Direct Detection Feedback for Preserving Quantum Coherence in an Open Cavity

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(Received 5 September 1996)

It is shown that the Yurke-Stoler coherent state of field in an open cavity preserves its nonclassical structure if the outgoing radiation is measured by a photodetector and the photocurrent is used for phase modulation of the intracavity field. [S0031-9007(96)02264-8]

PACS numbers: 42.50.Lc, 03.65.Bz

One of the most peculiar features of the quantum description of the world, most brightly distinguishing it from the classical description, is the notion of quantum superposition of two states of the system, which cannot be considered as arising from the lack of information about the system state. Schrödinger was perhaps the first who realized that quantum superposition can take place also in a macroscopic system, for example, a cat in a closed box, if described quantum mechanically, can in principle be in a superposition of the states of life and death [1]. In the last few years this phenomenon has been widely studied for one of the most simple quantum systems—single-mode field in a cavity, in which case the superposition of two coherent states with opposite amplitudes, $N(|\alpha\rangle + e^{i\theta}|-\alpha\rangle)$, where θ is arbitrary and N is a normalization constant, is usually called the Schrödinger cat state [2]. The density operator of this state has the form $\rho = \rho_{\mathrm{mix}} + \rho_{\mathrm{int}}$, where $\rho_{\mathrm{mix}} = N^2(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)$ corresponds to a mixture of two states while $\rho_{\mathrm{int}} = N^2(e^{i\theta}|-\alpha\rangle\langle\alpha| + e^{-i\theta}|\alpha\rangle\langle-\alpha|)$ describes the interference between these states. Such states have been widely studied in recent years in connection with their possible applications to quantum cryptography and quantum computation [3,4], but using quantum superpositions in quantum computers is highly complicated by fast decay of the interference part of the density operator in the presence of dissipation, the phenomenon generally known as quantum decoherence [5]. The rate of this decay for the case of a cat state in an open cavity is $2\gamma|\alpha|^2$, where γ is the energy decay rate [6], i.e., the larger the cat size $|\alpha|^2$, the faster decays quantum interference (quantum coherence). In this Letter we describe an experimentally realizable way for preventing quantum coherence from fast decay in a dissipative system.

Our approach is based on the properties of operators which may be called generalized photon creation and annihilation operators:

$$A_\varphi = e^{i\varphi a^+ a} a\,,$$
$$A_\varphi^+ = a^+ e^{-i\varphi a^+ a},$$

where a^+ and a are usual photon creation and annihilation operators, and φ is a c-number.

Operators A_φ and A_φ^+ obey usual boson commutation relations:

$$[A_\varphi, A_\varphi^+] = 1\,,$$

and therefore they can be considered as lowering and raising operators for a basic set of vectors $|N\rangle$, $N = 0, 1, 2, \ldots$ in the Hilbert space of harmonic oscillator [7]:

$$A_\varphi|N\rangle = \sqrt{N}|N-1\rangle\,, \qquad (1)$$
$$A_\varphi^+|N\rangle = \sqrt{N+1}|N+1\rangle\,.$$

As $A_\varphi^+ A_\varphi = a^+ a$, these vectors are eigenstates of both $A_\varphi^+ A_\varphi$ and $a^+ a$ and therefore they may differ from Fock states $|n\rangle$ only by phase:

$$|N\rangle = e^{i\Phi_n}|n\rangle\,.$$

Multiplying both sides of Eq. (1) by $\langle n-1|$ and substituting for A_φ and $|N\rangle$ we obtain the following relation:

$$e^{i\varphi(n-1)} e^{i\Phi_n} = e^{i\Phi_{n-1}}.$$

If we assume the phase shift for vacuum state to be zero, then

$$\Phi_n = \Phi_{n-1} - \varphi(n-1) = -\varphi\frac{n(n-1)}{2}\,.$$

The eigenstates of operator A_φ,

$$A_\varphi|\mathcal{A}\rangle = \mathcal{A}|\mathcal{A}\rangle\,,$$

have the following form:

