Noiseless Signal Amplification using Positive Electro-Optic Feedforward

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(Received 1 April 1997)

We propose an electro-optic feedforward scheme which can in principle produce perfect noiseless signal amplification (signal transfer coefficient of $T_s = 1$). We demonstrate the scheme experimentally and report, for a signal gain of 13.4 dB, a signal transfer coefficient of $T_s = 0.88$ which is limited mainly by detector efficiencies (92%). The result clearly exceeds the standard quantum limit, $T_s = 0.5$, set by the high gain limit of a phase insensitive linear amplifier. We use the scheme to amplify a small signal carried by 35% amplitude squeezed light and demonstrate that, unlike the fragile squeezed input, the signal amplified output is robust to propagation losses. [S0031-9007(97)03878-7]

PACS numbers: 42.50.Lc, 42.50.Dv, 42.60.Da

The size of amplitude fluctuations on a light beam limits its ability to detect or carry small amplitude signals [1]. In principle, coherent light with fluctuations at the quantum noise limit (QNL), or even squeezed light with fluctuations below the QNL, would be ideal for detection and transmission of small signals. However, such signals are very fragile to losses, which introduce fluctuations at the QNL that rapidly reduce the signal to noise ratio (SNR). A solution to this problem is to amplify the signal until it is much larger than the QNL and hence robust to losses [2]. However, this too has problems as phase insensitive amplifiers (PIA's), such as laser amplifiers, inevitably introduce excess quantum noise. In the case of coherent light, this excess noise halves (reduces by 3 dB) the signal to noise ratio in the high gain limit [3]. This is often referred to as the 3 dB penalty for PIA's.

The 3 dB penalty arises from the fact that a PIA amplifies the two conjugate observables, intensity and phase, simultaneously. If additional noise was not added in this process, the uncertainty relation for the variables would be violated. To avoid this penalty, amplification must be phase sensitive [4]. One method of phase sensitive amplification is to amplify one observable while deamplifying the conjugate observable. This normally requires a nonlinear optical process. For example, optical parametric amplification has been used to amplify intensity signals with almost no noise penalty [5]. Unfortunately such experiments are complex and difficult to control. Another method of phase sensitive amplification is to simply detect the light, electronically amplify the resulting photocurrent, and then reemit the light using a light emitting diode (LED) [6,7] or a diode laser. This method is phase sensitive as only the intensity is measured and amplified. The drawback to this method is that all phase information is destroyed by the detection process. The amplified output has no temporal or spatial coherence with the input beam.

In this Letter, we propose and demonstrate a simple, electro-optic, signal amplification scheme which retains optical coherence while not requiring any nonlinear optical process. Our scheme is based on partial detection of the light with a standard beam splitter and detector (Fig. 1). The light reflected from the beam splitter is detected and the resultant photocurrent is amplified and fed forward to an amplitude modulator in the transmitted beam. By correct choice of the electronic gain and phase, we show that intensity signals carried by the input light are amplified, while the vacuum fluctuations which enter through the empty port of the beam splitter are cancelled. Since not all of the input light is destroyed, the output is still coherent with the input beam.

The experimental setup is shown schematically in Fig. 1. A polarizing beam splitter taps off part of the input beam to the in-loop detector. The transmittivity of the beam splitter, ε_1, is controlled by a half-wave plate. On the in-loop beam, a balanced detector pair denoted by $D_{\rm il}$, is set up to enable self-homodyne measurements. The photon statistics of the beam can then be determined relative to the QNL. To achieve signal amplification, the detected photocurrents of the balanced detector pair are summed and passed through three stages of rf amplification and filtering. This is to ensure that sufficient rf gain can be achieved for the frequency bandwidth of interest, while maintaining relatively high transmittivity at the electro-optic modulator (EOM). An amplitude modulator is formed by using the EOM in conjunction with a

