

Nonlocal Effects on the Magnetic Penetration Depth in d -Wave Superconductors

Ioan Kosztin and Anthony J. Leggett

Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

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We show that, under certain conditions, the low temperature behavior of the magnetic penetration depth $\lambda(T)$ of a pure d -wave superconductor is determined by nonlocal electrodynamics and, contrary to the general belief, the deviation $\Delta\lambda(T) = \lambda(T) - \lambda(0)$ is proportional to T^2 and not T . We predict that the $\Delta\lambda(T) \propto T^2$ dependence, due to nonlocality, should be observable experimentally in nominally clean high- T_c superconductors below a crossover temperature $T^* = (\xi_0/\lambda_0)\Delta_0 \sim 1$ K. Possible complications due to impurities, surface quality, and crystal axes orientation are discussed. [S0031-9007(97)03539-4]

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There is a significant amount of experimental evidence that the pairing state in the cuprate high temperature superconductors (HTSC) is unconventional, most probably of $d_{x^2-y^2}$ symmetry [1–3]. One generic feature of a layered HTSC with any unconventional order parameter (OP) compatible with the underlying crystal symmetry is that the OP exhibits line nodes (point nodes in 2D) on the Fermi surface (FS) and, therefore, gapless quasiparticle excitations in the corresponding energy spectrum. These low-lying excitations dominate the low temperature thermodynamics and transport properties of these materials, and it is expected that the temperature dependence of the different thermodynamic quantities and transport coefficients will follow a power law rather than the conventional exponential behavior [4]. Direct experimental evidence for the existence of zeros of the gap function on the FS in HTSC has been found by angle resolved photo emission spectroscopy (ARPES) [5,6].

In particular, the low temperature behavior of the Meissner penetration depth $\lambda(T)$ is frequently regarded as an important probe of the morphology of the magnitude of the anisotropic OP, $\Delta(\hat{p})$, in the cuprates. In conventional s -wave superconductors, the deviation $\Delta\lambda(T)$ of $\lambda(T)$ from its zero temperature value $\lambda(0)$ exhibits activated behavior, i.e., $\Delta\lambda(T) \propto \exp(-\Delta/T)$ (throughout this paper we use units in which $k_B = \hbar = 1$), reflecting the existence of the isotropic BCS energy gap Δ at the FS. In contrast, in a pure d -wave superconductor, or any other unconventional superconductor with nodes in the gap, the London (local) penetration depth varies linearly with the temperature, i.e., $\Delta\lambda(T) \propto T$. Recently, by employing different high precision measurement techniques, such a linear T dependence of the in-plane $\Delta\lambda_{ab}(T)$ penetration depth (here the subscript refers to the axes along which the screening currents flow) has been observed experimentally in the Meissner state of several HTSC systems, such as: high quality single crystals of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO) [7–9] and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (BSCCO) [10–12], magnetically aligned powders of crystalline $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+\delta}$ [13] and high quality YBCO thin films [14–16]. However, below a certain sample dependent temperature T_{imp}^* the linear

T dependence of the penetration depth in HTSC crosses over into a higher power law, most probably T^2 . In the d -wave scenario of HTSC, the origin of the $\lambda(T) \propto T^2$ dependence has been explained by the presence of non-magnetic impurities which scatter in the unitary limit [17]. In this strong scattering limit a small amount of impurities can induce a finite residual density of states at the Fermi level which is sufficient to change the temperature dependence of the penetration depth from T to T^2 without lowering significantly the transition temperature. A direct experimental confirmation of such a crossover between pure and impurity dominated regimes was reported by Bonn *et al.* [18].

