Role of the Anomalous $U(1)_A$ for the Solution of the Doublet–Triplet Splitting Problem via the Pseudo-Goldstone Mechanism

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The anomalous $U(1)_A$ symmetry provides a generic method of getting accidental symmetries. Therefore, it can play a crucial role in solving the doublet-triplet splitting problem via the *pseudo-Goldstone* mechanism to all orders in M_P^{-1}. No additional discrete or global symmetries are needed. [S0031-9007(97)02321-1]

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One of the most difficult problems of the supersymmetric grand unified theories (GUTs) is the doublet-triplet splitting problem. It is difficult to understand how the theory, which knows only the very large scales $M_G \sim 10^{16}$ GeV and $M_P \sim 10^{19}$ GeV, arranges itself in such a way that a pair of essentially massless electroweak doublets $H, \overline{H}$ survive down to the low energies, not accompanied by their color-triplet partners. The natural logic is to attribute the lightness of the Higgs doublets to the smallness of the supersymmetry-breaking scale in the low energy sector $m_{3/2} \sim 100$ GeV. This requires a mechanism that would ensure masslessness of the doublets in the supersymmetric limit and at the same time guarantee that desired mass terms (μ and $B\mu$) of the right order of magnitude are generated by the supersymmetry (SUSY) breaking. As a guideline we will follow this strong criterion of naturalness, according to which the single mechanism must be responsible for both: (i) vanishing doublet mass in the SUSY limit and (ii) appearance of $\mu^2 \sim B\mu \sim m_{3/2}^2$ after its breaking. We also adopt the *minimality* requirement: both problems must be solved within the minimal set of the Higgs fields needed to break the GUT symmetry to the standard model. Besides the aesthetic problems, the nonminimal Higgs sector (additional adjoints, etc.) usually creates difficulties with asymptotic freedom. As far as we know, the only approach that can satisfy the above criterion is the "pseudo-Goldstone" idea [1–5]. The key point is that Higgs doublets can be identified as the zero modes of the compact vacuum degeneracy, which are massless to all orders in perturbation theory, because of supersymmetry. Once supersymmetry is broken, the flat directions are lifted and the doublets get masses of just the right order of magnitude: $\sim m_{3/2}$. On the way to constructing a realistic model along these lines, there are a few potential difficulties: (1) flat direction should not be a result of the fine tuning, but rather be guaranteed by the exact symmetries of the theory; (2) unless it is protected by the gauge symmetries, the flat direction can be lifted by the M_P-suppressed operators in the superpotential, which can destroy the original solution; (3) color-triplet partners must be heavy and decouple along the flat direction.

Closer to the realization of this program came the model of [2,3]. The crucial observation was that the desired compact degeneracy, automatically satisfying condition (3) above, could result if the different Higgs fields that break GUT symmetry are not correlated (have no cross couplings) in the superpotential. In this case, the vacuum has an accidental flat direction corresponding to the independent global rotations of the uncorrelated vacuum expectation values (VEVs). Since this rotation is not an exact symmetry of the theory (it is broken by the gauge and Yukawa couplings) the corresponding zero modes are not eaten up by any gauge field and are physical.

Thus, the central issue is to suppress the unwanted cross couplings by exact symmetries. Here one can identify the following problems: first, the symmetries, which forbid the cross couplings, also restrict the possible self-couplings of one of the fields, so that its VEV vanishes and the flat direction disappears; second, these symmetries are anomalous and cannot be ordinary gauge symmetries. Thus, there is no reason why they should be respected by the Planck scale suppressed, operators which would generate an unacceptably large mass for the doublets.

The key point of the present Letter is that the anomalous gauge $U(1)_A$ symmetry, usually present in string theories [6], can provide a simultaneous solution to the above problems. Cancellation of the anomalies by the Green-Schwarz mechanism [7] requires nonzero mixed anomalies and thus, some of the GUT fields must transform under $U(1)_A$. Since the symmetry is anomalous, the Fayet-Iliopoulos term (proportional to the sum of charges TrQ) is always generated [8] and, in strings, is given by [6]

$$\xi = \frac{g^2 \, \mathrm{Tr}Q}{192\pi^2} M_P^2 \, . \tag{1}$$

Since it is a gauge symmetry, the anomalous $U(1)_A$ can naturally uncorrelate the GUT VEVs in the superpotential to *all orders* in M_P^{-1} and at the same time induce the desired VEV $\sim\sqrt{\xi}$ through the Fayet-Iliopoulos D term. This gives an exciting possibility of solving the doublet-triplet splitting and the μ problems in all orders in M_P^{-1}, without any need of additional discrete or global symmetries, and within the minimal Higgs content. Incidentally it turns out that in this approach $U(1)_A$ plays the role of the matter parity also and can suppress all the dangerous baryon number violating operators.

