Is There an Excited State in ^{11}Li at $E_x = 1.3$ MeV?

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Inelastic scattering data of ^{11}Li from hydrogen at $68A$ MeV have been interpreted as the excitation of a state in ^{11}Li at $E_x = 1.3$ MeV, with an assignment of $J^\pi = \frac{3}{2}^+$. Analysis of those data in a distorted wave approximation assuming transitions to three candidates obtained in a $(0 + 2)\hbar\omega$ shell model suggests an alternative nuclear shakeoff mechanism. [S0031-9007(97)03915-X]

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The understanding of the structures of the light halo nuclei in terms of a spherical shell model has presented a challenge because of the very loose binding of the halo nucleons, which extend the matter distributions of those nuclei to large radii. The measurements of transfer reactions leading to excited states of ^{11}Li by Bohlen *et al.* [1] suggest excited states in ^{11}Li at 2.47, 4.85, and 6.22 MeV. There have been few shell-model calculations presented of halo nuclei: e.g., those of Poppelier, Wood, and Glaudemans [2] for several $0p$-shell neutron-rich halo nuclei, those of Warburton and Brown [3], and those of Sagawa, Brown, and Esbensen [4].

The measurements of ^{9,11}Li scattering from hydrogen taken by Moon *et al.* [5] at 60 and 62 MeV/nucleon and by Korsheninnikov *et al.* [6,7] at 68 and 75 MeV/nucleon provide elastic and inelastic proton scattering data from ^{9,11}Li in the inverse kinematics. These data not only provide energy spectra, by which comparison may be made with model predictions, but also cross section data with which to test the wave functions. The inelastic scattering data at $E_p = 68$ MeV of Korsheninnikov *et al.* [7] show a peak at $E_x = 1.3$ MeV and a broad distribution assumed to represent other reaction mechanisms. A coupled-channel optical model analysis of the differential cross section obtained from this peak suggested that the excitation is dominantly $L = 1$. For the ground state, $J^\pi; T = \frac{3}{2}^-; \frac{5}{2}$, and so an assignment of $J^\pi = \frac{3}{2}^+$ was made (although that analysis also would allow assignments of $\frac{1}{2}^+$ and of $\frac{5}{2}^+$). This is the starting point by which direct comparison with the shell model can be made.

A problem with the conventional optical model analysis of those data lies with the specification of the optical potential. Korsheninnikov *et al.* [6,7] used a phenomenological optical potential of conventional Woods-Saxon (WS) form that fitted the elastic scattering cross section found by Moon *et al.* [5] in an analysis of their 62 MeV data. Such a central mean field specification for the optical potential may be inappropriate for the scattering of protons from a halo nucleus. It may not take into account appropriately the density extending to large radii arising purely from the distribution of the halo nucleons. An alternative prescription is to use a fully microscopic optical potential formed by the folding of the scattering interaction with the nucleon occupancies and single-particle wave functions. When the halo single-particle wave functions are correctly specified that prescription of the optical potential appropriately takes into account the halo distribution.

An analysis of the data of Moon *et al.* [5] has been performed by Crespo *et al.* [8] assuming a few-body model for the ^{11}Li nucleus and specifically as a three-body system (^{9}Li + n + n). A Gaussian distribution model and a cluster model separately were assumed for the ^{9}Li core. Their calculations for the elastic proton scattering from both ^{9}Li and ^{11}Li underpredict the measured cross sections, and hence they concluded that the microscopic structure of ^{9}Li was important. Such structure is included naturally in the shell-model wave functions we consider herein.

We have performed a complete $(0 + 2)\hbar\omega$ shell-model calculation for the negative parity states of ^{11}Li, and a restricted $(1 + 3)\hbar\omega$ shell-model calculation of its positive parity states, using the code OXBASH [9]. A $0\hbar\omega$ calculation of the ground state spectrum of ^{9}Li was performed as well. The model contained all orbits from the $0s$ up to, and including, the $0f1p$ shell. Hence the restriction on the $(1 + 3)\hbar\omega$ model space is only the exclusion of the single-particle excitations up to the $0g1d2s$ shell. The interaction used was the WBP interaction of Warburton and Brown [3], while their P(5 − 16)T interaction [3] was used for the calculation of ^{9}Li in a pure $0\hbar\omega$ shell model. An energy shift was applied to the $2\hbar\omega$ and $3\hbar\omega$ components in each case to account for the neglect of higher $\hbar\omega$ components [10]. The energy shifts were $\Delta_{2\hbar\omega} = -2.00$ MeV and $\Delta_{3\hbar\omega} = -2.23$ MeV for ^{11}Li. [Those values are obtained from the calculated shift of the $n\hbar\omega$ configurations due to the $(n + 2)\hbar\omega$ admixtures.]