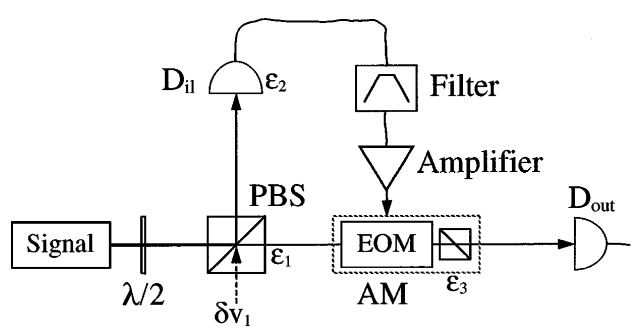

FIG. 1. Schematic of the experimental setup. $D_{\rm il}$: in-loop balanced detector pair; $D_{\rm out}$: out-of-loop detector; PBS: polarizing beam splitter; $\lambda/2$: half-wave plate; AM: amplitude modulator.

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polarizer. A spectrum analyzer is used to measure the noise and signal power spectrum of the output photocurrent of detector D_{out}. For small fluctuations the power spectrum is proportional to the amplitude fluctuation spectrum of the light $[V(\omega)]$ and can be written as the sum of contributions from classical amplitude modulations [signals, $V_s(\omega)$] and the quantum fluctuations [noise, $V_{\mathrm{n}}(\omega)$]; $V = V_{\mathrm{s}} + V_{\mathrm{n}}$.

The in-loop balanced detector pair has a total efficiency of $\varepsilon_2 = 0.92 \pm 0.02$. The out-of-loop detection efficiency, including the modulator losses, is $\varepsilon_3 = 0.80 \pm 0.05$. In our initial experiments the input light, at 532 nm, is within 0.2 dB of the QNL at a detection frequency of 20 MHz. The signal to noise ratio is defined by $\mathrm{SNR} \equiv V_{\mathrm{s}}/V_{\mathrm{n}} = (V/V_{\mathrm{n}}) - 1$. We use the subscripts in and out to designate properties of the input and output fields, respectively. At 20 MHz we impose an input signal with $\mathrm{SNR}_{\mathrm{in}} = 9.7 \pm 0.1$ dB. We define the signal gain of the system by $G \equiv V_{\mathrm{out}}(\omega_{\mathrm{s}})/V_{\mathrm{in}}(\omega_{\mathrm{s}})$, where ω_{s} is the signal modulation frequency. The signal transfer coefficient, T_{s}, is defined in the usual way as $T_{\mathrm{s}} = \mathrm{SNR}_{\mathrm{out}}/\mathrm{SNR}_{\mathrm{in}}$ [8]. The highest T_{s} of our scheme does not occur at arbitrarily large feedforward. There is an optimum magnitude and phase for the electronic gain, which corresponds to the complete cancellation of vacuum fluctuations introduced by the feedforward beam splitter. The half-wave plate is adjusted to tap half of the input light, $\varepsilon_1 = 0.5$, for feedforward. The signal transfer coefficient, T_{s}, and the signal gain, G, are obtained for various feedforward gains. As can be seen from Fig. 2, there is clearly an optimum feedforward gain where $T_{\mathrm{s}} = 0.86 \pm 0.02$ is a maximum at $G = 3.4 \pm 0.6$ dB. For higher signal gains, the T_{s} values degrade and asymptote to the T_{s} value corresponding to direct in-loop detection. This is because for high feedforward gains, the contribution from the reflected in-loop signal overwhelms the transmitted signal.

