Wave Function Scarring Effects in Open Stadium Shaped Quantum Dots

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(Received 14 November 1996)

In agreement with experiment, our calculations show that the low field magnetoconductance of stadium shaped quantum dots can be periodic, indicating that only a few regular orbits dominate the quantum transport, even though the structure is classically chaotic. Evidence for these orbits is seen in scarred wave functions that recur periodically in correspondence to selected peaks in the power spectrum. Crucial in exciting these orbits is the quantization of modes in the quantum point contacts that cause the electrons to be injected in collimated beams at well defined angles. [S0031-9007(97)03512-6]

PACS numbers: 73.23.Ad, 05.40.+j, 05.45.+b

Electron interference is an important process in mesoscopic devices and strongly influences their resulting electrical behavior. In disordered systems, electrons are diffusively scattered by a random impurity distribution and the resulting interference gives rise to universal conductance fluctuations (UCF) [1]. Such fluctuations are also observed in ballistic quantum dots, where the complex electron scattering is instead generated by the confining geometry [2–5]. If the mean free path and phase coherence length are larger than the dot dimensions, then transport should be dominated by the resolvable quantum spectrum, even in the presence of current flow [6–8]. The transport can be expected to involve just a few eigenstates that are excited by the collimation effect of the entrance quantum point contact (QPC). In contrast to UCF [1], in which the fluctuations are aperiodic, and to the semiclassical treatment of classically chaotic dot shapes, in which the assumption of ergodicity is made [9], this behavior leads to reproducible, periodic fluctuations, indicative of a *nonuniform* excitation of classical trajectories. The experimental results of Marcus *et al.* [2] for stadium shaped quantum dots showed this periodic behavior when no ensemble averaging was done and Fourier analysis of the conductance fluctuations revealed the presence of strong peaks at a few discrete frequencies. It was speculated that these resulted from wave function scaring by a few periodic orbits.

In this Letter, we simulate magnetotransport through stadium shaped, ballistic quantum dots and compute the wave functions. This is done by solving the problem on a discrete lattice using an iterative method that is a numerically stabilized variant of the transfer matrix approach [10,11]. The dot is enclosed inside a waveguide which extends a finite number of lattice sites in the transverse (y) direction. The structure is then broken down into a series of slices along the longitudinal (x) direction. Imposing an electron flux from the left, one translates across successive slices using the iterative method. On reaching the end, one obtains the transmission coefficients which enter the Landauer-Büttiker formula [11] to give the conductance. Consequently, we are able to reproduce the periodicity of the conductance fluctuations found experimentally [2]. Moreover, the wave functions can be scarred, with certain scars recurring periodically in field, in good correspondence with selected power spectrum peaks. Further investigations reveal that collimation plays a crucial role in the nonuniform excitation of orbits.

In Fig. 1(a), we plot the conductance fluctuations, δg vs perpendicular field B for a stadium shaped dot, with the same lead configuration used in experiment [2] [Figs. 1(b) and 1(c)]. Following Marcus *et al.* [2], the fluctuations themselves are obtained from the raw conductance by subtracting out a cubic polynomial background and we have used an electron density 3.6×10^{11} cm^{-2}. The radius of the end circles of the stadium is $R = 0.22$ μm and the total length is 0.88 μm, yielding an area of 0.35 μm^2. While smaller than the experimental estimate [2] by some 10%, we find that this size gives the best agreement and emphasize that there is always uncertainty in determining the experimental dot size, since it must be inferred indirectly from high magnetic field measurements of the dot conductance. The input QPC or lead allows four propagating modes to enter the dot. The resulting fluctuations have a quasiperiodic appearance, particularly if one looks at the spacing of successive minima. This characteristic is confirmed by the power spectrum of the fluctuations, which reveals well defined peaks at $f \sim$ 20, 33, and 65 T^{-1} [Fig. 1(a), right inset]. Compared to the experimental results [Fig. 1(a), left insert] this spectral content is seen to be strikingly similar. Making minor structural alternations (e.g., narrowing the QPCs), we find that the relative weighting of the peaks can be significantly altered but that their *positions* are quite stable, which is again consistent with the experimental results [2]

As noted above, Marcus *et al.* [2] suggested that these peaks may be associated with scarring. We have indeed found evidence of scarring in our simulations and