The purpose of this Letter is to show that at very low temperatures nonlocality may play an important role in the electromagnetic response of a d -wave superconductor (or any other unconventional superconductor with nodes in the gap), leading to a $\Delta\lambda(T) \propto T^2$ dependence even in the clean limit. Thus, besides impurities, nonlocality represents a second mechanism which leads to a T^2 dependence of the penetration depth sufficiently close to $T = 0$ K. To the best of our knowledge, all theoretical calculations and interpretations of the experimental measurements of the penetration depth in HTSC performed so far assume the validity of local electrodynamics (London limit) [19]. At first sight this is reasonable, since the zero temperature London penetration depth $\lambda_0 = \sqrt{mc^2/4\pi ne^2}$ is much larger than the corresponding coherence length ξ_0 in these materials. (In contrast to the penetration depth, which can be measured more or less directly, the coherence length ξ_0 cannot be determined experimentally and, in fact, is estimated in terms of the maximum value of the anisotropic gap function $\Delta_0 = \max\{\Delta(\hat{p})\}$ by using the usual BCS expression $\xi_0 = v_F/\pi\Delta_0$.) However, in the case of a clean, anisotropic superconductor it is more appropriate to introduce an anisotropic coherence length $\xi(\hat{p}) \equiv v_F/\pi|\Delta(\hat{p})|$. If the anisotropic OP has nodes on the FS, it is clear that sufficiently close to the nodes $\xi(\hat{p})/\lambda_0 = (\xi_0/\lambda_0)\Delta_0/|\Delta(\hat{p})| \geq 1$ holds and, therefore, the contribution of these regions of the FS to

the penetration depth $\lambda(T)$ must be determined by using nonlocal electrodynamics. The large value of λ_0/ξ_0 guarantees that the applicability of local electrodynamics is violated only on a very small fraction, of order $\alpha_0 \equiv \xi_0/\lambda_0$, of the FS. Since the whole FS contributes to the zero temperature penetration depth $\lambda(0)$, one expects no significant corrections to this quantity due to nonlocal effects. On the other hand, the low temperature dependence of $\lambda(T)$ must be dominated by nonlocal effects because this dependence is determined by a small region of the FS which is concentrated around the nodes of the OP. The crossover temperature below which nonlocal effects are important is given by $T^* = \alpha_0 \Delta_0$. Indeed, since the range of \hat{p} values corresponding to the thermally excited quasiparticles at a given temperature T is determined by the condition $|\Delta(\hat{p})| \leq T$, for $T < T^*$ one obtains $\xi(\hat{p})/\lambda_0 = \alpha_0 \Delta_0/|\Delta(\hat{p})| \geq T^*/T > 1$. For $T \gg T^*$ the local limit is applicable. As a typical example consider a YBCO single crystal with $\Delta_0 \approx 250$ K, $\xi_0 \approx 14$ Å, and $\lambda_0 \approx 1400$ Å; this yields $\alpha_0 \approx 10^{-2}$ and $T^* \approx 2.5$ K.

To demonstrate the effect of nonlocal electrodynamics on $\lambda(T)$ let us consider the case when a weak, uniform and static magnetic field $\mathbf{H} = \nabla \times \mathbf{A}$ is applied along the c axis of a semi-infinite HTSC with a plane surface which is perpendicular to the b axis. For this particular geometry, both the vector potential \mathbf{A} and the screening supercurrent density \mathbf{j} are oriented parallel to the a axis, while the direction of penetration is along the b axis. We model the HTSC as a quasi-two-dimensional d -wave superconductor in which the motion of the electrons is confined for our purposes to the Cu-O planes. In principle, to calculate $\lambda(T)$, one must first solve self-consistently the relevant Maxwell equation $\nabla \times \nabla \times \mathbf{A} = (4\pi/c)\mathbf{j}$ together with the equation which relates \mathbf{j} to \mathbf{A} , subject to some properly chosen boundary conditions. In a weak magnetic field (Meissner state) linear response theory yields for our geometry $j(y) = -\int dy' K(y, y') A(y')$, where y is the coordinate along the b axis ($y = 0$ gives the position of the boundary) and the nonlocal electromagnetic response kernel $K(y, y')$ must be calculated by using some microscopic theory. Once $H(y)$ is determined, the penetration depth can be calculated according to the standard definition (valid for a semi-infinite superconductor with plane boundary) $\lambda = H(0)^{-1} \int_0^\infty dy H(y)$. Furthermore, we assume that the boundary reflects the electrons either specularly or diffusively. In both these limiting cases $\lambda(T)$ can be expressed in terms of the Fourier transform of the bulk response kernel $K(q; T)$. For a specular boundary one has [20]

