Comment on "Anomalous Hall Effect in $YBa_2Cu_3O_7$"

In a recent Letter Stojković and Pines (SP) presented a model addressing the anomalous behavior of the Hall effect in $YBa_2Cu_3O_{7-\delta}$ (YBCO) [1]. We wish to comment on the extent to which their model describes the relevant experiments.

It is convenient to summarize the Hall problem in cuprates as follows. (I) In almost all the hole cuprates (close to optimum doping) the Hall resistivity ρ_H varies nominally with temperature (T) as $1/T$ up to 500 K, or higher for some systems. (II) The in-plane resistivity ρ displays a dependence that is remarkably linear in T to 320 K (untwinned YBCO) [2] and 1000 K (LaSrCuO) [3]. (III) The Hall conductivity σ_H varies as [4] $1/T^3$. (IV) In the hole-type cuprates, plots of $\cot\Theta_H$ (Hall angle) vs T^2 provide strong evidence [5,6] for an additional time scale τ_H that varies as $1/T^2$, as predicted [7].

While the $\cot\Theta_H$ behavior uncovers an important aspect of the Hall problem, we remark that the central anomaly resides in the directly measured quantities ρ_H and ρ (the other two, σ_H and $\cot\Theta_H$, are derived). A model that gives a T^2 dependence for $\cot\Theta_H$ but ignores the problematic behavior of ρ_H and ρ hardly constitutes a solution. As a simple example, we consider a conventional 2D metal in which $\tau(\mathbf{k})$ varies as $1/T^2$ everywhere on the Fermi surface (FS). Clearly, $\cot\Theta_H$ goes as T^2, but σ_H and ρ will vary as $1/T^4$ and T^2, respectively, in disagreement with (III) and (II). Worst of all, the Hall resistivity ρ_H in this model is T independent.

In the model of SP, the lifetime varies (essentially) as T^{-2} at the corners of the FS, but as $\sim 1/T$ over the flat portions (hot spots). We first consider their resistivity. The assumption that $\tau(\mathbf{k}) \sim T^{-2}$ over a significant fraction of the FS should have observable consequences. Indeed we find that the calculated ρ (at 0.25 doping) in Fig. 1 of SP is well described by $T^{1.8}$ below 300 K [see also their Fig. 1(b)]. Plots of $(\rho - \rho_0)/T$ vs T [Fig. 1(a) of SP] may be made to appear flat by the expedience of taking an impurity term ρ_0 that is *negative*. Experimentally, however, the derivative $d\rho/dT$ is unambiguous. Measurements show that $d\rho/dT$ is almost independent of T up to 500 K (for YBCO [4]) and 1000 K (optimally doped LaSrCuO [3]). Thus an exponent of 1.8 may be excluded in optimum, as well as slightly underdoped, cuprates.

Next we consider the Hall response. In SP's Fig. 2, the strong T dependence of σ_H tends to deemphasize the important disagreement from the displayed data (note that σ_H varies as T^{-4} in the isotropic-τ model discussed above). Our fits to the Hall data of Ginsberg *et al.* shows that $\sigma_H \sim T^{-3.08}$ (similar to Chien *et al.* [4]), whereas σ_H, as calculated by SP, varies as $T^{-3.6}$. This difference is important when the Hall resistivity is computed. Measurements [6] of ρ_H in YBCO with oxygen content in the range 6.7 to 7.0 show strong temperature dependence that becomes *steeper* with decreasing T. In contrast, SP's curves for ρ_H are weakly T dependent especially below 200 K (inset in their Fig. 2). While acknowledging the poor agreement, SP also sidestep the difficulty by invoking the effects of CuO chains and in-plane mass anisotropy. Since the point of the paper was to explain the anomalous Hall effect, we are left baffled by the intent of the calculation. We remark that similar, steep variation of ρ_H is observed in HgBaCuO [8] and optimum LaSrCuO [9] where chains are not an issue.

A simple argument shows why SP's model reproduces the right behavior for $\cot\Theta_H$, but the wrong behavior for ρ and ρ_H. At very low temperatures the FS corners must dominate the current, so $\rho \rightarrow T^2$. In the intermediate internal 120 to 300 K, we expect $\rho \sim T^\alpha$, with α just slightly less than 2. In 2D, a geometric argument strongly constrains the weak-field σ_H, regardless of the form for $\tau(\mathbf{k})$. σ_H equals the directed area swept out by the mean-free-path vector $\vec{\ell}(\mathbf{k})$ as $\mathbf{k}$ goes around the FS [10]. In SP's FS, the swept area is always dominated by contributions from the corners where $\tau(\mathbf{k})$ varies as $1/T^2$ (the flat segments contribute very little). Therefore σ_H varies as $T^{-\beta}$, with β just slightly less than 4. Thus $\cot\Theta_H = \sigma_H \rho$ varies as T^γ, with γ close to 2. (Fits to SP's curves between 120 and 300 K give $\alpha = 1.8$ and $\beta = 3.6$, so that $\gamma = 1.8$, consistent with their Fig. 3.) However, $\rho_H = \sigma_H \rho^2$ varies as $T^{2\alpha-\beta} \sim T^0$. The near cancellation remains even if α and β are both slightly less than 2.

The effects of strong correlation on the normal-state transport are now well documented. SP's model provides yet another demonstration of the inadequacy of the Bloch-Boltzmann approach in describing transport experiments on the cuprates.

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Received 12 June 1996 [S0031-9007(96)02191-6]
PACS numbers: 74.25.Fy, 74.25.Ha, 74.72.Bk

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