Direction of Optical Energy Flow in a Transverse Magnetic Field

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In this Letter, we report a theoretical and experimental study of the direction of optical energy flow in homogeneous media subject to a transverse magnetic field. For transparent media we verify experimentally for the first time the existence of magnetic deflection of circularly polarized light. In absorbing media the calculated directions of the Poynting vector and of a wave packet do not coincide. Experimentally we demonstrate that the Poynting vector result is not correct. [S0031-9007(97)02307-7]

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Recently, it was shown both theoretically [1] and experimentally [2] that light propagating in a scattering medium subject to a transverse magnetic field is deflected in a direction perpendicular to both the incident light beam and the magnetic field. This new effect bears a phenomenological resemblance to the electronic Hall effect, where the Lorentz force causes a deflection of the electron trajectories. The "photonic Hall effect" finds its origin in the magnetically induced change of the optical properties of the medium. This mechanism is not limited to scattering media, and therefore the question naturally arises whether light propagating in nonscattering, homogeneous media is also deflected by a static transverse magnetic field. Some aspects of deflection of light by a magnetic field have already been discussed by Landau, Lifshitz, and Pitaevskii [3]. Those effects have been estimated to be very small [4], which may explain why, to our knowledge and surprise, they have never been observed experimentally. A different kind of magnetodeflection has been reported in absorbing homogeneous media [5]. The question whether light is bent by magnets in homogeneous media has recently been raised again by 't Hooft and van der Mark [6], in reviewing our work on scattering media [2]. This topic is part of a much broader discussion on the properties of macroscopic electromagnetic fields inside dielectrics that, despite its long history, still yields new results [7,8]. In this Letter we study the direction of optical energy flow in a homogeneous medium subject to a transverse magnetic field, both theoretically and experimentally.

Arguments based on energy conservation and macroscopic Maxwell equations show that the energy flow of an electromagnetic wave in media with a local response is given by

$$\tilde{\mathbf{S}} \equiv \frac{c_0}{4\pi} \mathbf{E} \times \mathbf{H} + \nabla \times \mathbf{T}, \tag{1}$$

where $\mathbf{T}$ is some vector field, c_0 is the vacuum velocity of light, and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of the wave [9]. The ambiguity in the energy flow, reflected by the existence of the second term in Eq. (1), is well appreciated in text books on electrodynamics [9–12], but is always discarded, leading to the standard form for the energy flow, the so-called Poynting vector $\mathbf{S}$, which reads, after cycle averaging [13],

$$\mathbf{S} = \frac{c_0}{8\pi} \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*). \tag{2}$$

Justifications of this choice are the consistency with all experimental observations so far and with theorems for energy flow under restricted conditions [3,14]. Only in media with nonlocal optical response (i.e., spatial dispersion) an extra material contribution in Eq. (2) arises [3,15]. For media with *local* optical response, we will present the first theoretical and experimental evidence that the choice made in Eq. (2) is not generally correct.

The direction of wave propagation can be unequivocally established by following the propagation of a wave packet in space and time. In general this requires considerable computational effort, but the situation can be greatly simplified by considering a wave packet of sufficiently narrow frequency spread in a homogeneous medium without absorption. In this case distortion of the wave packet form can be neglected. Such media are characterized by a real-valued dispersion law $\omega(\mathbf{k})$. If the initial wave packet amplitude is denoted by $\Psi_0(\mathbf{r})$, the evolution of the density $|\Psi(\mathbf{r},t)|^2$ can be shown to be given by [3,9,11] $|\Psi(\mathbf{r},t)|^2 \approx |\Psi_0(\mathbf{r} - \mathbf{v}_g t, 0)|^2$, which leads to the well-known conclusion that the velocity of energy flow, including its direction, is given by the group velocity $\mathbf{v}_g \equiv \nabla_{\mathbf{k}} \omega(\mathbf{k})$. Under the same conditions of homogeneity and transparency, the directions of the Poynting vector and the group velocity can be shown to coincide [3,14]. This still holds true in anisotropic media, where the direction of energy flow or the Poynting vector will in general not be parallel to the wave vector. Also in inhomogeneous but periodic media, group velocity and energy flow of a Bloch wave packet are parallel [16].