For ^{9}Li, the ground and first-excited state (3.14 MeV in our model calculation) agree with the experimental assignments. The calculated energies and spin-parity assignments for ^{11}Li are 0 ($J^\pi; T = \frac{3}{2}^-; \frac{5}{2}$), 1.49 ($\frac{3}{2}^-$), 1.83 ($\frac{3}{2}^+$), 1.87 ($\frac{1}{2}^-$), 2.68 ($\frac{1}{2}^+$), and 3.25 MeV ($\frac{5}{2}^+$), of which the second, third, and fourth states are likely candidates for the excitation of ^{11}Li in the inelastic proton scattering experiment [7]. The calculation by Poppelier *et al.* [2] had the excited $\frac{3}{2}^-$ state at 21.96 MeV. Their result also showed an excited $\frac{5}{2}^+$ state at 2.68 MeV, a $\frac{1}{2}^-$ state at 4.58 MeV, and a $\frac{3}{2}^+$ state at 3.13 MeV. It is important to note that all excited states in ^{11}Li are broad continuum states, and their energies in our shell model are accurate to about 1 MeV.

The ground state wave function of ^{11}Li in our model is $62.71\%|0\hbar\omega\rangle + 37.29\%|2\hbar\omega\rangle$. This wave function contains a substantial admixture of $2\hbar\omega$ components, of which 19.62% come from the pure $(0d)^2$ configurations and a further 10.02% arise from the pure $(1s)^2$ configurations.

Our methods of analyses of the elastic and inelastic proton scattering data follow those we used in analyses of the elastic and inelastic scattering data from 200 MeV protons on ^{12}C [11] and on ^{6,7}Li [12]. Those analyses are based upon an effective nucleon-nucleon (*N*-*N*) interaction in coordinate space that has been obtained from an accurate mapping of the (*N*-*N*) g matrices of the Paris *N*-*N* interaction [13] for infinite nuclear matter obtained from solving the Bruckner-Bethe-Goldstone equations [14]. That complex interaction is both energy and density dependent. Folding the effective interaction with the target density matrix elements then yields energy dependent, complex, and nonlocal nucleon-nucleus (*N*-*A*) optical potentials in which is contained the density dependence required to describe well both elastic and inelastic scattering data [11]. The latter have been calculated in the distorted wave approximation (DWA) in which the same effective interaction is the transition operator and the distorted waves are obtained from the microscopic optical potentials. The interaction at 65 MeV also has been used in analyses of proton elastic and inelastic scattering from diverse targets [15], wherein very good agreement with cross section and polarization data has been obtained. The code DWBA91 of Raynal [16] has been used to calculate all of the elastic and inelastic scattering cross sections. Note that these calculations are entirely parameter-free. There are no *a posteriori* scalings of the results obtained from the calculations in their comparisons with data, and so our results are purely predictive. For complete details, see Refs. [11,12].

Specification of the single particle wave functions is important in analyses of scattering data. This is especially true for the scattering from ^{11}Li, as that halo nucleus requires single particle wave functions that reproduce the density extending to large radii. Such is not the case for ^{9}Li. We have used HO and WS single particle wave functions in the calculations. As there are no electron scattering data by which to set the wave functions, the WS parameter values were determined from fits to the longitudinal elastic electron scattering form factors for either ^{7}Li [12] (a choice predicated on the similarity of charge) or ^{9}Be [17] (a choice predicated on mass). However, for the scattering with ^{11}Li the WS functions were adjusted to define the halo nature of that nucleus. Specifically, we used WS wave functions with a binding energy of 500 keV for the halo neutron orbits, namely the $0p_{1/2}$ orbit and the $0d1s$ and $0f1p$ shells in the complete $(0 + 2)\hbar\omega$ shell-model space. The problem of choosing appropriate radial wave functions for transitions between loosely bound states has been illuminated (and resolved) by Millener *et al.* [18] for the case of ^{11}Be.

