Stable Coulomb Bubbles?

L. G. Moretto, K. Tso, and G. J. Wozniak
Nuclear Science Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
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Coulomb bubbles, though stable against monopole displacement, are unstable at least with respect to quadrupole and octupole distortions. We show that there exists a temperature at which the pressure of the vapor filling the bubble stabilizes all the radial modes. In extremely thin bubbles, the crispation modes become unstable due to the surface-surface interaction. [S0031-9007(97)02348-X]

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The possibility of stable or metastable nonspherical nuclear configurations, like bubbles or tori, has been occasionally considered [1–6]. Earlier studies, based upon the liquid drop model, showed the presence of a bubble monopole minimum above a certain fissility parameter (Coulomb bubble) [4]. However, the higher deformation modes of the bubble appeared to be unstable. A recent calculation using the generalized rotating liquid drop model has shown the appearance of metastable bubblelike minima at high angular momentum [7]. Similarly, finite temperature Hartree-Fock and Thomas-Fermi calculations give indications of the onset of bubble formation [6]. Recent simulations of nuclear collisions by means of transport equations indicate the possibility of bubble formation [8–11].

Coulomb bubbles, their formation, stability, and eventual demise are of broad interest, and are relevant not only to nuclei, but also to highly electrified fluids when the Coulomb interaction becomes dominant over the surface tension.

In what follows, we will show how the vapor pressure solves the outstanding problem of the secular stability of Coulomb bubbles. Furthermore we shall illustrate the role of a recently discovered surface instability (sheet instability) [12] in their eventual demise.

Within the framework of the liquid drop model, the energy E of a bubble in units of twice the surface energy of the equivalent sphere (constant volume) can be easily written down as a function of the bubble monopole coordinate x:

$$\begin{aligned} E = {} & \frac{1}{2}x^2 + \frac{1}{2}(1 + x^3)^{2/3} \\ & + X\left[(1 + x^3)^{5/3} + \frac{3}{2}x^5 - \frac{5}{2}x^3(1 + x^3)^{2/3}\right] \\ & + \frac{R}{[(1 + x^3)^{5/3} - x^5] - x^3 P}\,. \end{aligned} \tag{1}$$

Here x is defined as the ratio of the inner sphere radius R_1 over the radius of the equivalent sphere R_o. The *Coulomb, angular momentum* and *pressure* terms are defined in terms of the fissility parameter X, rotational energy R, and reduced pressure P, respectively:

$$X = \frac{E_c^o}{2E_s^o}, \quad R = \frac{E_R^o}{2E_s^o}, \quad P = \frac{pV_o}{2E_s^o}\,.$$

Here the common denominator $2E_s^o$ is twice the surface energy of the equivalent sphere, E_c^o and E_R^o are the Coulomb and rotational energies, and p and V_o are the actual pressure and equivalent sphere volume, respectively.

At zero pressure and angular momentum, the surface energy increases as a bubble develops from a sphere, but the Coulomb energy decreases as the charges are brought farther apart due to the bubble expansion. Therefore, an interplay between the Coulomb and surface energies may generate a minimum energy point along the monopole coordinate. The bubble minimum appears first at a value of the fissility parameter $X = 2.022$, and becomes the absolute minimum at $X = 2.204$ [4,13]. How can such large values of X be accessible, if the value of X for ^{238}U is only 0.714, and even for the nucleus arising from the fusion of two nuclei of ^{238}U, $X = 1.427$? The obvious possibility lies in higher temperatures, which decrease the surface energy coefficient (which must go to zero at the critical temperature). For instance, within the framework of a Thomas-Fermi calculation [14,15], a nucleus like $^{238}\mathrm{U} + {}^{238}\mathrm{U}$ achieves the critical value $X = 2.204$ for bubble formation at $T = 8.13$ MeV.

The solid line in the upper inset of Fig. 1 plots the dimensionless monopole coordinate of the bubble minimum as a function of the fissility parameter X. The radius of a Coulomb bubble is found to increase with the fissility parameter X. The spherical minimum and the bubble minimum are separated by a barrier whose maximum value $\Delta E_b = 0.0306E_s^o$ is attained at $X = 2.022$.