No such clear relation between Poynting vector and group velocity has been obtained when the medium is either disordered or absorbing. Nevertheless both quantities are widely assumed to give the direction of energy flow and therefore to be parallel. Absorbing media can still be described by a dispersion law, albeit complex valued. This means that the group velocity is still defined and has become complex valued as well. By choosing a Gaussian wave packet for $\Psi_0(\mathbf{r})$ one can establish that the propagation direction of the maximum of the energy density is determined by the *real* part of the group velocity. Inhomogeneous, strongly scattering media can be regarded as absorbing media as long as the ensemble-averaged amplitude is considered; the relevance of the group velocity for the coherent wave has been demonstrated experimentally for light in absorbing, dispersive media [17,18] and recently for acoustic waves in strongly scattering media [19].

In this paper we consider the direction of electromagnetic energy flow in a local, isotropic, homogeneous medium. The linear effect of a static magnetic field $\mathbf{B}$ on the optical properties of such a medium is described by the dielectric tensor $\varepsilon_{ij}(\mathbf{B})$ [3],

$$\varepsilon_{ij}(\mathbf{B}) = n^2\delta_{ij} + i\gamma\epsilon_{ijk}B_k\,, \tag{3}$$

where n is the complex refractive index of the medium, $\mathrm{Re}\gamma$ determines the strength of magnetic circular birefringence (the Faraday effect, a difference in refractive index for left- and right-handed circularly polarized light), and $\mathrm{Im}\gamma$ is that of magnetic circular dichroism—MCD—(a difference in absorption α for left- and right-handed circularly polarized light) [3]. The dispersion relation for the two circularly polarized eigenmodes can be determined from the Helmholtz equation and reads

$$\omega_\pm(\mathbf{k}) = \frac{kc_0}{n} \pm \frac{\gamma c_0}{2n^3}\mathbf{B}\cdot\mathbf{k}\,. \tag{4}$$

For simplicity we leave aside the interesting complication that n and γ may depend on frequency. The resulting group velocity reads

$$\mathbf{v}_g^\pm = \frac{c_0}{n}\hat{\mathbf{k}} \pm \frac{\gamma c_0}{2n^3}\mathbf{B}\,. \tag{5}$$

On the basis of this equation, the optical energy flow can be deflected by a magnetic field, as predicted in Ref. [3] (section 101) [4]. We note that the deflection is only in the direction of the magnetic field, and no magnetotransverse term, perpendicular to both magnetic field and wave vector, is present.

Using Maxwell's equations, we can calculated the Poynting vector, as defined in Eq. (2), for circularly polarized light. For a plane wave one readily finds

$$\mathbf{S}_\pm(\mathbf{B}) \propto \mathrm{Re}n\hat{\mathbf{k}} \pm \frac{1}{2}\mathrm{Re}\frac{\gamma\overline{n}}{n^2}\mathbf{B} + \frac{1}{2}\mathrm{Im}\frac{\gamma\overline{n}}{n^2}\hat{\mathbf{k}}\times\mathbf{B}\,. \tag{6}$$

Upon comparing the real part of the group velocity in Eq. (5) with the Poynting vector in Eq. (6), one finds that without absorption (n and γ real valued) or without magnetic field $\mathbf{B}$ the two are parallel. However, a nonzero value for $\mathrm{Im}n$ or $\mathrm{Im}\gamma$ will cause differences between the directions of Poynting vector and group velocity in the presence of a magnetic field. In particular, a magnetotransverse term appears. Later, we will consider whether these discrepancies can be removed by an appropriate choice of $\mathbf{T}$ in the more general formulation of energy flow as given by Eq. (1).

In a magnetic field, linearly polarized waves are not eigenmodes of the system. Therefore a group velocity cannot be defined, and one has to resort to the full wave packet approach. The details of this lengthy but straightforward calculation will not be given. The main result is that, when calculating the propagation of the energy density $U(\mathbf{r},t) = (\mathbf{E}\cdot\mathbf{D} + \mathbf{H}^2)/4\pi$ of a linearly polarized Gaussian wave packet in a homogeneous, absorbing medium subject to a transverse magnetic field, we find no deflection, i.e., the direction of energy flow is parallel to the central $\mathbf{k}$ vector of the packet, independent of magnetic field.