We model the scheme as follows. Suppose the beam splitter has a transmittivity ε_1 and negligible losses. The reflected beam is directed to a detector of efficiency ε_2. We can write the input laser beam in the linearized form

$$\hat{A}_{\mathrm{in}}(t) = A_{\mathrm{in}} + \delta\hat{A}_{\mathrm{in}}(t), \tag{1}$$

where $\hat{A}_{\mathrm{in}}$ is the field annihilation operator; A_{in} is the classical steady state value of the field; and $\delta\hat{A}_{\mathrm{in}}$ is a zero-mean operator which carries all the classical and quantum fluctuations. The detected output field is given by

$$\hat{A}_{\mathrm{out}} = \sqrt{\varepsilon_3}\,(\sqrt{\varepsilon_1}\,A_{\mathrm{in}} + \sqrt{\varepsilon_1}\,\delta\hat{A}_{\mathrm{in}} + \sqrt{1-\varepsilon_1}\,\delta\hat{v}_1 + \delta\hat{r}) + \sqrt{1-\varepsilon_3}\,\delta\hat{v}_3\,, \tag{2}$$

where ε_3 is the combined efficiency due to the transmittivity of the modulator and the quantum efficiency of the out-of-loop detector. As usual vacuum fluctuations from the unused port of the beam splitter, $\delta\hat{v}_1$, and due to out-of-loop losses, $\delta\hat{v}_3$, appear on the transmitted beam. We have assumed that the feedforward does not affect the steady state value of the field but just adds a small fluctuating term $\delta\hat{r}$ which can be written as a convolution over time [9],

$$\begin{aligned}\delta\hat{r} = \int_{-\infty}^{\infty} & k(\tau)\sqrt{(1-\varepsilon_1)\varepsilon_2} \\ & \times A_{\mathrm{in}}[\sqrt{(1-\varepsilon_1)\varepsilon_2}\,\delta\hat{X}_A(t-\tau) \\ & \quad - \sqrt{\varepsilon_1\varepsilon_2}\,\delta\hat{X}_{v1}(t-\tau) \\ & \quad + \sqrt{(1-\varepsilon_1)}\,\delta\hat{X}_{v2}(t-\tau)]\,d\tau\,,\end{aligned} \tag{3}$$

that expresses changes in the phase and amplitude of the feedforward signal due to the electronics by a function $k(t)$.

The amplitude fluctuations of the input field and its accompanying vacuum fluctuations from the beam splitter δv_1, and the nonunity detector efficiency δv_2, are defined by $\delta\hat{X}_{A\mathrm{in}} = \delta\hat{A}_{\mathrm{in}} + \delta\hat{A}_{\mathrm{in}}^{\dagger}$ and $\delta\hat{X}_{vi} = \delta\hat{v}_i + \delta\hat{v}_i^{\dagger}$. Note that energy conservation requires that the vacuum fluctuations introduced on the reflected beam are anticorrelated with those on the transmitted beam. The amplitude fluctuation spectrum of the output field is the expectation value of the Fourier transform of the absolute squared amplitude fluctuations, i.e., $V_{\mathrm{out}}(\omega) = \langle|\delta\tilde{X}_{A\mathrm{out}}|^2\rangle$. Note that experimentally V_{out} is obtained by normalizing the power spectrum from the spectrum analyzer to the QNL for the same optical power. We find

$$\begin{aligned}V_{\mathrm{out}}(\omega) = {} & \varepsilon_3\,|\,\sqrt{\varepsilon_1} + \lambda\sqrt{(1-\varepsilon_1)\varepsilon_2}\,|^2\,V_{\mathrm{in}}(\omega) \\ & + \varepsilon_3\,|\,\sqrt{(1-\varepsilon_1)} - \lambda\sqrt{\varepsilon_1\varepsilon_2}\,|^2\,V_1 \\ & + \varepsilon_3\,|\,\lambda\sqrt{(1-\varepsilon_2)}\,|^2\,V_2 \\ & + (1-\varepsilon_3)V_3\,,\end{aligned} \tag{4}$$

where various parameters have been rolled into the electronic gain $\lambda(\omega)$, which is in general a complex number. $V_{\mathrm{in}}(\omega) = \langle|\delta\tilde{X}_{A\mathrm{in}}|^2\rangle = V_{\mathrm{s,in}}(\omega) + V_{\mathrm{n,in}}(\omega)$ is the amplitude fluctuation spectrum of the input field. The