The predictions made for the elastic scattering of 62 and 68 MeV protons from ^{11}Li, and of 62 MeV protons from ^{9}Li are shown in Fig. 1. Therein the data for 62*A* MeV [5] and 68*A* MeV [7] ^{11}Li scattering from hydrogen are compared in the top panel of Fig. 1 to the result at 62 MeV made using the ^{9}Be WS wave functions (solid line), and also to that at 68 MeV (dashed line). The prediction of the scattering from ^{11}Li made at 62 MeV using harmonic oscillators ($b = 1.65$ fm) is displayed as the dot-dashed line. It overestimates the cross section significantly, and illustrates the need for specifying the halo density distribution appropriately. The results of the calculations using the WS single-particle wave functions are insensitive to changing the binding energy of the halo orbits to as low as 50 keV. The data for the elastic scattering of 62*A* MeV ^{9}Li from hydrogen [5] and the predictions made using the WS wave functions are compared in the bottom panel of Fig. 1, wherein the results obtained using the ^{9}Be and ^{7}Li sets are displayed by the solid and dashed lines, respectively. There the use of the ^{9}Be set of WS functions is closer in agreement with the data, although the ^{7}Li set provides a reasonable representation. In the case of ^{11}Li, the results using both

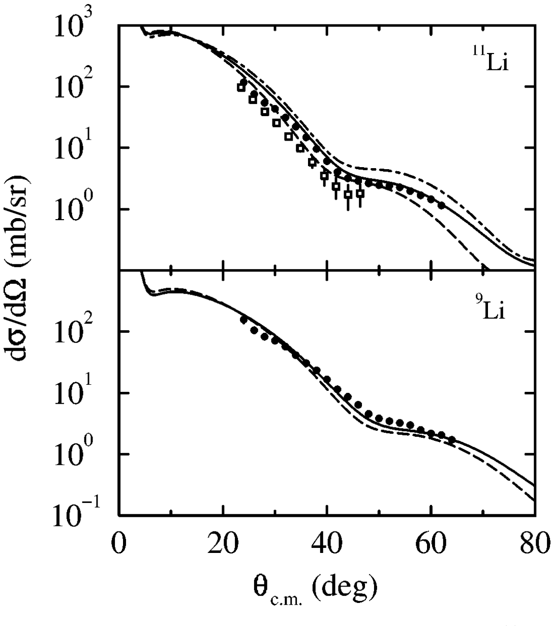

FIG. 1. Elastic scattering of 60 and 68 MeV from ^{11}Li (top) and ^{9}Li (bottom). The data of Moon *et al.* [5] (circles) and of Korsheninnikov *et al.* [7] (squares) are compared to our predictions made using the $(0 + 2)\hbar\omega$ and $(1 + 3)\hbar\omega$ shell-model wave functions. The predictions for the scattering from ^{11}Li at 60 MeV used both the ^{9}Be WS (solid line) and HO single particle wave functions (dot-dashed line). The prediction at 68 MeV using the WS wave functions is displayed by the dashed line. The prediction for the scattering from ^{9}Li using the ^{7}Li set of WS wave functions is displayed by the dashed line.

WS sets are quite similar, and hence only the ^{9}Be results are displayed. The excellent agreement with experiment for both nuclei confirms the conclusion by Crespo *et al.* [8] of the need for the specification of the full structure of the ^{9}Li core.

The total nuclear elastic scattering cross sections from ^{9}Li and ^{11}Li are 281 and 393 mb, respectively. This is in disagreement with Moon *et al.* [5], who interpreted their experiment as indicating a significant decrease in cross section from ^{9}Li to ^{11}Li. Our result is due to the contributions made from the cross section values at the (unobserved) forward angles for ^{11}Li and is what should be expected: The larger size of ^{11}Li leads to a narrower and more intense diffraction pattern.