Similarly, at zero fissility and pressure, there exists a critical value ($R = 0.953$) of the rotational parameter at which a bubble first appears, and a second critical value ($R = 1.055$) at which the bubble minimum becomes the deeper minimum.

The pressure, on the other hand, does not give rise to a bubble minimum on its own. At constant pressure, zero fissility, and zero angular momentum, the sphere minimum is the only minimum. When x increases, a barrier is encountered beyond which there is a runaway expansion of the bubble. At constant Px^3, like at constant

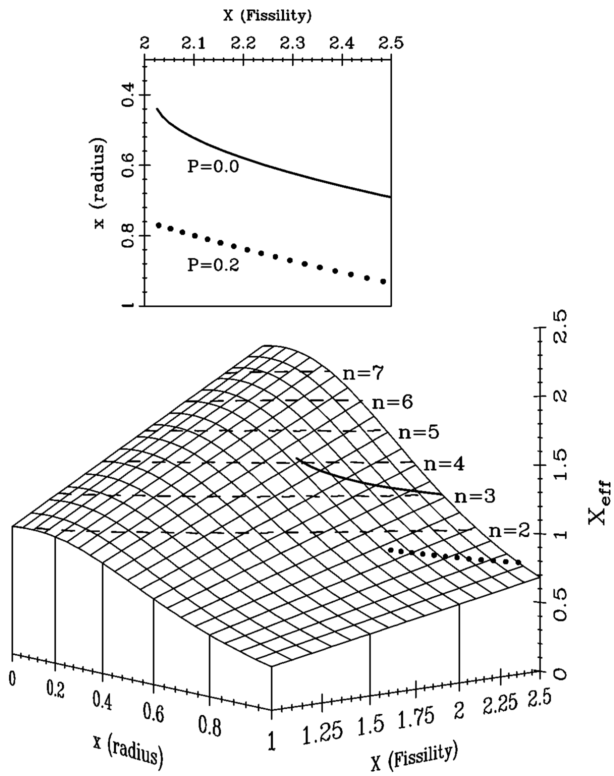

FIG. 1. Effective fissility parameter X_{eff} as a function of the fissility parameter X of the equivalent sphere and the inner radius x of the bubble. The dashed lines indicate the onset of instability for specified modes. The solid and dotted curves plot the value of X_{eff} as a function of X for reduced pressures at 0.0 and 0.2, respectively. (Upper inset) The projections of the solid and dotted curves on the x-X plane.

temperature and molar number, the pressure term becomes a constant energy shift, and the energy rises indefinitely with x like the total surface energy.

A Coulomb bubble that is stable against monopole oscillations may be subjected to higher order perturbations. The higher deformation modes of the bubble can be divided into two classes [13]: the *radial modes* and the *crispation modes*. The deformations on the two surfaces are in phase with each other for a radial mode, and they are out of phase for a crispation mode.

The monopole oscillation obviously belongs to the class of radial modes. On the other hand, the lowest order crispation mode is the dipole mode which corresponds to a rigid displacement of the two spheres, one with respect to the other. Notice that this mode, in the absence of the Coulomb and rotational terms, is indifferent, and leads to the eventual puncturing of the bubble. The introduction of the Coulomb term tends to stabilize a bubble against crispation dipole oscillation. The radial dipole mode, however, is trivial since it involves only the motion of the center of mass. Hence, a nuclear bubble is always stable with respect to a dipole perturbation within our present description.