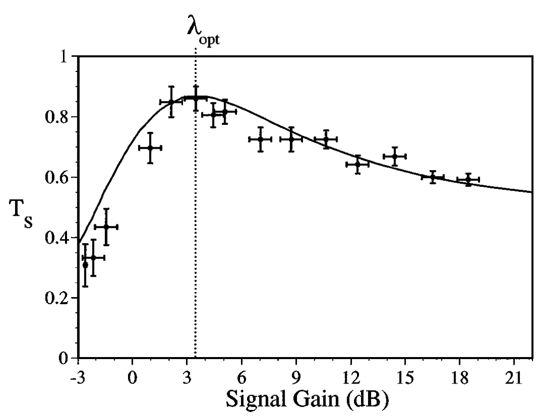

FIG. 2. Signal transfer coefficient, T_{s}, vs signal gain, G. $\varepsilon_1 = 0.5$. The optimum value of $T_{\mathrm{s}} = 0.86 \pm 0.02$ occurs at a gain of $G = 3.4 \pm 0.6$ dB. Increasing the gain beyond this point degrades the T_{s}.

vacuum noise spectra due to the beam splitter V_1, the in-loop detector efficiency V_2, and the out-of-loop losses V_3 are shown explicitly to emphasize their origins. All vacuum input are quantum noise limited, i.e., $V_1 = V_2 = V_3 = 1$. Because of the opposite signs of the feedback parameter λ in Eq. (4), it is possible to amplify the input noise (first term), while canceling the vacuum noise from the feedforward beam splitter (second term). The third and fourth terms of Eq. (4) represent unavoidable experimental losses. In particular, if we choose

$$\lambda = \frac{\sqrt{1 - \varepsilon_1}}{\sqrt{\varepsilon_1 \varepsilon_2}}, \tag{5}$$

the vacuum fluctuations from the beam splitter, V_1, are exactly canceled. Then, under the optimum condition of unit efficiency detection and negligible out-of-loop losses ($\varepsilon_2 = \varepsilon_3 = 1$), we find

$$V_{\mathrm{out}}(\omega) = \frac{1}{\varepsilon_1} V_{\mathrm{in}}(\omega) = \frac{1}{\varepsilon_1} [V_{\mathrm{s,in}}(\omega) + V_{\mathrm{n,in}}(\omega)]. \tag{6}$$

That is, the fluctuations are noiselessly amplified by the inverse of the beam splitter transmittivity. The signal and quantum noise are amplified by the same amount, and there is no noise added, hence there is no degradation of the signal to noise ratio. Thus our system ideally can attain a transfer coefficient of $T_s = 1$ for a signal gain of $G = 1/\varepsilon_1$. The effect of nonunity in-loop detector efficiency is to limit the optimum signal transfer coefficient to $T_{\mathrm{s}}^{\mathrm{max}} = \varepsilon_2$. Extra losses downstream from the feedforward affect the output in the same way as losses due to the out-of-loop efficiency ε_3.

A theoretical curve calculated from the experimental parameters is also plotted on Fig. 2. In particular, the optimum feedforward gain λ_{opt} corresponds to a signal gain of $G = 3.4$ dB and a signal transfer coefficient of $T_{\mathrm{s}} = 0.87$, in good agreement with the experimental values.

To obtain optimum performance at higher signal gain requires a greater reflectivity at the beam splitter [see Eq. (6)]. For higher beam splitter reflectivity, the transmitted beam is dominated by the vacuum fluctuations. Thus, higher feedforward gain is required to completely cancel the vacuum fluctuations, resulting in a shift of the optimum operating point λ_{opt} to a higher value of G. This is demonstrated in Fig. 3 where the beam splitter reflectivity was increased to 90%, i.e., $\varepsilon_1 = 0.1$. With maximum available feedforward gain, we achieve $T_{\mathrm{s}} = 0.88 \pm 0.02$ with a signal gain of $G = 13.4 \pm 0.5$ dB. We have also calculated the T_{s} as a function of signal gain for a PIA, as shown by curve (*a*). A PIA with the same signal gain would be limited to a transfer coefficient of $T_{\mathrm{s}} = 0.51$. Our system clearly exceeds this limit.

