation represented by Fig. 1(c):

$${\rm Im}\;\Sigma^0_{\rm imp}(\omega) = c\left(\frac{v}{z}\right)^2 {\rm Im}\sum_k G'(\mathbf{k},\omega)\,, \tag{9}$$

where we include in $G'(\mathbf{k},\omega)$ the self-energy Σ' which includes $\Sigma(\omega,k)$ as well as $\Sigma^0_{\rm imp}(\omega)$.

Assuming a constant density of states $N(0)$ over a bandwidth $\approx 2\epsilon_{\rm F}$ in the pure limit (other assumptions about the pure density of states do not produce any essential difference), Eq. (9) gives

$${\rm Im}\;\Sigma^0_{\rm imp}(\omega) = \left(\frac{1}{2\tau_0}\right)\frac{1}{z^2}\,\frac{2}{\pi}\tan^{-1}\left(\frac{\epsilon_{\rm F}}{{\rm Im}\;\Sigma'(\omega)}\right), \tag{10}$$

where $\tau_0 = \ell_0/v_{\rm F}$. For z evaluated from the marginal self-energy and for $\epsilon_{\rm F}/{\rm Im}\;\Sigma^0_{\rm imp}(\omega) \gg 1$,

$${\rm Im}\;\Sigma^0_{\rm imp}(\omega) \approx \frac{1}{2\tau_0}\left(1 + \lambda\ln\frac{\omega_c}{\omega}\right)^2. \tag{11}$$

This was the result derived earlier [3]. However, this crosses over in the opposite limit to

$${\rm Im}\;\Sigma^0_{\rm imp}(\omega) \approx \lambda\sqrt{\frac{\epsilon_{\rm F}}{\pi\tau_0}}\,\ln\left(\frac{\omega_c}{\omega}\right). \tag{12}$$

The above results, Eqs. (11) and (12), are valid for $\omega \gg T$. For $\omega \ll T$, πT should be substituted for ω in Eqs. (10) and (11). The crossover from (11) to (12) occurs for

$$\left(\ln\frac{\omega_c}{\max(\omega_x,\pi T_x)}\right)^2 \approx \frac{2\epsilon_{\rm F}\tau_0}{\lambda^2} \tag{13}$$

or at $\omega = 0$ at

$$T_x \approx \frac{\omega_c}{\pi}\exp(-\lambda^{-1}\sqrt{k_{\rm F}\ell_0})\,. \tag{14}$$

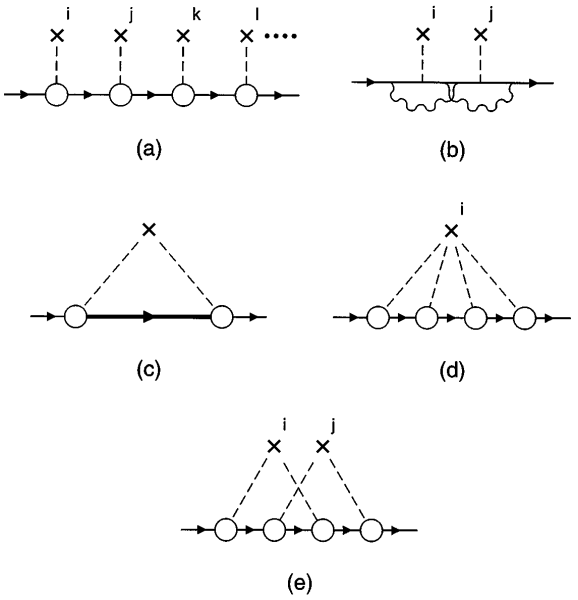

FIG. 1. Graphs describing the calculation of the impurity self-energy: (a) The impurity self-energy for a given configuration $(i,j,\ldots)$ of impurities. The line with arrows is the single particle propagator including renormalization due to electron-electron interaction and the circle connected to the dashed lines is the impurity vertex renormalized for the effect of interactions. (b) Vertex corrections due to interactions neglected in (a). The wavy line represents electron-electron interactions. (c) Impurity self-energy (after configuration averaging) in the self-consistent Born approximation. The thick line includes the self-energy due to interactions as well as due to impurities. (d) The t matrix with renormalized vertices of interaction from an impurity at site i. (e) The crossed graphs for interference effects between two impurities at site i and at site j.

Note that below ω_x the density of states near the Fermi energy

$$\frac{1}{\pi}\mathrm{Im}\sum_k G'(k,\omega) = N(0)/\ln(\omega_c/\omega)\,, \qquad (15)$$

which approaches zero at the chemical potential.

The real part of the self-energy corresponding to Eq. (12) is a constant whose value depends on the low and high-energy cutoffs and particle-hole asymmetry of the band structure. This can be absorbed as a change in the chemical potential, just as for impurity scattering in Fermi liquids, and does not affect the results. The two major omissions in evaluating (9) in the self-consistent Born approximation are (i) possible strong renormalized scattering from a given impurity and (ii) interference effects between scattering at different impurities. Consider the former. For Fermi liquids the singular scattering from a given impurity in the Born approximation is remedied by evaluating the t matrix [Fig. 1(d)]:

$$\tilde{t}_i = \frac{v_i/z}{1 - v_i z^{-1}\sum_k G(k,\omega)}\,. \qquad (16)$$

The $\tilde{t}$ matrix obeys the unitary limit $i\pi\tilde{t}N(0) \to 1$ as z becomes very small (but not zero), as in heavy fermions. This is because for Fermi liquids $\sum_k G(k,0) = i\pi N(0)$, the bare density of states, independent of the value of z.