The data for the inelastic scattering of 68 MeV protons from ^{11}Li [7] are compared with our predictions in Fig. 2(a). Therein, the cross sections to the $\frac{3}{2}^-$ (1.49 MeV), $\frac{3}{2}^+$ (1.83 MeV), and $\frac{1}{2}^-$ (1.87 MeV) states are displayed by the solid dashed and dot-dashed lines, respectively. The transitions to the negative parity states are mostly $E2$ in character, while the transition to the $\frac{3}{2}^+$ state is dominantly $E1$. However, in all cases, neither the shapes nor the magnitudes agree with the data. It is a possibility that a problem may lie in our choice of wave functions for the excited states. It is also possible that our approach does not guarantee proper treatment of excitations in the continuum. Also, as the excited (continuum) states are broad, the excitation of many higher lying states may contribute to the inelastic cross section. Hence we propose a simpler, semiquantitative model, incorporating the excitation of the continuum as a whole.

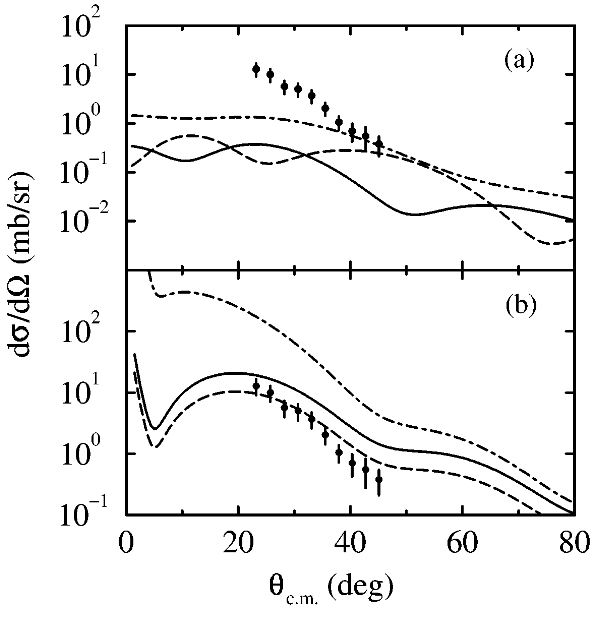

FIG. 2. Comparison of the inelastic 68 MeV proton scattering data [7] with (a) the results of the calculations made assuming the transition to the $\frac{3}{2}^-$ (1.49 MeV) (solid line), the $\frac{3}{2}^+$ (1.83 MeV) (dashed line), and the $\frac{1}{2}^-$ (1.87 MeV) (dot-dashed line) states, and (b) the result of the calculation assuming the shakeoff mechanism (solid line). The dashed line in (b) is the same result scaled by a factor of 0.5 corresponding to the strength associated with the 1.3 MeV peak in the spectrum. The elastic scattering of 68 MeV protons from ^{9}Li is displayed by the dot-dashed line.

The basic process is elastic scattering of the proton from the ^{9}Li core [19]. The momentum imparted to the halo in the new center-of-mass (c.m.) system entails a certain probability of breakup into the constituents $^9\text{Li} + n + n$. Such processes commonly are encountered in atomic physics, where they are referred to as "shakeoff." Another analogy is the recoilless absorption of photons in the Mössbauer effect, where the probability that the struck system remains in its ground state is referred to as the Debye-Waller (DW) factor.

Since the ^{11}Li halo has no bound excited states, we may calculate the shakeoff probability P_s as unity minus the DW factor. This is equivalent to a (non-energy weighted) sum rule. Assume, for a simple estimate, that the spatial wave function of the ground state with energy E_0 may be written as a product of two neutron wave functions (neglecting the c.m. corrections and nn correlations), i.e., $|\Psi\rangle = |1s\rangle_1|1s\rangle_2$ and that the momentum transferred to the ^{9}Li core after elastic scattering of a proton is $\vec{Q}$. In the c.m. system after the scattering, the change in momentum of each of the two neutrons is $-\vec{q} = -\vec{Q}/(A+2)$ and that of the core is $+2\vec{q}$. In the sudden approximation, the wave function after the scattering is

$$|\vec{Q}\rangle = \exp(-i\vec{q}\cdot\vec{r}_1 - i\vec{q}\cdot\vec{r}_2)|1s\rangle_1|1s\rangle_2\,. \qquad (1)$$