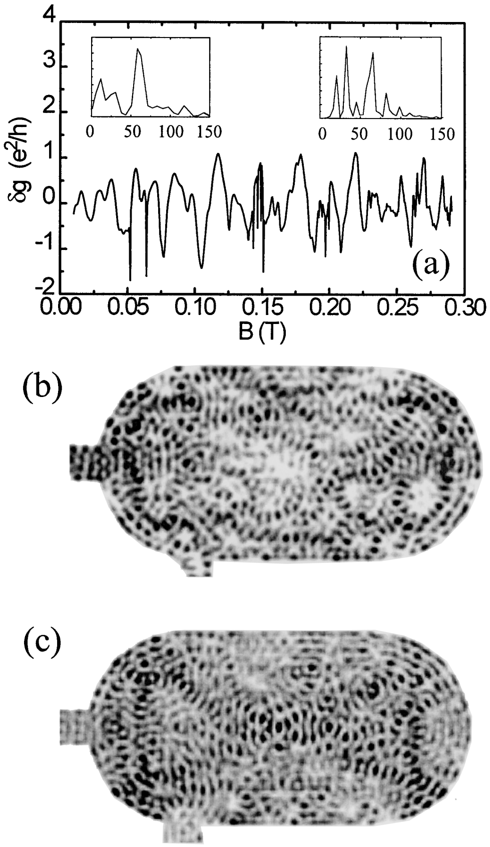

FIG. 1. (a) Conductance fluctuations (δg) are plotted as a function of magnetic field for a stadium shaped quantum dot. The right inset shows the corresponding power spectrum, in arbitrary units. For comparison, the left inset shows a power spectrum obtained from the stadium shaped dot of Ref. [2]. Probability density ($|\psi(x,y)|^2$) versus x and y is plotted for (b) $B = 0.229$ T and (c) $B = 0.2515$ T. Darker shading corresponds to higher amplitude. Note the positions of the leads.

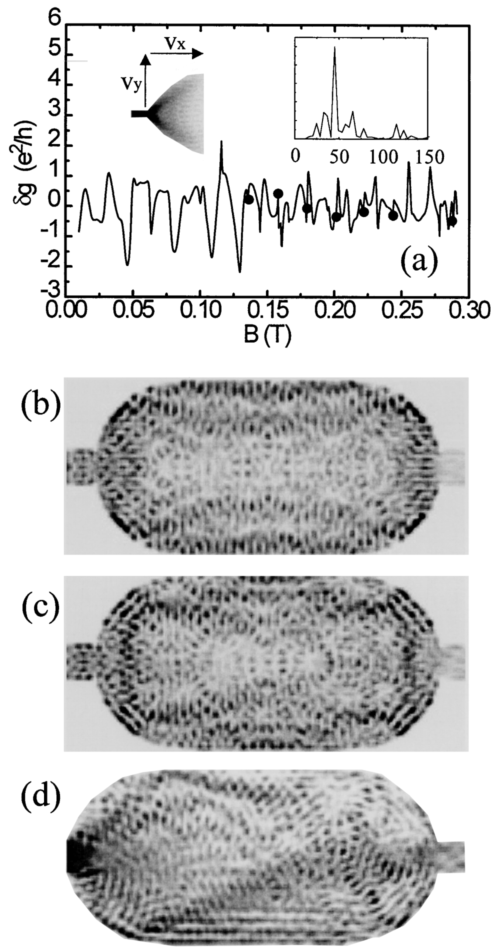

FIG. 2. (a) Conductance fluctuations (δg) are plotted as a function of magnetic field for a stadium quantum dot with aligned leads. The right inset shows the corresponding power spectrum and the left inset the collimated beams emerging from a QPC. ($|\psi(x,y)|^2$) versus x and y is plotted for (b) $B = 0.135$ T and (c) $B = 0.288$ T, showing a rectangular scar in both cases. (d) also corresponds to $B = 0.288$ T, but now phase breaking is introduced to isolate the effects of the input lead.

show two representative examples. In Fig. 1(b), we plot $|\psi(x,y)|^2$ vs x and y for $B = 0.2515$ T. A roughly rectangular scar is evident, with the corners and ends being clearly resolved. In Fig. 1(c), a stronger "bow-tie" scar is apparent at $B = 0.229$ T and is strikingly similar to a scar observed in the *closed* stadium (see Fig. 2 of Ref. [12]). Unfortunately, it is difficult to establish a specific period to these scars and thus establish a connection with a particular power spectrum peak since similar looking scars can be resolved only at a few other values of field.