We may check the validity of the self-consistent Born approximation by evaluating the corrections to the self-energy due to (16) by using $G = G'$ in (16). Now we note that on impurity averaging, one gets correction terms

$$\mathrm{Im}\,\Sigma_{\mathrm{imp}}(\omega) = \mathrm{Im}\,\Sigma^0_{\mathrm{imp}}(\omega)\left[1 + \left(\frac{v}{z}\sum_k G'(k,\omega)\right)^2 + \left(\frac{v}{z}\sum_k G'(k,\omega)\right)^4 + \cdots\right]. \qquad (17)$$

Using (15), we see that the singularity due to z^{-1} is canceled out to all orders leaving an analytic correction of $0(vN(0)) \ll 1$.

We may similarly consider correction due to crossed graphs [Fig. 1(e)]. Again the renormalized density of state cancels the singularity in v/z and the corrections are successive powers of $cvN(0)$. Since interference between scatterers is not important, the wave functions are not localized; the important effect is the vanishing of the density of states at the chemical potential, as in Eq. (15).

For $\alpha < 1$, Eq. (12) is modified to

$$\mathrm{Im}\,\Sigma^0_{\mathrm{imp}}(\omega) \approx \lambda\sqrt{\frac{\epsilon_{\mathrm{F}}}{2\tau_0}}\left(\frac{\omega_c}{\omega}\right)^{1-\alpha}, \qquad (18)$$

so the effects of impurities are more singular.

The most important effect neglected above is the renormalization by impurities of the fluctuation which produced the non-Fermi-liquid state in the pure limit. This cannot be discussed without a microscopic theory for such fluctuations. A theory of such fluctuations has been constructed [9]. The corrections to the fluctuations due to impurities have been briefly examined which appear not to change the results here in an essential way. This matter is however far from being settled. At this point it is best to leave this as an assumption and to point out that the results thus obtained appear to agree with experiments, as discussed below.

A prediction following from Eq. (15) is that the specific heat $C(T)$ and the magnetic susceptibility $\chi(T)$ have the forms $C(T)/T \sim \chi(T) \sim (\ln T)^{-1}$, for $T \ll T_x$. For s-wave scattering by impurities, there are no backward scattering (vertex) corrections for the calculation of resistivity. The resistivity can then be calculated from the single particle Green's function alone and is proportional to $\ln\frac{\omega_c}{T}$ for $T \ll T_x$.

Consider now the anisotropic resistivity of quasi-two-dimensional materials. Two distinct temperature scales need to be defined to discuss the anisotropic resistivity of non-Fermi-liquids. One of them is $T_{x,a}$ defined in Eq. (1), and the other is T_x defined in Eq. (14). $T_{x,a}$ is enough to discuss Fermi liquids. For $T \gg T_{x,a}$, the layers are mutually phase incoherent and momentum in is not conserved in the process of electron transfer between the planes. One can calculate ρ_c^{-1} by a tunneling rate calculation. The tunneling rate (if the tunneling matrix element has some momentum dependence) is proportional to the (in-plane) inelastic scattering rate. Then $\rho_c^{-1}(T) \sim \rho_{a,b}(T)$. For $T \ll T_{x,a}$ a coherent propagation must prevail in a Fermi liquid and $\rho_c(T) \sim \rho_{a,b}(T)$. This behavior is indeed observed, for example, in Sr_2RuO_4 [15].

For non-Fermi-liquids two limiting cases can easily be distinguished: (i) $T_{x,a} \ll T_x$, which occurs for highly anisotropic and fairly dirty materials. The resistivity in the c direction increases with decreasing temperature for all temperatures but below $T_{x,a}$. Both $\rho_{a,b}$ and ρ_c have similar temperature dependences below $T_{x,a}$ but for $T \gg T_{x,a}$, $\rho_c^{-1} \sim \rho_{a,b}$. (ii) $T_x \gg T_{x,a}$: very high quality samples of $YBa_2Cu_3O_{6.9}$ appear to fall in this class if we assume that in this low anisotropy material $T_{x,a}$ is above the measured temperature range. The more anisotropic dirty materials are consistent with class (i) but in the high quality samples studied T_x is quite low, not too far from $T_{x,a}$. For instance, in the measurements of Boebinger *et al.* [15] on $La_{1.85}Sr_{0.15}CuO_4$, $k_{\mathrm{F}}\ell_0$ is estimated to be about 15 which with $\omega_c \approx 2 \times 10^3$ K and $\lambda \approx 1$, estimated from the slope of the linear resistivity and the optical conductivity [7] gives $T_x \approx 0(20\ \mathrm{K})$ from Eq. (17). This is consistent with the temperature at which $\rho_{a,b}(T)$ has a minima in the experiments and asymptotically below which a logarithmic temperature dependence is observed. Systematic estimations of $k_{\mathrm{F}}\ell_0$ are not yet available to test Eq. (14).

The results here have possible applications to other situations where interactions lead to non-Fermi-liquid properties in the pure limit. These include the quantum critical points in itinerant ferromagnets and antiferromagnets as well as the mysterious transitions in $CeCu_{6-x}Au_x$ [16]. It would appear from the results here that disorder is strongly relevant at metal-insulator transitions otherwise driven by electronic correlations. The effect of disorder in driving $N(0) \rightarrow 0$ appears stronger than the localization effect which sets in only for $k_F \ell_0 \sim O(1)$. The pseudoparticle Green's function at the $\nu = \frac{1}{2}$ quantum Hall effect also has a marginal Fermi-liquid form [17]. But since in this case the Green's function is a gauge dependent object, a separate investigation is required for physical quantities.

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