Previously the implications of the anomalous $U(1)_A$ were considered for the fermion [9] and sfermion [10] masses, for mediating the supersymmetry breaking, and for the flavor problem [11]. Here we show that it is a new and crucial role that $U(1)_A$ can play for the solution of the doublet-triplet splitting and the μ problems. We want to stress that the idea of solving the μ problem through $U(1)_A$ has been considered in a different context [12]; our main result is a *simultaneous* solution of these two problems.

Problem and the solution.—To illustrate the problem and our solution we will consider the model of Refs. [2,3]. Consider the minimal supersymmetric SU(6) GUT. In order to break the symmetry down to the standard model group $G_W = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, a minimum of two Higgs representations are necessary: an adjoint Σ_i^k and a fundamental-antifundamental pair $\overline{\Phi}^i, \Phi_i$ $(i,k = 1,2,\ldots,6)$. The relevant D-flat VEVs are

$$\Sigma = \mathrm{diag}(1,1,1,1,-2,-2)\sigma,$$
$$\Phi_i = \overline{\Phi}^i = (\phi,0,0,0,0,0)\,, \tag{2}$$

which leave unbroken $G_\Sigma = SU(4)_c \otimes SU(2)_L \otimes U(1)$ and $G_\Phi = SU(5)$ symmetries, respectively, so that the intersection gives unbroken G_W. Assume now that these two sectors have no cross couplings in the superpotential

$$W = W(\Sigma) + W(\Phi)\,. \tag{3}$$

Thus, it effectively has $G_{gl} = SU(6)_\Sigma \otimes SU(6)_\Phi$ symmetry. Since for the VEVs given in Eq. (2) this global symmetry is broken to $G_\Sigma \otimes G_\Phi$, there are compact flat directions in the vacuum that do not correspond to any broken gauge generator; thus, the corresponding zero modes are physical fields. Note that the SU(6) D terms cannot lift this degeneracy, since the contributions of Φ, $\overline{\Phi}$, and Σ are *independently* zero. By a simple counting of the Goldstone states and of the broken gauge generators, we find that leftover zero modes are two linear combinations of the electroweak doublets from Σ and Φ ($\overline{\Phi}$):

$$H = \frac{H_\Sigma\langle\phi\rangle - H_\Phi 3\langle\sigma\rangle}{\sqrt{\langle\phi\rangle^2 + 9\langle\sigma\rangle^2}}, \quad \overline{H} = \frac{\overline{H}_\Sigma\langle\phi\rangle - \overline{H}_\Phi 3\langle\sigma\rangle}{\sqrt{\langle\phi\rangle^2 + 9\langle\sigma\rangle^2}}\,. \tag{4}$$

All other states are heavy and the doublet-triplet splitting problem is solved. The main difficulty is to justify the absence of the possible cross couplings in the superpotential up to a sufficiently high order in M_P^{-1}, by some exact symmetry. This is very difficult to do without also forbidding the possible self-couplings of the Higgs fields, so that usually one ends up either with one of the VEVs being zero, or with an enormous degeneracy of the vacuum, with many new, massless, colored, and charged superfields. More importantly, perhaps, the global symmetries under which the cross coupling $\Sigma\overline{\phi}\phi$ is noninvariant are anomalous and need not be respected by M_P^{-1} suppressed operators. Any such mixed operator with dimensionality less than 6–7 would destroy the solution completely. (Note, the higher operators are safe only if $\phi \ll M_P$, which is an additional input of the theory.) This consideration indicates that we are naturally lead, in the problem of separating the two sectors, to the concept of anomalous gauge symmetry. As we now show, the $U(1)_A$ symmetry provides a natural loophole due to the simple reason that it is "anomalous." It is enough to assume that $\Phi, \overline{\Phi}$ fields carry negative charges q and $\overline{q}$ and all the other fields, and in particular quarks and leptons, carry non-negative charges so that the total trace $\mathrm{Tr}Q > 0$. As we will see below, this assumption naturally fits in the structure of Yukawa couplings and also avoids dangerous charge and color breaking flat directions. We also assume that Σ carries zero charge. Then Φ and $\overline{\Phi}$ are simply left out of the most general $SU(6) \otimes U(1)_A$-invariant Higgs superpotential [15]