$$\frac{\lambda_{\text{spec}}(T)}{\lambda_0} = \frac{2}{\pi} \int_0^\infty \frac{d\tilde{q}}{\tilde{q}^2 + \tilde{K}(\tilde{q}; T)}, \quad (1)$$

while for a diffuse boundary [20]

$$\frac{\lambda_{\text{diff}}(T)}{\lambda_0} = \pi \left\{ \int_0^\infty d\tilde{q} \ln[1 + \tilde{K}(\tilde{q}; T)/\tilde{q}^2] \right\}^{-1}, \quad (2)$$

where the dimensionless quantities \tilde{q} and \tilde{K} are given by $\tilde{q} = q\lambda_0$ and $\tilde{K} = (4\pi\lambda_0^2/c)K$, respectively.

For a weak-coupling, anisotropic superconductor the nonlocal bulk response kernel is similar to the corresponding expression for a conventional s -wave superconductor [21] and can be written as

$$\tilde{K}(\tilde{q}; T) = 2\pi T \times \sum_{n=-\infty}^{\infty} \left\langle \hat{p}_{\parallel}^2 \frac{\Delta_p^2}{\sqrt{\omega_n^2 + \Delta_p^2}(\omega_n^2 + \Delta_p^2 + \alpha^2)} \right\rangle, \quad (3)$$

where ω_n are fermionic Matsubara frequencies, $\Delta_p \equiv \Delta(\hat{p})$, \hat{p}_{\parallel} is the projection of \hat{p} on the boundary, $\langle \dots \rangle$ means averaging over the circular 2D Fermi surface, and $\alpha = (qv_F/2)\hat{q}\hat{p}$. Here \hat{q} is a unit vector perpendicular to the boundary, and it gives the direction in which the penetration of the magnetic field takes place. Note that in a different geometry where the boundary is parallel to the a - b plane (\mathbf{H} parallel to the boundary), the direction of penetration \hat{q} would be along the c axis, i.e., perpendicular to \hat{p} , yielding $\alpha = 0$. Thus, we may conclude that the effect of nonlocal electrodynamics on $\lambda_{ab}(T)$ is relevant only when \mathbf{H} is parallel to the c axis. Furthermore, at sufficiently low temperatures, the OP in Eq. (3) can be approximated with its limiting expression close to the nodes, i.e., $\Delta_p = \Delta_0 \Phi(\hat{p}) \approx \Delta_0 \Phi'(0) \varphi$, where φ is the angular deviation of \hat{p} from the given node direction in the basal plane. In the case of a model d -wave OP with $\Phi(\hat{p}) = \hat{p}_x^2 - \hat{p}_y^2$ one has $\Phi'(0) = 2$.

Let us calculate first the nonlocal correction to the zero temperature penetration depth $\lambda(0)$. For $T = 0$ the frequency sum in (3) goes into an integral which can be evaluated exactly with the result

$$\tilde{K}(\tilde{q}; 0) = 1 - \left\langle 2\hat{p}_{\parallel}^2 \left[1 - \frac{\sinh^{-1}(\alpha/\Delta_p)}{(\alpha/\Delta_p)\sqrt{1 + (\alpha/\Delta_p)^2}} \right] \right\rangle. \quad (4)$$