For a linearly polarized plane wave, where ϕ is the angle between polarization vector and magnetic field $\mathbf{B}$, one finds from Maxwell's equations for the direction of the Poynting vector,

$$\mathbf{S}(\phi,\mathbf{B}) \propto \mathrm{Re}n\hat{\mathbf{k}} + \frac{1}{2}\mathrm{Im}\frac{\gamma\overline{n}}{n^2}[\sin 2\phi\mathbf{B} + (1-\cos 2\phi)\hat{\mathbf{k}}\times\mathbf{B}]\,. \tag{7}$$

This result shows magnetic deflection both in the directions of $\mathbf{B}$ and $\mathbf{k}\times\mathbf{B}$ in the presence of absorption, but contrary to the case of circularly polarized light, both terms are proportional to $\mathrm{Im}\gamma\overline{n}/n^2$, i.e., deflection is only predicted in absorbing media. As a special case, we recover the deflection term proportional to $\mathrm{Im}\gamma(\hat{\mathbf{k}}\times\mathbf{B})$ that was discussed by Schlesser and Weis [5]. Again, we establish that the direction of the Poynting vector is not consistent with the wave packet analysis in the presence of both absorption *and* a magnetic field.

To experimentally test the various predictions for the direction of energy flow we have determined the deflection of light upon propagation in several homogeneous dielectrics in a transverse magnetic field. The setup is shown schematically in Fig. 1. A light beam of a given polarization state is normally incident on the sample, placed in a transverse magnetic field, alternating at 8 Hz. The transmitted light is detected by two-quadrant split photodiodes, whose interconnecting axis can be directed along the $\mathbf{B}$ axis or the $\mathbf{k}\times\mathbf{B}$ axis. The difference between the photodiode signals is then phase sensitively detected at the magnetic field frequency and represents a magnetic field induced lateral displacement of the beam after passage through the sample. Calibration of this displacement signal is done by means of moving the photodiode assembly over controlled distances. It is experimentally advantageous to determine the difference between the deflection signals for two different polarization states, as this

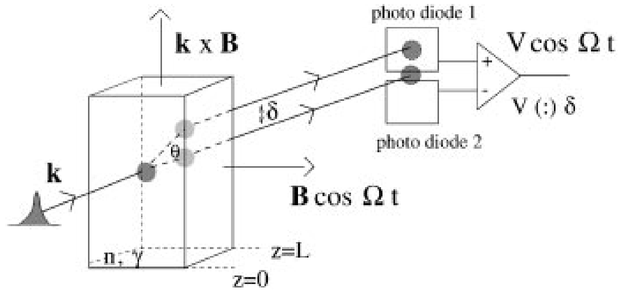

FIG. 1. Schematic setup. As shown, the displacement of the beam in the $\mathbf{k} \times \mathbf{B}$ direction is detected. By rotating the photodiode assembly over 90° along $\mathbf{k}$, the $\mathbf{B}$ axis displacement is detected.

eliminates spurious signals induced directly in the photodiodes by the alternating magnetic field.

First we will describe the experiments for the case where no theoretical controversy exists, i.e., deflection in transparent media of circularly polarized light in the $\mathbf{B}$ direction, proportional to γ (n and γ both being real valued). From Eq. (6) it is clear that this deflection can be determined by subtracting the $\mathbf{B}$ axis displacement signals for left and right circularly polarized light, according to

$$\frac{\mathbf{S}_{+}(\mathbf{B}) - \mathbf{S}_{-}(\mathbf{B})}{S(B = 0)} = \frac{\gamma}{n^2}\,\mathbf{B}\,. \tag{8}$$

In transparent materials is $\gamma = n\lambda V/\pi$, V being the Verdet constant of the medium and λ the vacuum wavelength of the light. Under these conditions also the group velocity is well defined [Eq. (5)] and predicts the same deflection between energy flow and $\mathbf{k}$ vector, quantified by the deflection angle $\theta = VB/k$. Figure 2 shows the measured displacement in the $\mathbf{B}$ direction of the light beam on the photodiodes, as a function of sample length. The linear dependence observed shows that the magnetic field indeed induces a *constant* deflection angle between wave vector and energy flow inside the sample. The inset shows that this deflection angle is linear in magnetic field strength. It was checked that the deflection angle was independent of light intensity. Figure 3 shows the results obtained for several materials with different Verdet constants and refractive indices. Good agreement is obtained with the theoretical prediction for θ, shows as the solid line. These results constitute, to our knowledge, the first experimental verification of the magnetic deflection of circularly polarized light, implicitly predicted by Landau, Lifshitz, and Pitaevskii [3] on the basis of the group velocity.