$$|\mathcal{A}\rangle = e^{-\frac{1}{2}|\mathcal{A}|^2}\sum_{N=0}^{+\infty}\frac{\mathcal{A}^N}{\sqrt{N!}}|N\rangle$$
$$= e^{-\frac{1}{2}|\mathcal{A}|^2}\sum_{n=0}^{+\infty}\frac{\mathcal{A}^n e^{i\Phi_n}}{\sqrt{n!}}|n\rangle\,,$$

and represent a subclass of generalized coherent states [8].

The most interesting case arises for $\varphi = \pi$. Taking into account that for an ordinary coherent state $|\alpha\rangle$,

$$e^{i\varphi a^+ a}|\alpha\rangle = |\alpha e^{i\varphi}\rangle\,,$$

it is easy to verify that

$$e^{i\pi a^+ a} a(|\alpha\rangle + i|-\alpha\rangle)/\sqrt{2} = -i\alpha \times (|\alpha\rangle + i|-\alpha\rangle)/\sqrt{2}\,;$$

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that is, the eigenstate of A_π is the so-called Yurke-Stoler (YS) coherent state [9]:

$$|\mathcal{A}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + i|-\alpha\rangle),$$

where $\alpha = i\mathcal{A}$. Using close analogy between the operators A_π and a we can find master equations and corresponding processes preserving the structure of the YS coherent state. For example, the density operator ρ of a single mode of an open cavity with decay rate γ obeys the following master equation [10]:

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}(2a\rho a^+ - a^+a\rho - \rho a^+a),$$

and if the initial state of the field is a coherent state $|\alpha_0\rangle$, then with time the state remains coherent with decreasing amplitude $\alpha(t) = \alpha_0 e^{-\gamma t/2}$. Therefore, if the dynamics of the cavity field is governed by the master equation

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}(2A_\pi\rho A_\pi^+ - A_\pi^+A_\pi\rho - \rho A_\pi^+A_\pi),$$

or equivalently

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}(2e^{i\pi a^+a}a\rho a^+e^{-i\pi a^+a} - a^+a\rho - \rho a^+a), \tag{2}$$

then the initial YS state $(|\alpha_0\rangle + i|-\alpha_0\rangle)/\sqrt{2}$ will remain a YS state with decreasing amplitude $\alpha(t) = \alpha_0 e^{-\gamma t/2}$.

The structure of Eq. (2) is exactly that, typical for a system with measurement mediated feedback, which gives a natural way for practical application of the formalism developed above. Using feedback for manipulating quantum properties of the field has become recently a widely investigated problem [11]. Possibility of creating [12] and preserving [13] quantum superpositions by means of quantum nondemolition measurement mediated feedback has been illustrated in several recent theoretical works. However, our approach is quite different and much more simple, as it does not require a highly complicated intracavity quantum-nondemolition technique but uses direct detection of external radiation.

According to the theory of feedback, based on the continuous photodetection theory [14,15], if the external field is being measured by a photodetector and each photocount is followed by fast, compared to cavity photon lifetime, interaction between the feedback loop and the cavity field, which interaction is described by equation

$$\left(\frac{\partial\rho}{\partial t}\right)_{fb} = \mathcal{L}\rho,$$

where $\mathcal{L}$ is some superoperator, then the master equation of the cavity field reads as

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}(2e^{\mathcal{L}\tau}a\rho a^+ - a^+a\rho - \rho a^+a),$$

where τ is the feedback interaction time, $\tau \ll \gamma^{-1}$. In our case, to achieve the dynamics described by Eq. (2) we need $\mathcal{L}\rho = i\omega[a^+a, \rho]$ and $\omega\tau = \pi$, which corresponds to shifting the phase of the cavity field by π in each act of feedback interaction. Such a shifting can be realized by increasing the optical length of the cavity by means of an intracavity electro-optical modulator.