Unlike the dipole oscillation, higher multipole perturbations tend to increase the surface energy, and thus stabilize the unperturbed bubbles. This surface effect is the same for the radial and crispation modes, since the two modes differ only in the relative orientation of their surfaces. On the other hand, the Coulomb effect is drastically different for the two modes. The Coulomb perturbation energy is always negative for the radial mode, since the average distance between charges is increased slightly due to the perturbation. A similar effect of Coulomb destabilization is observed for the crispation mode in case of thick bubbles. In fact, the two modes are indistinguishable for a solid sphere. However, this destabilization effect becomes progressively weaker as the bubble expands. When a bubble is sufficiently thin, the Coulomb perturbation energy becomes positive, and stabilizes the crispation modes. This is because the Coulomb force tends to resist the attempt to concentrate the charge in "clumps" distributed on the surface of the thin bubble, as required by the higher order crispation modes. In general, the Coulomb destabilization effect is always stronger for the radial modes. Therefore, a bubble that is stable with respect to radial perturbations is always stable against crispation perturbations within our present description.

To see the role of the Coulomb term on the stability of radial modes, let us recall that for a charged drop, the reduced frequency of the nth modes is given by [16]

$$\omega^2 = \frac{1}{8}\, n(n-1)[(n+2) - 4X]. \qquad (2)$$

Notice that for $X = 1$ the frequency goes to zero for $n = 2$. This is the onset of quadrupole instability, or the well known fission instability. For $X > 1$ progressively higher modes are destabilized. The *last unstable* mode is $n_{\mathrm{last}} = 4X - 2$. For instance, n_{last} increases from 10 to 14 as X is incremented from 3 to 4. This shows that an increase of the Coulomb force destabilizes a larger number of radial modes. In addition, Eq. (2) allows one to define the *most unstable* mode (negative minimum of ω^2). For example, the most unstable modes are 7 and 10 for $X = 3$ and 4, respectively. Hence, a highly charged sphere will not merely fission, but will break up into many droplets through an instability associated with a high multipole mode. Interestingly, the most unstable mode does not coincide with n_{last}, nor with the lowest (fission) mode either.

Equation (2) can be applied to the radial modes of the bubble as well, provided that, at any given value of the monopole coordinate x, an "effective" fissility parameter is defined

$$X_{\mathrm{eff}} = \frac{E_c(x)}{2E_s(x)}.$$

Since the Coulomb term decreases with x, while the corresponding surface term increases, the value of X_{eff} decreases as the bubble expands at a given fissility parameter, as shown in Fig. 1. If the original nucleus ($x = 0$)

is unstable up to the multipole of order n, as it develops into a bubble ($x > 0$) it starts stabilizing the higher order radial modes. The dashed lines in Fig. 1 show that the last unstable mode decreases with increasing x.

The solid curve in Fig. 1 indicates the values of X_{eff} associated with the bubble minima at different fissilities. At the threshold fissility of $X = 2.022$, the value of X_{eff} lies just about at the $n = 4$ stability line, indicating that the bubble is unstable up to the $n = 4$ mode. As more charge is brought into the bubble with increasing values of X, the Coulomb bubble expands and it becomes stable with respect to the $n = 4$ and even to the octupole mode ($n = 3$) at $X = 2.5$. However, the Coulomb bubble is still unstable with respect to the quadrupole mode ($n = 2$). In fact, a further increase of X does not stabilize the quadrupole mode.

Yet, it may be possible to have a stable nuclear bubble. If the bubble is warm, it fills up with vapor arising from the fluid itself. The effect of pressure on the stability of the radial modes is most remarkable! The resulting pressure acts only upon the monopole mode, by displacing outwards the Coulomb minimum. The effect on the other radial modes is nil, since only changes in volume are relevant to pressure. Consequently, the positions in x of the last unstable modes for a fixed value of X do not change. The dotted curve in the inset of Fig. 1 shows the expansion of the Coulomb bubble provided by a reduced pressure of 0.2. When this dotted curve is projected onto the surface of X_{eff}, it appears below the quadrupole stability line. *This shows that the bubble has become secularly stable with respect to all the modes.*

To study this pressure effect in combination with the fissility parameter, a contour plot indicating the inner radius at the bubble minimum is shown as a function of P and X in the top panel of Fig. 2. The lower limit of X is 2.022, the fissility at which a bubble minimum first appears. The dashed line indicates the onset of instability for the quadrupole mode, which also defines the boundary conditions of bubble stability against all the radial modes. It can be seen that at a given value of X, it is always possible to find a pressure large enough to shift the bubble minimum to a thinner and stable configuration.