The average over the FS in (4) can be evaluated analytically for both London (local) and Pippard (extreme nonlocal) limits. In the London limit, when $\alpha_0 \tilde{q} = q\xi_0 \ll 1$, one obtains $\tilde{K}(\tilde{q}; 0) = 1 - (\pi^2\sqrt{2}/16)\alpha_0 \tilde{q}$, while in the Pippard limit, when $\alpha_0 \tilde{q} \gg 1$, one has $\tilde{K}(\tilde{q}; 0) = (2/3) \ln(\alpha_0 \tilde{q})/(\alpha_0 \tilde{q})^2$. Note that in both limiting cases the response kernel for a d -wave superconductor decreases with \tilde{q} more rapidly than for a conventional s -wave superconductor [20]. Now the correction to the zero temperature penetration depth due to nonlocality can be obtained from Eqs. (1) and (2) for both specular and diffuse boundaries. The results are $\Delta\lambda_{\text{spec}}(0)/\lambda_0 = \lambda_{\text{spec}}(0)/\lambda_0 - 1 = \pi\sqrt{2}\alpha_0/16$, and $\Delta\lambda_{\text{diff}}(0)/\Delta\lambda_{\text{spec}}(0) = \ln(\alpha_0^{-2})/2 \approx 4.6$. Thus for both type of boundaries, due to the very small value of α_0 , the nonlocal correction to $\lambda(0)$ is less than 1% and therefore it can be obviously neglected, especially because this correction is situated within the experimental errors of the

most accurate measurements of the absolute value of the penetration depth.

We turn now to calculate $\Delta\lambda(T)$. At low temperatures, $\delta\tilde{K}(\tilde{q}; T) \equiv \tilde{K}(\tilde{q}; T) - \tilde{K}(\tilde{q}; 0)$ represents a small correction to the zero temperature response kernel $\tilde{K}(\tilde{q}; 0)$. Therefore, by using our previous result $\lambda(0) \approx \lambda_0$, from Eqs. (1) and (2) one obtains

$$\frac{\Delta\lambda_{\text{spec}}(T)}{\lambda_0} = \frac{2}{\pi} \int_0^\infty d\tilde{q} \frac{-\delta\tilde{K}(\tilde{q}; T)}{(\tilde{q}^2 + 1)^2} \quad (5)$$

and

$$\frac{\Delta\lambda_{\text{diff}}(T)}{\lambda_0} = \frac{1}{\pi} \int_0^\infty d\tilde{q} \frac{-\delta\tilde{K}(\tilde{q}; T)}{\tilde{q}^2 + 1}. \quad (6)$$

Furthermore, a convenient expression for $\delta\tilde{K}$ can be obtained by evaluating the Matsubara sum in Eq. (3) by means of complex contour integration

$$-\delta\tilde{K}(\tilde{q}; T) = 2 \int_0^\infty f(\omega) d\omega \times \left\langle 2\hat{p}_\parallel^2 \text{Re} \frac{\Delta_p^2}{\sqrt{\omega^2 - \Delta_p^2} (\Delta_p^2 + \alpha^2 - \omega^2)} \right\rangle. \quad (7)$$

Note that in the $\alpha \rightarrow 0$ limit one recovers the familiar local limit expression for $\delta\tilde{K}$ [17]. Because of the presence of the Fermi function $f(\omega)$ in (7) the main contribution to the frequency integral comes from the interval $\omega \lesssim T$. Therefore, in the average over the FS the relevant regions are determined by $|\Delta_p| \leq \omega \lesssim T$ and are obviously located around the nodes of the OP. By using the expression for the OP close to a node one arrives, after some straightforward algebra, at the following result:

$$\delta\tilde{K}(\tilde{q}; T) = \delta\tilde{K}(0; T) F(\tilde{q}/t), \quad (8)$$

where $t \equiv T/T^*$, $\delta\tilde{K}(0; T) = -2 \ln 2 T/\Delta_0$ is the well known local limit expression of $\delta\tilde{K}$ for a d -wave superconductor [1], the expression

$$F(z) = 1 - \frac{1}{\ln 2} \int_0^{(\pi\sqrt{2}/4)z} dx f(x) \sqrt{1 - \frac{8}{\pi^2} \left(\frac{x}{z}\right)^2} \quad (9)$$