The physical meaning of such a feedback can be understood in the following way. If the field going out of a cavity with the decay rate γ is measured by a photodetector and in the time interval $[0, t)$ exactly n photocounts occur at times $t_1, t_2, ..., t_n$, then the conditional state of the field is given by the following expression [16,17]:

$$|\psi_c(t)\rangle = \gamma^{\frac{n}{2}}e^{-\frac{\gamma}{2}a^+a(t-t_n)}ae^{-\frac{\gamma}{2}a^+a(t_n-t_{n-1})}a\cdots ae^{-\frac{\gamma}{2}a^+at_1} \times|\psi(0)\rangle. \tag{3}$$

This expression shows that when the initial state of the field is a YS state $|\psi(0)\rangle = (|\alpha_0\rangle + i|-\alpha_0\rangle)/\sqrt{2}$, the evolution of the state consists of two processes: between two counts the amplitude of state decays: $(|\alpha_{t_n}\rangle + i|-\alpha_{t_n}\rangle)/\sqrt{2} \to (|\alpha_{t_{n+1}}\rangle + i|-\alpha_{t_{n+1}}\rangle)/\sqrt{2}$ (here we omit normalization factors), where α_t is defined as above, while each photocount brings about a shifting by π the relative phase of states $|\alpha_{t_n}\rangle$ and $|-\alpha_{t_n}\rangle$: $(|\alpha_{t_n}\rangle + i|-\alpha_{t_n}\rangle)/\sqrt{2} \to (|\alpha_{t_n}\rangle - i|-\alpha_{t_n}\rangle)/\sqrt{2}$. We see that if the number of photons detected in the time interval $[0, t)$ is known exactly, the conditional state of the field remains a YS coherent state. However, if this number is unknown, the interference of two states is destroyed due to phase shifting after time of the order of average half-distance between two successive counts: $t_{\rm decoh} \sim 2^{-1}\gamma^{-1}|\alpha_0|^{-2}$. Feedback allows us to obtain an unconditional decaying YS state, restoring the phase of the intracavity field after a photocount occurs. The effect of feedback is easy to see from Eq. (3), rewritten in the presence of feedback as

$$|\psi_c(t)\rangle = \gamma^{\frac{n}{2}}e^{-\frac{\gamma}{2}a^+a(t-t_n)}e^{i\pi a^+a}ae^{-\frac{\gamma}{2}a^+a(t_n-t_{n-1})}e^{i\pi a^+a} \times a\cdots e^{i\pi a^+a}ae^{-\frac{\gamma}{2}a^+at_1}|\psi(0)\rangle.$$

It follows from the above that the proposed method for preserving quantum coherence in an open cavity is very sensitive to quantum efficiency η of the photodetector. Below we calculate the influence of the detector inefficiency, introducing an additional channel of losses with the rate $\gamma(1-\eta)$, and rewriting Eq. (2) as

$$\frac{\partial\rho}{\partial t} = \gamma\eta D(A_\pi)\rho + \gamma(1-\eta)D(a)\rho, \tag{4}$$

where the superoperator $D(x)$ for any operator x is defined in the following way:

$$D(x)\rho = \frac{1}{2}(2x\rho x^+ - x^+x\rho - \rho x^+x).$$

To solve Eq. (4) we will use the positive P representation [18], where the density operator is represented as

$$\rho(t) = \int P(\alpha,\beta,t)\frac{|\alpha\rangle\langle\beta|}{\langle\beta\,|\,\alpha\rangle}d^2\alpha d^2\beta. \tag{5}$$