It is important to note that the absolute power of the amplified output signal, as measured by the spectrum analyzer, is not necessarily larger than that of the input signal. This is because the absolute signal power is scaled by the intensity of the light, and in our scheme this is unavoidably decreased. The reduction in intensity also reduces the QNL of the output beam such that the size of the amplified signal with respect to the QNL is increased. It is this relative amplification of the signal (as measured by G) which reduces the fragility of the signal.

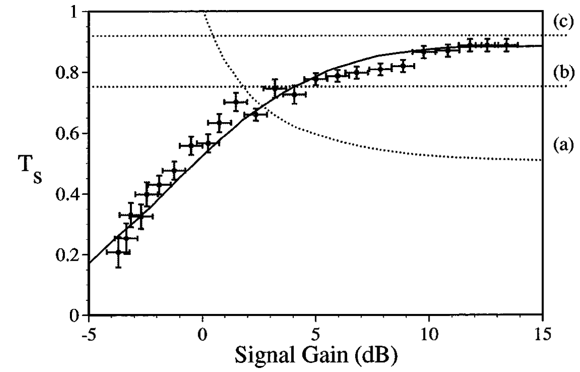

FIG. 3. Signal transfer coefficient, T_{s}, vs signal gain, G. $\varepsilon_1 = 0.1$. Dotted curves are limiting cases: (*a*) is the best possible performance of a PIA, points above this curve are evidence of phase sensitive amplification. (*b*) is the T_{s} value when feedforward signal dominates. Points above this line are evidence of vacuum fluctuations cancellation. (*c*) is the $T_{\mathrm{s}}^{\mathrm{max}}$ of the scheme set by the efficiency of the in-loop detector $\varepsilon_2 = 0.92 \pm 0.02$.

To illustrate this, we use our system to amplify squeezed light, which is notoriously sensitive to losses. As our squeezed source, we use the second harmonic output from a singly resonant frequency doubler as described in [10]. The doubler produces squeezing in the amplitude quadrature which can then be amplified by our feedforward scheme. The top half of Fig. 4 shows the input noise spectrum. This is obtained from the in-loop balanced detector pair by setting the beam splitter to total reflection. Trace (i) shows the QNL, which is obtained by subtracting the photocurrents in the balanced detector pair. Trace (ii) is the sum of the photocurrents, which gives the noise spectrum of the input light. Regions where (ii) is below (i) are amplitude squeezed. The maximum measured squeezing of 1.6 dB is observed in the region of 8–10 MHz on a 26 mW beam. The inferred value after taking into account the detection efficiency and electronics noise floor is 1.8 dB. A small input modulation signal (2.80 dB observed) is introduced at 10 MHz which, allowing for detection losses, has $\mathrm{SNR_{in}} = 1.10 \pm 0.03$. Other features of the spectra include the residual 17.5 MHz locking signals of the frequency doubling system [10] and the low frequency roll-off of the photodetector, introduced to avoid saturation due to the large relaxation oscillation of the laser at $\approx$0.5 MHz.

The bottom half of Fig. 4 shows the noise spectra obtained from the single output detector. Setting the beam splitter reflectivity to zero, $\varepsilon_1 = 1$, the transmitted beam is made to experience 86% downstream loss, $\varepsilon_3 = 0.14$, after the feedforward loop. As trace (*a*)