is a universal function, and $f(x) = (e^x + 1)^{-1}$. It is remarkable that, within the above mentioned approximations, the kernel $\delta\tilde{K}$ depends only on the ratio \tilde{q}/t and not separately on its two arguments. A careful quantitative analysis of Eq. (9) motivates the following reasonable approximation: $F(z) \approx 1 - c_1 z$, for $z < 2$, and $F(z) \approx c_0/z^2$, for $z > 2$, where $c_0 = 6\zeta(3)/\pi^2 \ln 2 \approx 1.05$, and $c_1 = (1 - c_0/4)/2 \approx 0.37$. Note that c_1 is somewhat smaller than the absolute value of the slope of $F(z)$ at the origin, i.e., $|F'(0)| = \pi^2\sqrt{2}/32 \ln 2 \approx 0.63$. The temperature dependence of the penetration depth can now be calculated by inserting (8) in Eqs. (5) and (6).

For $t \gg 1$ (i.e., $T \gg T^*$) and for a specularly reflecting boundary one obtains $\Delta\lambda_{\text{spec}}(T)/\lambda_0 = \ln 2 (T/\Delta_0) - (\pi\sqrt{2}/16) \alpha_0 + \mathcal{O}(1/t)$. Here, the leading term is the well known linear in T local expression for $\Delta\lambda(T)$ for a d -wave superconductor, i.e., $\Delta\lambda_L(T) = \ln 2 (T/\Delta_0) \lambda_0$. The second, small negative constant term in the expression of $\Delta\lambda_{\text{spec}}(T)$ is due to nonlocality and shows clearly that the linear T dependence cannot extend all the way down to $T = 0$ K; it must cross over to a higher power law at some $T \sim T^*$. In the case of a diffuse boundary one obtains a similar result, namely, $\Delta\lambda_{\text{diff}}(T) = \Delta\lambda_L(T) - (\pi\sqrt{2}/16) \alpha_0 \lambda_0 \ln t + \mathcal{O}(1/t)$. Note that the magnitude of the nonlocal correction to the local penetration depth is larger than in the case of the specular boundary by a factor of $\ln t$.

In the opposite limit $t \ll 1$ (i.e., $T \ll T^*$) one obtains for a specular boundary $\Delta\lambda_{\text{spec}}(T) = \beta \Delta\lambda_L(T) T/T^* \propto T^2$, where $\beta = 8(1 - c_1 + c_0/4)/\pi \approx 2.2$. Thus, due to nonlocal electrodynamics, for $T \ll T^*$ the temperature dependence of a *pure* d -wave superconductor is proportional to T^2 and not T , regardless of how small is $\alpha_0 = \xi_0/\lambda_0$. This conclusion is one of the main results of the present paper. It should be noted that the above value for the coefficient β is just an approximation; a more accurate value of β can be obtained by approximating $F(z)$ by a polynomial of degree $N > 1$ for $z < z_0$, and by its large z asymptotic form for $z > z_0$, where z_0 is a conveniently chosen value. By reevaluating the integrals in $\Delta\lambda(T)$ one obtains again the leading term proportional to T^2 but with a slightly different numerical value for β . A similar calculation in the case of a diffuse boundary yields $\Delta\lambda_{\text{diff}}(T) \approx \Delta\lambda_{\text{spec}}(T)/2$.

For arbitrary temperatures $\Delta\lambda(T)$ must be calculated numerically by employing the exact expression (9) for the function $F(z)$. In Fig. 1(a) the ratio $\Delta\lambda(T)/\Delta\lambda_L(T)$ is plotted, for both specular and diffuse boundaries, as