$$W_{\mathrm{Higgs}} = \frac{M}{2}\Sigma^2 + \frac{h}{3}\Sigma^3 + \lambda_n \frac{\Sigma^n}{M_P^{n-3}}\,, \tag{5}$$

which fixes the VEV as in Eq. (2) with $\sigma = (M/h)[1 + O(M_G/M_P)]$. The VEV of the ϕ is fixed from the D terms

$$\frac{g^2}{2}(\Phi^* T^a \Phi - \overline{\Phi} T^a \overline{\Phi}^* + [\Sigma^*\Sigma]T^a + \text{matter fields})^2 + \frac{g_A^2}{2}[q|\Phi|^2 + \overline{q}|\overline{\Phi}|^2 + \xi + q_i|S_i|^2]^2, \tag{6}$$

where T^a are SU(6) generators and $q_i|S_i|^2$ is a sum over all the positively charged fields with $q_i > 0$. Minimization gives $\phi^2 = -\xi/(q + \overline{q})$ [the equality $\Phi = \overline{\Phi}$ is demanded from the SU(6) D term]. The only allowed cross couplings between Σ and Φ sectors are the ones that involve positively charged matter field (see Yukawa couplings below). These couplings, however, can never affect the vacuum degeneracy, since all the positively charged fields have *zero* VEVs. Thus, the doublet-triplet splitting problem is solved in all order in M_P^{-1} without need of any extra symmetries.

μ and $B\mu$.—Assuming the conventional [16] gravity-mediated hidden sector supersymmetry breaking, both $B\mu \sim \mu^2$ of the desired magnitude are automatically generated in this scenario and we end up with the following tree-level relation among the electroweak Higgs doublet mass parameters

$$m_H^2 \simeq m_{\overline{H}}^2 \simeq B\mu = \mu^2 + m^2,$$
$$\mu = (3A_{(3)}/h - 2A_{(2)})\,, \tag{7}$$

where m is a soft mass of the Σ field and $A_{(3)}$ and $A_{(2)}$ are coefficients of the soft trilinear and bilinear couplings, respectively. The above relation is given at M_P and holds up to the corrections of order $\epsilon = \xi/M_P^2$. This is a standard pseudo-Goldstone relation of [1–3] for the minimal soft terms and is due to the fact that for the minimal Kähler potential at the tree level there should be one exactly massless state

$$H_+ = \frac{H + \overline{H}^*}{\sqrt{2}}. \tag{8}$$

This is because for the minimal Kähler potential

$$K = |\Sigma|^2 + |\overline{\Phi}|^2 + |\Phi|^2 + \cdots, \tag{9}$$

the universal scalar soft terms (except for the "Yukawa" trilinears with matter scalars, which vanish anyway) respect the $G_{gl} = \mathrm{SU}(6)_\Sigma \otimes \mathrm{SU}(6)_\Phi$ symmetry. Therefore, by the Goldstone theorem the tree-level mass matrix of Higgs doublets must have one exactly massless eigenstate Eq. (8), leading to the first relation in Eq. (7). Explicit minimization (in series of $m_{3/2}/M_G$ and $m_{3/2}/\sqrt{\xi}$) just confirms this result and provides the second relation in Eq. (7).

For a generic nonminimal Kähler potential

$$K = \alpha_1|\Sigma|^2 + \alpha_2|\overline{\Phi}|^2 + \alpha_3|\Phi|^2 + \frac{\alpha_4}{M_P^2}\Phi^*\Sigma^*\Sigma\Phi + \frac{\alpha_5}{M_P^2}\overline{\Phi}\Sigma^*\Sigma\overline{\Phi}^* + \text{other terms}, \tag{10}$$

where α_i are some dimensionless functions of the hidden sector fields that break SUSY, the relation in Eq. (7) may be disturbed (although order of magnitudewise it is still valid). A potential disturbance appears because of the nonuniversal soft masses of Φ and $\overline{\Phi}$ ($\alpha_2 \neq \alpha_3$) and because of the nonzero cross couplings ($\alpha_4, \alpha_5 \neq 0$); this is 100% important for both $\phi \sim M_P$ and $\phi \sim M_G$. The only regime in which it can be suppressed is $M_P \gg \phi \gg M_G$. This is precisely the situation in our case and Eq. (7) holds for the arbitrary nonminimal Kähler potential and is essentially a prediction of the model. This is because in our model the scale $\phi \sim \sqrt{\xi}$ is predicted to be just halfway between M_P and M_G and the light pseudo-Goldstones predominantly reside in Σ [see Eq. (4)]. Because of this, both contributions from the nonuniversal soft terms of Φ and $\overline{\Phi}$ and contributions from the cross couplings in the Kähler potential are suppressed. Thus, we have in this model one less free parameter than in minimal supergravity; hence it can predict, for instance, $\tan\beta$ in terms of the other masses [1,3,17].