The DW factor is now $|\langle\vec{Q}|\Psi\rangle|^2$, the square of the elastic overlap amplitude, which has been evaluated with WS wave functions for a $1s$ state. Noting that the shakeoff probability to lowest order in q is $\frac{2}{3}\langle r^2\rangle q^2$, we adjusted the WS potential to reproduce the experimental root-mean-square radius of 7 fm for the halo and the corresponding binding energy of each neutron is 0.56 MeV.

The differential cross section for shakeoff, shown in Fig. 2(b), was calculated as the product of P_s and the differential cross section for elastic scattering from ^{9}Li. Its total amounts to 20 mb.

The energy spectrum for the shakeoff process is expected to show an asymmetric peak with a "tail" extending towards higher energies. It is possible to obtain an estimate of the average excitation energy from the ratio of the energy-weighted to the non-energy-weighted sum rule. The expectation value of the energy measured from the ^{11}Li ground state is $\langle\vec{Q}|H|\vec{Q}\rangle - E_0 = q^2/m$ so that it simply is increased by the amount that would be imparted classically to the two neutrons (an example of Ehrenfest's theorem). We now obtain the average excitation energy by dividing this result by the shakeoff probability, which in the perturbation limit as given above also scales as q^2. Therefore the average excitation energy in this limit is independent of momentum transfer and hence of the angle.

The result is $3\hbar^2/2m\langle r^2\rangle \approx 1.3$ MeV, in qualitative agreement with the observation. (The energy should actually be augmented by 20% from the recoil energy of the charged core; a contribution well below the accuracy of this estimate. More accurate estimates of P_s would furthermore make the average excitation energy increase slowly with angle.)

Unfortunately, it is not possible to draw more precise conclusions about the shape of the low-energy excitation spectrum from the results reported by Korsheninnikov *et al.* [7]. Although the spectra show data up to 20 MeV excitation energy, the acceptance in the missing-mass experiment drops rapidly [20] to very small values above roughly 6 MeV. The spectra reported in [6,7] give little information beyond this point.

We suspect that in many other reactions involving neutron halo nuclei, the low-energy neutrons observed are the result of shakeoff. Coulomb excitation to the continuum is a particularly good example of this. The interaction acts on the charged core alone so that we may identify the impulse $\vec{Q}$ of the present paper with the momentum transfer in a sudden Coulomb collision. This has been discussed for the cases of ^{11}Be [21] and ^{11}Li [22]. In both those cases, the energy spectrum was dependent purely on the properties of the initial state. There was no dependence on the momentum transfer, in agreement with our simple estimate based on sum rules. It is interesting that Pushkin *et al.* [22] used a three-body model with plane waves for the final states and found a spectrum that peaks at 1 MeV and decreases with a tail towards higher energies. From this calculated spectrum we estimate that roughly half of the intensity of the shakeoff cross section will fall inside the window attributed to the symmetric 1.3 MeV peak, and the corrected cross section is displayed in Fig. 2(b) by the dashed curve. A more detailed calculation by Esbensen and Bertsch [23] assumed that the neutron-^{9}Li interaction was dominated by a p state resonance at 0.8 MeV. They also found the main strength to be at a low energy. Also they used the ratio of sum rules to estimate the mean excitation energy to be near to 2 MeV and found that it was not very model dependent.

In conclusion, there does not seem to be any compelling evidence from the proton scattering experiments of Korsheninnikov *et al.* [6,7] for a 1.3 MeV excited level in ^{11}Li. Our interpretation of the observed structure as an effect of nuclear shakeoff is not in conflict with a shell model nor an alternative to it. Dynamic effects of this kind should appear naturally if the transition in the spectrum from bound to continuum states is included explicitly. The shakeoff effect should manifest itself in many other reactions involving halo nuclei. Nevertheless, for ^{11}Li, states are predicted in the shell model in the region 1–2 MeV. Such states may be observed and interpreted in single-nucleon transfer experiments on ^{12}Be.

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