Much stronger evidence for the association between a specific power spectrum peak and scarring is provided in Fig. 2, which shows results for a slightly smaller stadium with centrally *aligned* leads. Here, $R = 0.2$ μm and the total length is 0.8 μm, yielding an area of 0.29 μm^2. The density in this case is 4×10^{11} cm^{-2}. In Fig. 2(a), we plot the conductance fluctuations for this configuration and the corresponding power spectrum is shown in the inset. A single peak at 47 T^{-1} clearly dominates. The corresponding magnetic field period is 0.021 T, in good agreement with the spacings between successive minima in the fluctuations. The solid circles correspond to values of B where a roughly rectangular scar was observed in the wave function. These appear at very nearly periodic intervals, with the period also being 0.021 T, though one period is missed (a scar is expected at $B \sim 0.26$ T). In Figs. 2(b) and 2(c), we plot $|\psi(x,y)|^2$ vs x and y for the cases corresponding to the first and last dots, $B = 0.1355$ and 0.288 T, respectively. Both wave functions reveal similar looking but not identical scars [Fig. 2(c) clearly shows extra "bounces"], following trajectories running along the periphery of the stadium. We return to this point below.

Insight into the formation of the scars can be obtained by considering the left hand inset to Fig. 2(a), which shows a simulation of an isolated QPC that supports a single mode. Given an opening of width w, this mode has a quantized transverse wave number, $k_{1y} = \pi/w$ at zero field. With scattering boundaries absent, two collimated beams can be clearly seen exiting the QPC, with exit angles set by the transverse velocities, $v_y = \pm\hbar k_{1y}$. It is beams such as these that excite the particular periodic orbits. Isolating this collimation effect inside the actual dot can be done phenomenologically by introducing an imaginary potential $V_{\mathrm{in}} = -i\hbar/2\tau_\phi$, where τ_ϕ is the inelastic scattering time [13]. One introduces a large enough τ_ϕ so that the inelastic path length $l_\phi = v_F\tau_\phi$, where v_F is the Fermi velocity, is about the length of the stadium, so that the electron waves are damped before multiply reflecting off the boundaries. The wave function corresponding to such a calculation is shown in Fig. 2(d), which was done for the same value of B as Fig. 2(c). Here we have used $\tau_\phi = 5$ ps, which corresponds to an $l_\phi = 1.4$ μm. Note the tendency for the electrons to enter the dot at very sharp angles with respect to the axis formed by the aligned leads, behavior similar to the QPC shown in the inset. The first few reflections of the electron waves are also evident, with one beam bouncing off the bottom and appearing to exit the dot, and the second taking a more peripheral trajectory more in keeping with the scar. The differences in the trajectories are accounted for by the Lorentz force on the electrons generated by the magnetic field. Importantly, the "rectangular" scar shown in Fig. 2(c) only becomes truly visible above $\tau_\phi \sim 0.05$ ns. This corresponds to an inelastic path length $l_\phi = \nu_F\tau_\phi$ of about 14 μm, sufficient to make about six complete circuits of the scarred loop, demonstrating the need for long term electrons storage to permit the observation of well defined scars. This result and the clear differences between the scars in Figs. 2(b) and 2(c) are consistent with an observation of Heller *et al.* [14], who found that, in closed stadiums, there is a difficulty in making a correspondence between a scarred wave function and a *specific* orbit, as each simple orbit has a large number of similar but more complex "cousins," consisting of progressively more bounces, only matching up with themselves after several trips around the stadium. Thus, the observed periodic scarring may be the result of a whole series of such cousins.

The scaring features discussed thus far all occurred above $B = 0.1$ T. One might argue that we have crossed over into the regular regime, as a transition from chaotic to regular behavior occurs classically as the magnetic field is increased. However, using the formula $B^* = \phi_0/(\rho_{\min}\lambda_F)$, where $\rho_{\min}$ is the smallest radius of curvature of the device and λ_F is the Fermi wavelength, Marcus *et al.* [2] estimated this to occur at $B^* = 0.45$ T, for the stadium dots he examined. For the example used in Fig. 2, $B^* = 0.48$ T, since the dot is smaller and λ_F is larger. Thus, with this criteria, the scarring effects we have shown are at fields in which the classical scattering is expected to be chaotic.