Fermion masses and proton stability The fermion masses in the above scheme were analyzed in more detail in [4], where it was shown that the model admits a realistic (within uncertainties in coupling constants of order 1) description of the fermion mass hierarchy in terms of the hierarchy of scales $M_P \gg \phi \gg M_G$ without invoking flavor symmetries. The most interesting result is that only the top quark has a renormalizable Yukawa interaction at the tree level. This happens if besides the three chiral families in $15_\alpha + \overline{6}'_\alpha + \overline{6}_\alpha$ (the minimal anomaly-free set that accommodates $10 + \overline{5}$ of SU(5) per family) [18] one assumes an odd number of real 20-plets with invariant M_P mass terms. A decomposition of these multiplets in terms of SU(5) representations gives

$$15_\alpha = 10_\alpha + 5_\alpha, \quad 20 = 10 + \overline{10}. \tag{11}$$

The important group-theoretical fact is that no invariant mass term can be formed from the symmetric product of two 20-plets; thus, a single 20-plet will survive as light and can get a mass only after SU(6) symmetry breaking. Up to a field redefinition, the most general renormalizable couplings are (coupling constants are neglected)

$$\Sigma 2020 + \Phi 15_3 20 + \overline{\Phi} 15_\alpha \overline{6}'_\beta. \tag{12}$$

The last coupling simply gives SU(5)-invariant masses to the extra heavy states $(5, \overline{5}')$ from 15-s and $\overline{6}'$-s, mixing them with each other. The second term combines a 10_3-plet from 15_3 with $\overline{10}$ from 20, and they become heavy as well. The remaining light 10, predominantly residing in 20, gets a tree-level Yukawa coupling with H through the first term, giving mass to the top. The masses of lighter fermions are generated through the higher-dimensional operators

$$\frac{1}{M_P^{n+1}}\Phi\Sigma^n\Phi 1515 + \frac{1}{M_P^n}\overline{\Phi}\Sigma^n 15\overline{6} + \cdots \tag{13}$$

with different possible Σ insertions. This gives us the possibility to account for the fermion mass hierarchy in terms of two ratios of the scales M_G/M_P and ϕ/M_P; for more details we refer the reader to [4]. The only new point in our case is that the necessary condition $M_G \ll \phi \ll M_P$, which was an additional input of the theory in the case of [4], is now a natural outcome since the scale ϕ is generated from the Fayet-Iliopoulos D term. It is easy to show that our $\mathrm{U}(1)_A$ charge assignment (necessary to solve the doublet–triplet splitting problem) is automatically compatible with the above structure of Yukawa couplings. The simplest possibility is not to invoke any flavor dependence in the spirit of Ref. [4]. Then the flavor-blind $\mathrm{U}(1)_A$ charges are constrained as

$$q_{15} = -q, \quad q_{\overline{6}} = q - \overline{q}. \tag{14}$$

The additional constraint comes from the neutrino masses. For example, if we generate the right-handed neutrino masses from the operator

$$\frac{(\Phi\overline{6})^2}{M_P^3}(\Phi\overline{\Phi}), \tag{15}$$

then charges are fixed as $q_{15} = -q$, $q_{\overline{6}} = -4q$, $\overline{q} = 5q$. This assignment automatically kills any baryon number violating operator trilinear in the matter fields $\overline{6}15\overline{6}$ to all orders in M_P^{-1}. Thus, $U(1)_A$ can play the role of the matter parity. Family-dependent charge assignment, along the lines of [9], is also possible without altering any of our conclusions. The novel feature in such a construction, not attempted here, will be that in contrast to [9] the Higgses that break $U(1)_A$ are not the GUT singlets. Thus their Yukawa couplings will be constrained by both the GUT symmetry and the anomalous $U(1)_A$. This can offer the the possibility of generating specific (and hopefully predictive) textures for the fermion masses.

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