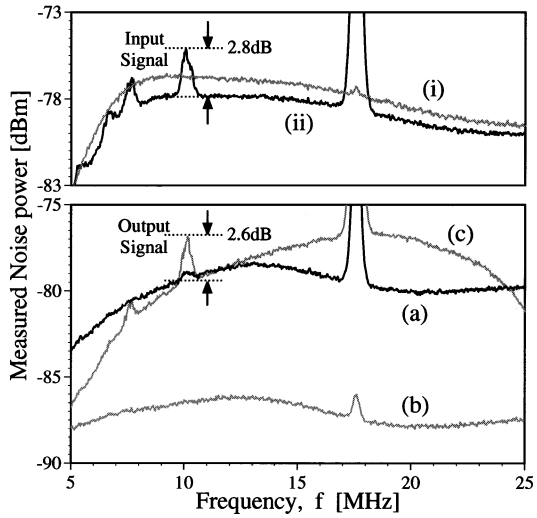

FIG. 4. Top: Noise spectra of the squeezed input beam. Traces (i) and (ii) are the difference and sum of the balanced photocurrents, respectively. A modulation signal of $SNR_{in} = 2.8$ dB is introduced at 10 MHz. Bottom: Noise spectra of the output beam. Trace (*a*): Direct detection of the input light using the single output detector. Because of the presence of loss $\varepsilon_3 = 0.14$, the signal degrades to SNR = 0.4 dB. Trace (*b*): Output noise spectrum without feedforward and with large loss $\varepsilon_{tot} = 0.014$. The signal is completely destroyed. Trace (*c*): With optimum feedforward gain, the signal is reconstructed with $SNR_{in} = 2.6$ dB. This corresponds to $T_s = 0.75 \pm 0.02$; and $G = 9.3 \pm 0.2$ dB.

shows, the SNR is strongly degraded by the attenuation such that the signal is now barely visible above the noise. We now perform signal amplification by setting the beam splitter reflectivity to 90%, $\varepsilon_1 = 0.1$. This further attenuates the output beam to $\varepsilon_{tot} = \varepsilon_1 \varepsilon_3 = 0.014$. With no feedforward gain, as trace (*b*) shows, the modulation signal is now too small to be seen above the noise. Because of the large amount of attenuation, trace (*b*) is quantum noise limited to within 0.1 dB over most of the spectrum. Finally, by choosing the optimum signal gain, $G = 9.3 \pm 0.2$ dB, trace (*c*) shows the amplified input signal with $SNR_{out} = 0.82 \pm 0.03$. Traces (*b*) and (*c*) are of the same intensity, hence we can see that the output signal is significantly above the QNL. This is the reason why the amplified output is far more robust to losses than the input. This result corresponds to a signal transfer coefficient of $T_s = 0.75 \pm 0.02$, again in good agreement with the theoretically calculated result of $T_s = 0.77$. This is to be compared with the best performance of a PIA, with similarly squeezed light, of $T_s \approx 0.4$. Note that trace (*c*) has a different shape than traces (*a*) and (*b*) due to the transfer function of the in-loop electronics and the phase variation of the feedforward across the frequency spectrum. The bandwidth of the rf gain is from 7 to 21 MHz. However, the optimum feedforward gain and phase are only satisfied in a limited region of the spectrum around 10 MHz.

In conclusion, we have shown that an electro-optic feedforward scheme can be used as a noiseless signal amplifier. The scheme does not employ any nonlinear optical process and preserves optical coherence. It is phase sensitive as it only amplifies the amplitude quadrature. The optimum performance is explained in terms of the cancellation of vacuum fluctuations that are introduced during the measurement process. We have demonstrated the effectiveness of our scheme by amplifying signals carried by squeezed light with minimal loss of signal to noise, even in the presence of large (86%) losses. The scheme does cause a reduction of the optical power of the signal beam; however, this is not in principle a disadvantage as injection locking can be used to restore or even increase the output intensity without affecting the fluctuations [11]. In fact, as the signal is well above the QNL after amplification, it can be further amplified by a standard PIA, such as a laser amplifier without serious degradation of the signal to noise ratio.

We wish to acknowledge many useful discussions with A. G. White, M. B. Gray, D. E. McClelland, B. C. Buchler, and C. C. Harb. We thank M. S. Taubman and A. G. White for the setup of the frequency doubler and J. Lye for her technical assistance. This research is supported by the Australian Research Council.

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