A natural source for this pressure, in the case of nuclei or other fluids in vacuum, is the pressure of the saturated vapor, which spontaneously fills up the bubble if $T > 0$. As the outer surface is looking into vacuum, one might think that no pressure is exerted on it. However, since the outer surface is constantly evaporating, an ablation pressure is generated. Since the average impulse brought in by a vapor particle is equal at equilibrium to that of the evaporated particle, it follows that the ablation pressure is exactly equal to one-half of the vapor pressure.

Using the Thomas-Fermi model [17], a temperature can always be found at which the vapor pressure stabilizes the bubble against all the radial mode perturbations. An example for the system of $^{238}U + ^{238}U$ is shown in the bottom panel of Fig. 2. The dashed line is equivalent to the dashed line in the top panel, which defines the boundary conditions of bubble stability against all the radial modes. The solid line shows the temperature effect on both the reduced pressure and the fissility parameter. In this case, a nuclear temperature of about 10 MeV is sufficient to stabilize a bubble configuration against perturbations of all radial modes.

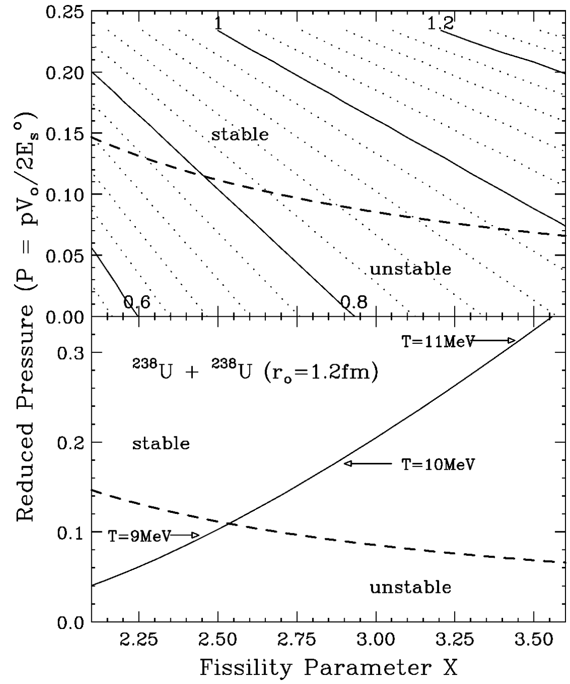

FIG. 2. (Top) The linear contour plot (dotted and solid lines) shows the inner radius of a bubble minimum as a function of reduced pressure P and fissility parameter X. The dashed line indicates the onset of instability for the quadrupole mode. (Bottom) For the system of $^{238}U + ^{238}U$, the line plots the increasing values of P and X with temperature. The dashed line is the dashed line from the top panel.

Thus far, we have considered the effects of surface, charge, and pressure on distorted bubbles, and found that (a) stability against radial perturbations can be achieved, and (b) that it is a sufficient condition for the overall bubble stability. However, when a bubble becomes rather thin, a possible demise of the bubble may be associated with the sheet instability which has not been treated here so far. The sheet instability [12] is a new kind of Rayleigh-like surface instability associated with the crispation modes. A nuclear sheet of any thickness tends to escape from the high surface energy by breaking up into a number of spherical fragments with less overall surface. However, any perturbation of finite wavelength increases the surface area, and consequently the energy of the sheet, independent of the sheet thickness. Clearly, this barrier prevents the sheet from reaching the more stable configurations. However, when a nuclear sheet becomes sufficiently thin, the two nuclear surfaces interact with each other. This proximity interaction may become sufficiently strong to overcome the sharp barrier and causes the sheet to puncture into numerous fragments. Using the expression in Ref. [18] for the proximity potential, a critical wavelength is determined for the onset of this surface instability for a flat sheet: $\lambda_c = 1.1b \exp(2d/3b)$, where b is the range of the proximity interaction and d is the thickness of the sheet.