For the case of linearly polarized light in absorbing media (n and γ complex valued), a controversy exists between the predictions based on the Poynting vector and on wave packet analysis. The Poynting vector in Eq. (7) predicts the existence of a magnetotransverse deflection of linearly polarized light which is proportional to $\mathrm{Im}\gamma$. Schlesser and Weis, in observing such a deflection, have presented experimental evidence in favor of this Poynting vector prediction. Here we measure such a possible deflection by subtracting the displacement signals along

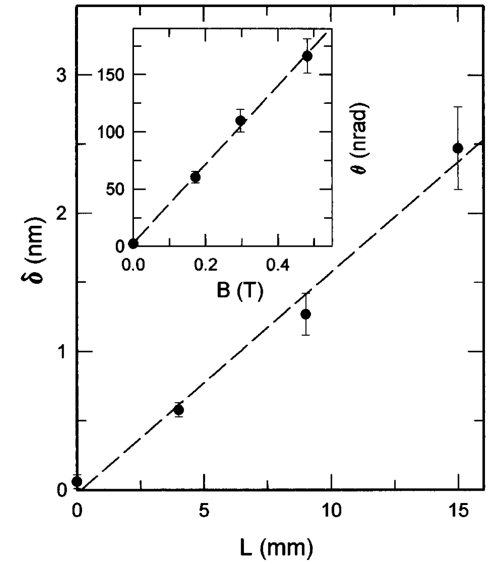

FIG. 2. $\hat{\mathbf{B}}$ axis displacement δ versus sample length L (laser wavelength 632.8 nm, sample material is Plexiglas ($n = 1.49, V = 4.5$ rad/T m) and magnetic field strength $B = 0.48$ T.) Dashed line is a linear fit to the data points. Inset shows the dependence of the deflection angle $\theta = \delta/L$ on magnetic field strength, also for Plexiglas. Dashed line is a linear fit to the data points.

the $\mathbf{k} \times \mathbf{B}$ axis for light polarized parallel ($\phi = 0$) and perpendicular ($\phi = \pi/2$) to the magnetic field,

$$\frac{\mathbf{S}(\phi = \pi/2, \mathbf{B}) - \mathbf{S}(\phi = 0, \mathbf{B})}{S(B = 0)} = \frac{\mathrm{Im}(\gamma\bar{n}/n^2)}{\mathrm{Re}n}\,\hat{\mathbf{k}} \times \mathbf{B}\,. \tag{9}$$

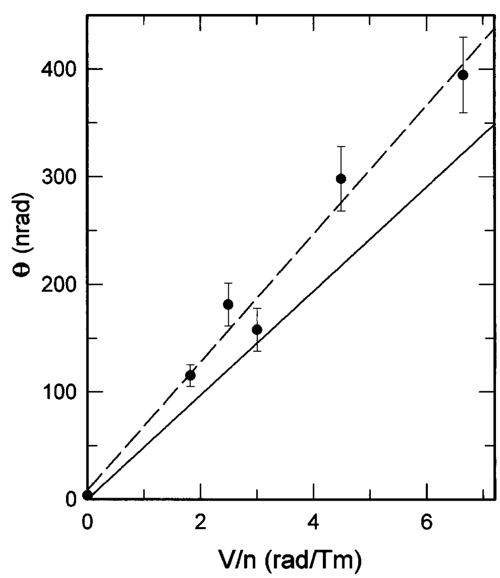

FIG. 3. $\hat{\mathbf{B}}$ axis deflection angle θ versus V/n for several materials. (In order of increasing V/n; air, methanol, water, Plexiglas, toluene, and 1-methyl naphthalene. Laser wavelength 632.8 nm, sample length 13 mm, and magnetic field strength $B = 0.48$ T.) Dashed line is a linear fit to the data points; solid line is the theoretical prediction (see text).