Equation (4) gives the following equation for the quasiprobability density $P(\alpha,\beta,t)$:

$$\frac{\partial}{\partial t}P(\alpha,\beta,t) = \frac{\gamma}{2}\left(\frac{\partial}{\partial\alpha}\alpha + \frac{\partial}{\partial\beta^*}\beta^*\right)P(\alpha,\beta,t) + \gamma\eta\alpha\beta^*[P(-\alpha,-\beta,t) - P(\alpha,\beta,t)],$$

which splits into two independent equations:

$$\frac{\partial}{\partial t}P_+(\alpha,\beta,t) = \frac{\gamma}{2}\left(\frac{\partial}{\partial\alpha}\alpha + \frac{\partial}{\partial\beta^*}\beta^*\right)P_+(\alpha,\beta,t), \quad (6)$$

$$\frac{\partial}{\partial t}P_-(\alpha,\beta,t) = \frac{\gamma}{2}\left(\frac{\partial}{\partial\alpha}\alpha + \frac{\partial}{\partial\beta^*}\beta^*\right)P_-(\alpha,\beta,t) - 2\gamma\eta\alpha\beta^*P_-(\alpha,\beta,t), \quad (7)$$

by introducing new functions

$$P_+(\alpha,\beta,t) = P(\alpha,\beta,t) + P(-\alpha,-\beta,t), \quad (8)$$

$$P_-(\alpha,\beta,t) = P(\alpha,\beta,t) - P(-\alpha,-\beta,t). \quad (9)$$

For the initial YS state with the amplitude α_0 the solutions of Eqs. (6) and (7) read as

$$P_+(\alpha,\beta,t) = \delta(\alpha - \alpha_t)\delta(\beta - \alpha_t) + \delta(\alpha + \alpha_t)\delta(\beta + \alpha_t), \quad (10)$$

$$P_-(\alpha,\beta,0) = i\{\delta(\alpha + \alpha_t)\delta(\beta - \alpha_t) - \delta(\alpha - \alpha_t)\delta(\beta + \alpha_t)\} \times e^{2|\alpha_0|^2 - 2\eta|\alpha_0|^2(1-e^{-\gamma t})}, \quad (11)$$

where α_t is defined as above, giving according to Eqs. (8), (9), and (5) the following evolution of the density operator:

$$\rho(t) = \frac{1}{2}(|\alpha_t\rangle\langle\alpha_t| + |-\alpha_t\rangle\langle-\alpha_t|) + \frac{i}{2}e^{-2(1-\eta)|\alpha_0|^2(1-e^{-\gamma t})}(|-\alpha_t\rangle \times \langle\alpha_t| - |\alpha_t\rangle\langle-\alpha_t|).$$

This expression shows that the additional channel of losses results in the decoherence with the characteristic time $t_{\mathrm{decoh}} \sim 2^{-1}\gamma^{-1}(1-\eta)^{-1}|\alpha_0|^{-2}$, which corresponds to one-half of the mean time interval between two successive photons in this channel. So the inefficiency of the photodetector in the feedback loop restricts the maximum size $|\alpha_0|^2$ of a cat, which can be preserved from fast decoherence by the proposed method. The importance of high efficiency photodetectors in our scheme is demonstrated by the following example: even for $\eta = 0.9$ the decoherence will be slowed down to the rates of the order of the energy decay rate only for $|\alpha_0|^2 = 5$. However, the effect of feedback can be observed with more optimism for realization quantum efficiency of $\eta = 0.5$ in which case the rate of decoherence will be twice less than without feedback for any α_0.

In conclusion we want to remark that our approach can be considered as an example of a more general principle of possibility to preserve the nonclassicality of the field inside an open cavity, using the information obtained from the measurement of the external field for manipulating intracavity field characteristics. In the considered case, where the information obtained from direct detection of external field is used for intracavity phase modulation, though the mean number of photons inside the cavity decreases exponentially, the state preserves its highly nonclassical form—it remains a Schrödinger-cat-like state.

We thank Professor W. Schleich for helpful discussions of the problem. This work was supported in part by Belarus Basic Research Foundation under Grant No. MΠ 96-38 and in part by Belarus Soros Foundation.

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