A bubble behaves much like a sheet, and is subject to the sheet instability. Since a bubble, like a sheet, must rely on the proximity interaction to become unstable, it will retain its surface stability until the range of the surface-surface interaction is of the order of its thickness. Thus a critical range of proximity interaction for the onset of bubble instability against crispation perturbation can be defined as $b_c = f(x, X, n)$.

Figure 3(a) plots the value of b_c for the onset of dipole instability at the indicated values of fissility. Notice that the line for $X = 0$ is missing, since the dipole mode of a neutral bubble is indifferent, and any finite proximity effect is sufficient to trigger the instability. Recall that the introduction of charge stabilizes a bubble against dipole oscillation, and thus offsets the proximity effect. Consequently, the value of b_c at any given bubble radius increases with X as shown in Fig. 3(a).

Unlike the dipole mode, the surface energy of higher multipole perturbations increases monotonically with the bubble radius. To study the interplay between this surface effect and the proximity interaction, a neutral bubble is considered. The solid lines in Fig. 3(b) plot b_c as a function of x for progressively higher order modes ($n = 2-10$). Clearly, the quadrupole instability is most easily triggered among the multipole modes. As the proximity interaction becomes stronger (larger b_c), higher order multipoles are gradually destabilized. The dashed line in Fig. 3(b) shows the onset of quadrupole instability for a charged bubble with $X = 1.5$. Interestingly, the dashed and the corresponding solid lines cross at about $x = 0.6$, reflecting different Coulomb effects mentioned earlier for thin and thick bubbles undergoing multipole crispation perturbations. An increase in charge stabilizes a bubble against higher order modes and offsets the proximity effect (larger b_c) until it becomes sufficiently thick ($x < 0.6$ for the quadrupole mode at $X = 1.5$).

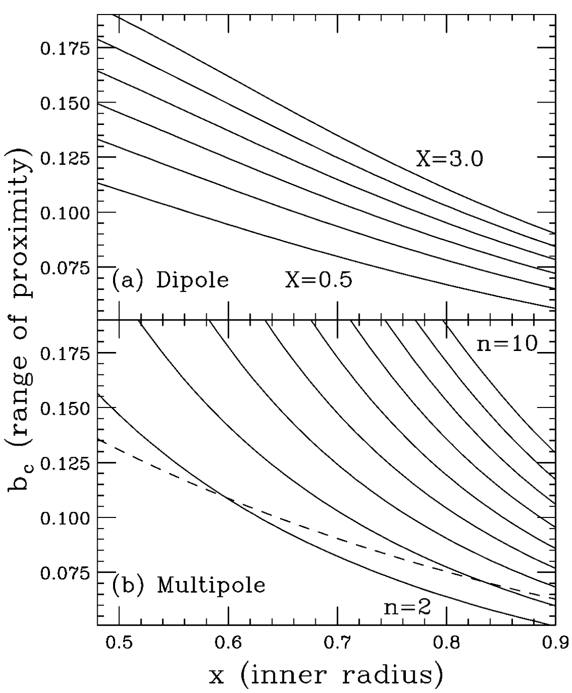

FIG. 3. (a) Critical range of proximity interaction (b_c) as a function of inner radius (x) for the dipole mode at various fissility parameters ($X = 0.5-3.0$). (b) b_c as a function of x for multipole modes ($n = 2-10$) of a neutral bubble. The dashed line indicates values of b_c for a charged bubble ($X = 1.5$) undergoing quadrupole perturbation.

In conclusion, the depletion of charge in the central cavity of nuclear bubbles reduces the Coulomb energy significantly and thus stabilizes "Coulomb" bubbles against monopole oscillations. These Coulomb bubbles, however, are at least unstable to perturbation of the quadrupole radial mode. On the other hand, a sufficiently high temperature generates a vapor pressure in the central cavity which drives the bubble to a thinner configuration that is stable against all the radial modes. Finally, a thin Coulomb bubble behaves like a sheet, and becomes susceptible to a proximity surface instability via the crispation modes when its thickness is comparable to the range of the proximity interaction.

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