We have selected a weakly absorbing material, where $\text{Im}n \ll \text{Re}n$, i.e., $\overline{n} \simeq n$. $\text{Im}\gamma$ can be independently determined from the MCD $\Delta\alpha(B) \equiv \alpha_+(B) - \alpha_-(B)$ according to $\text{Im}\gamma = n\Delta\alpha/2kB$. As a medium we have used an aqueous Nd^{3+} solution, which around $\lambda = 791$ nm shows a transition with a weak absorption $\alpha \simeq 10$ molar^{-1} cm^{-1} and a large relative MCD $\Delta\alpha/\alpha \simeq 5 \times 10^{-2}$ T^{-1} [20]. From our own MCD measurements, at a Nd^{3+} concentration of 0.1 molar, we determine $\text{Im}\gamma = (1.0 \pm 0.1) \times 10^{-6}$ T^{-1} at $\lambda = 790.0$ nm, somewhat smaller than reported in Ref. [20], which according to Eq. (9) should correspond to a deflection angle of $(29 \pm 3) \times 10^{-8}$ rad at $B = 0.5$ T. Our setup should have no difficulty detecting such a deflection angle, as witnessed by Fig. 3. However, experimentally we observe at this field strength a deflection angle of $(0.3 \pm 0.3) \times 10^{-8}$ rad, i.e., *no significant deflection.* This result is in quantitative agreement with the prediction based on the wave packet calculation, and in strong disagreement with the Poynting vector prediction. To our knowledge, this is the first time that the widely accepted Poynting vector definition [Eq. (2)], and, in particular, its direction, is experimentally proven to be incorrect. Our measurements disagree with the findings by Schlesser and Weis in cesium vapor. Since their observed deflection was found to be strongly nonlinear in magnetic field and light intensity, probably due to population and coherence effects, these authors themselves suggested that linear electromagnetic theory, as expressed in Eq. (3), may not apply to their medium. The absence of a magnetotransverse deflection in homogeneous absorbing media emphasizes the different impact of a magnetic field on absorbing and scattering media, as the latter have been shown to exhibit such a deflection [2].

Our final task is to determine whether a choice for $\mathbf{T}$ exists that reconciles the energy flow as expressed by Eq. (1) with our wave packet analysis, which in turn is consistent with our experimental observations. In vacuum, evidently $\nabla \times \mathbf{T} = 0$. This is no longer true inside our medium, and the requirement of energy flux conservation at the interface between vacuum and medium [14] therefore implies that the normal component of $\nabla \times \mathbf{T}$ must vanish at the interface inside the medium. As the energy flow decays in the medium as $\exp[-2\text{Im}(n)kz]$, we can expect a similar dependence for $\nabla \times \mathbf{T}$. It can be easily shown that the $\mathbf{k} \times \mathbf{B}$ terms in the Poynting vector expressions can be canceled by $\mathbf{T} \propto \exp[-2\text{Im}(n)kz]\mathbf{B}$ [21]. Similarly, terms proportional to $\mathbf{B}$ could be canceled by $\mathbf{T} \propto \exp[-2\text{Im}(n)kz]\hat{\mathbf{k}} \times \mathbf{B}$. Both these choices fulfill the energy flow conservation at the interface, and an appropriate linear combination will make the energy flow vector of Eq. (1) parallel to the propagation direction of the wave packet. We emphasize that these choices are not necessarily unique. Although we have not clarified the physical significance of such remarkable albeit unavoidable choices for $\mathbf{T}$, in our view, our findings contribute to the ongoing discussion on the interpretation of the Poynting vector as the electromagnetic energy flow [22].

In conclusion, we have reported for the first time the observation of magnetic deflection of circularly polarized light in transparent media, in accordance with longstanding predictions. The magnetic deflection of linearly polarized light in absorbing media, predicted by the Poynting vector, was not observed. This proves that the conventional identification of the Poynting vector with the energy flow is not generally correct.

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