We have also seen other scars familiar from the literature on closed stadiums. Examples for the aligned lead case are shown in Fig. 3(a), in which the classic "bouncing-ball" scar appears quite strongly at $B = 0.145$ T, and in Fig. 3(b), which shows a double V (compare this with Fig. 2 of Ref. [12]) for $B = 0.13$ T. As mentioned above, some scars do recur, while many others do not. When there is recurrence, typically it is not easy to assign a periodicity (if it in fact exists) since the scars do not recur with the same intensity each time and some periods are apparently missed. In this regard, we note that the energy levels are broadened in *open* dots, so there is a question of whether the scars correspond to eigenstates of particular energy levels, or averages over broadened and overlapping levels. It is difficult to answer this question conclusively. It is possible that some levels are still being resolved since the level spacing is not even and indeed all the scars shown here correspond to points in the conductance where there was some sharp resonance feature. Thus, the missing of periods may be due to uneven broadening effects, which smear some resonances but not others. That being said, we have found that narrowing the leads and, thus, reducing the expected broadening, does not necessarily give sharper scars. We also point out that while, for example, the scarred stadium wave functions plotted by Heller *et al.* [14] do correspond to individual eigenstates, Bogomolny [15] has shown that scarring can be a property of *energy averaged* wave functions. The effects of magnetic field on the energy level spacing has been studied in the context of a closed stadium [16].

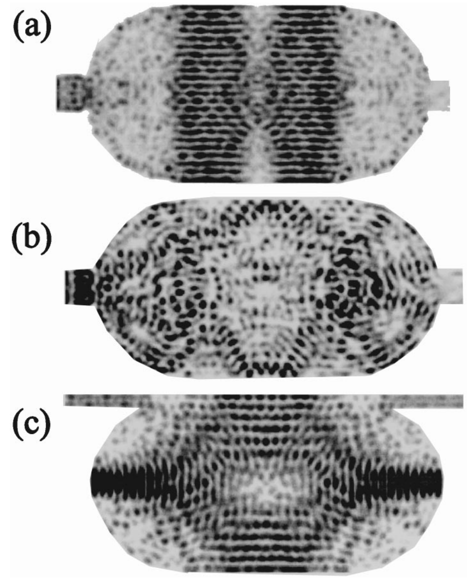

FIG. 3. (a) The bouncing-ball scar. (b) The "double-V" scar. (c) A "diamond" scar that occurs when the leads are placed at the top edges of the stadium.

In Fig. 3(c), a diamond-shaped scar is strongly apparent at $B = 0.2095$ T. The parameters are the same as in Fig. 2 but now the leads are aligned at the *top* of the stadium, and two modes propagate in the leads. This example underlines the importance of lead *placement* in determining which orbits are excited. Here a collimated beam pointed *downward* (and upward modes are reflected) is what generates the diamond scar. We note also that the position of the output lead is also of great importance. There are scars apparent in the aligned case (Fig. 2) that *do not appear* in the nonaligned lead case (Fig. 1) and vice versa. This may be due to the exit leads allowing the electron to escape too quickly before the wave function is built enough to show a particular scar.

Analogous effects to those shown here have been seen in experimental [5,17] and theoretical [5,11] studies of square quantum dots. As in the stadium, the calculations reveal "scarlike" wave functions, with periodicities in correspondence to the power spectrum peaks. Similar wave functions have also been seen in circular dots [18]. Thus, while the individual power spectra depend on specific details such as the dot shape and lead position, we conclude that the regular behavior is in fact *universal*. Moreover, it has been found that the positions of the peaks scale inversely with a dot's linear dimension [18], indicating the period is actually related to the length of the orbit. From this and noting that scars like the bow tie and double V reoccur but do not appear to enclose any Ahoronov-Bohm flux (there is flux cancellation in the case of the bow tie), it seems it is the symplectic area, which is swept out as the electrons make several circuits around the dot, that is most relevant with regards to the period and not the simple area apparent inside the scars.

In conclusion, we have shown that the low field magnetoconductance of open stadium shaped quantum dots can be periodic, indicating that only a few regular orbits dominate the quantum transport, even though the structure is classically chaotic. Evidence for these orbits is seen in scarred wave functions, which recur periodically in magnetic field in correspondence with selected peaks observed in the power spectra of the experimental and simulated magnetoconductance. Crucial in exciting these orbits is the quantization of modes in the QPCs, which inject electrons in collimated beams directed at well defined angles.

The authors express their appreciation to Professor C. M. Marcus for allowing them to use his data in this paper. The work at ASU was supported in part by DARPA.

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