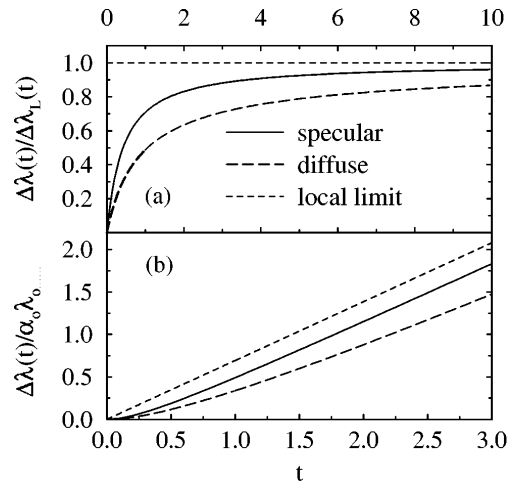


FIG. 1. Plot of $\Delta\lambda(T)$ [in units (a) $\Delta\lambda_L(T)$, and (b) $\alpha_0 \lambda_0$, respectively] vs $t = T/T^*$ for both specular (solid line) and diffuse (long-dashed line) boundary. For comparison, the local limit result is also shown (dashed line).

a function of the reduced temperature t . The deviation from the standard result obtained in the local limit is evident. The clear linear dependence in the vicinity of the origin indicates a quadratic T dependence of $\Delta\lambda(T)$. For $t \gg 1$, $\Delta\lambda(T)$ approaches asymptotically its local limit (minus a small constant correction of order α_0). Note that the deviation of $\Delta\lambda(T)$ from the corresponding local expression is much more pronounced for a diffuse boundary than for a specular one. The same $\Delta\lambda(T)$, this time in units of $\alpha_0\lambda_0$, is shown as a function of $t = T/T^*$ in Fig. 1(b). The deviation from linearity becomes visible around $t = 1$ ($t = 2$) for the specular (diffuse) boundary. For the numerical example considered above for a clean YBCO single crystal ($\alpha_0\lambda_0 = \xi_0 \approx 14 \text{ \AA}$) one finds that the deviation from $\Delta\lambda(T) \propto T$ takes place somewhere between 2 to 5 K, depending on the surface quality of the crystal. Such a crossover is seen experimentally in nominally clean YBCO crystals [22]; in the existing literature it has been attributed to impurities which scatter in the unitary limit [17].

In principle, there is a simple experimental test to determine whether this crossover in $\Delta\lambda(T)$ is due to nonlocal electrodynamics or to impurities. The main idea is to estimate experimentally the crossover temperature in $\Delta\lambda_{ab}(T)$ for the same nominally clean HTSC for two different magnetic field orientations: (i) \mathbf{H} parallel to the c axis, and (ii) \mathbf{H} parallel to the a - b plane. As we have already mentioned, nonlocality is expected to be relevant only when the applied magnetic field is oriented parallel to the c axis (so that the penetration direction lies in the a - b plane), while the effect of impurities should not depend on the orientation of the field. Thus, if T^* is noticeably smaller in case (ii) than in case (i) one may conclude that the observed effect is mainly due to nonlocal electrodynamics and not to impurities. Otherwise, the conclusion is that nonlocal effects are in fact completely masked by impurities. The suggested experiment can be made even more conclusive by using two different nominally clean YBCO samples with the relevant faces having: (a) (1,0,0) orientation (i.e., the geometry considered in this paper), and (b) (1,1,0) orientation, respectively. It can be shown that by moving away from the most favorable (1,0,0) surface orientation, the nonlocal effects become less important, and they eventually vanish for the (1,1,0) orientation [23]. Thus, if the electromagnetic response of the cuprates is indeed governed by nonlocal effects at very low temperatures, in case (a) one would expect a noticeable difference between the T dependence of λ_{ab} for the two different field orientations, while in case (b) the orientation of the applied magnetic field should be completely irrelevant.

In conclusion, we have shown that nonlocal electrodynamics dominate the low temperature behavior of the in-plane magnetic penetration depth of a clean d -wave

high- T_c superconductor. At temperatures $T \ll T^* \sim 1 \text{ K}$ the penetration depth $\lambda(T)$ has a quadratic temperature dependence, while above the crossover temperature T^* , but still well below T_c , $\lambda(T)$ has the well known linear T dependence. Thus, nonlocality represents a second possible mechanism, beside strongly scattering impurities, which may account for the experimentally observed deviation from the linear T dependence of the penetration depth at the lowest measured temperatures in nominally clean HTSC. A simple experiment to probe the viability of this mechanism has been proposed.

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