Probing the QCD Pomeron in e^+e^- Collisions

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The total cross section for scattering virtual photons at high energy is sensitive to pomeron physics. If the photons are sufficiently virtual, QCD perturbation theory applies, so that the reaction probes the short distance pomeron. We study this reaction for present and future $e^\pm e^-$ colliders. [S0031-9007(96)02266-1]

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The behavior of scattering in the limit of high energy and fixed momentum transfer is one of the outstanding open questions in the theory of strong interactions. A useful description of this behavior was developed in the 1960s, based on analytic continuation in the complex J plane, where J represents the angular momentum of exchanged particles [1]. In the late 1970s, Lipatov and collaborators established a connection of the high energy behavior with the quark and gluon degrees of freedom, at least for special situations in which perturbation theory applies [2]. These papers still form the core of our knowledge of QCD scattering at high energies. The physical effect they describe is often referred to as the QCD pomeron, or BFKL effect.

Experimental studies of the QCD pomeron are at present carried out mainly at the HERA ep collider in deeply inelastic scattering in the region of low values of the Bjorken variable x [3]. In this case, QCD pomeron effects are expected to give rise to a power-law growth of the structure functions as x goes to zero, and an increase in the scaling violation at fixed values of x (for small enough x). However, studying BFKL pomeron effects in deeply inelastic scattering is made difficult by the fact that the low x behavior is influenced by both short distance and long distance physics. As a result, predictions at photon virtuality Q depend on a set of nonperturbative inputs at a scale $Q_0 < Q$. This makes it difficult to untangle perturbative BFKL predictions from nonperturbative effects.

Other processes have been suggested in which the ambiguity related to large distance physics is expected to be reduced. One such process is the production of two jets in hadron-hadron collisions [4]. One looks for jets produced by inelastic parton scattering at fixed momentum p_T transferred to the parton in the limit of large parton energy $\sqrt{\hat{s}}$. In order to avoid entanglement with the parton distributions, one has to keep $\hat{s}/s$ fixed as $\hat{s} \to \infty$. Unfortunately, this is not readily done at a single hadron collider. An essentially equivalent process is to look for a high p_T jet with a large fraction of the proton's longitudinal momentum in the final state of a deeply inelastic scattering event at small x.

A related possibility is to look for elastic parton scattering at large $\hat{s}$ with fixed p_T with the demand that the exchanged quanta be in a color singlet state. The signature for this is the existence of a gap in the rapidity interval between the two jets produced by the scattered partons [5]. Unfortunately the gap signal is affected by various long distance processes, including spectator collisions.

In principle, one could avoid the difficulties mentioned above by measuring the cross section for the scattering at large s and fixed t of two colorless bound states of a heavy quark and a heavy antiquark ("quarkonia") [6]. This process is perturbative because of the small size of the quarkonium state. Unfortunately, until one can build a quarkonium accelerator, this must remain a gedanken experiment.

In this Letter we consider the possibility of studying BFKL effects by measuring the total cross section for off-shell photons at $e^\pm e^-$ colliders. Essentially, we replace the quarkonia by virtual photons. The size of the wave function for finding quarks in a virtual photon is controlled by the photon virtuality instead of the heavy quark mass. An exciting feature of such an experimental investigation would be that the virtualities of each of the two photons could be tuned by measuring the momenta of the recoil leptons.

The process that we discuss, virtual photon scattering, has recently been used by Balitskii to reformulate the BFKL problem in terms of an expansion using Wilson line operators [7]. This method may enable theory to get beyond the leading logarithm approximation. In the present Letter, we are not concerned with the foundations of the theory, but with the practical possibilities for using virtual photon scattering as a probe of the short distance pomeron.

There have been other investigations of the high energy regime in the context of $e^\pm e^-$ collisions. These have mainly focused on photon structure functions (see, for instance, Ref. [8] for a recent overview of this subject) and diffractive meson production [8,9]. In these cases, either nonperturbative parton distributions in the photon or nonperturbative meson wave functions enter the theoretical predictions. The results presented in this Letter appeared in preliminary form in [10]. Similar results obtained independently by Bartels, De Roeck, and Lotter may be found in Ref. [11].

We will describe the total cross section for the scattering of two transversely polarized virtual (spacelike) photons $\gamma^*(q_A)$ and $\gamma^*(q_B)$, with virtualities $q_A^2 = -Q_A^2$ and $q_B^2 = -Q_B^2$, in the high energy region where $s = (q_A + q_B)^2$ is much larger than Q_A^2 or Q_B^2. We suppose that the photon virtualities are in turn large with respect to the QCD scale Λ_{QCD}^2, so that the process occurs at short distances (much smaller than $\Lambda_{\text{QCD}}^{-1} \approx 1$ fm) and QCD perturbation theory applies.

It is simple to see that the two-gluon exchange mechanism gives rise to a constant $\gamma^* \gamma^*$ total cross section at large s, $\sigma^{(0)}(s, Q_A^2, Q_B^2) \sim \alpha^2 \alpha_S^2 f(Q_A^2, Q_B^2)$ [12]. To higher orders in perturbation theory, the iteration of gluon ladders promotes this constant to logarithms, and the perturbative expansion of the cross section at high energy has the form

$$\sigma(\gamma^* \gamma^*) \sim \sigma^{(0)} \left[1 + \sum_{k=1}^{\infty} a_k (\alpha_S L)^k + \ldots \right],$$
$$L = \ln(s/Q^2), \tag{1}$$

where Q^2 is a scale of the order of the initial photon virtualities, the sum represents the series of the leading logarithms to all orders in the strong coupling α_S, and the dots stand for nonleading terms. To study the high energy behavior, it is convenient to analyze the cross section in its Mellin moments, defined by

$$\sigma(s, Q_A^2, Q_B^2) = \int_{a-i\infty}^{a+i\infty} \frac{dN}{2\pi i} e^{NL} \sigma_N(Q_A^2, Q_B^2), \tag{2}$$

where the N integral runs parallel to the imaginary axis and to the right of any singularities in σ_N. We see from this definition that a constant behavior of the cross section with the energy s is generated by a simple pole in σ_N at $N = 0$, while powers of logarithms are generated by multiple poles at $N = 0$.

To sum the leading logarithmic terms, we employ the method developed in Ref. [13] for computing hard scattering cross sections at high energy. As illustrated in Fig. 1, we write σ_N as a convolution of three factors,

$$\sigma_N(Q_A^2, Q_B^2) = \int \frac{d^2 \mathbf{k}_A}{\pi \mathbf{k}_A^2} \int \frac{d^2 \mathbf{k}_B}{\pi \mathbf{k}_B^2}$$
$$\times G(\mathbf{k}_A^2/Q_A^2) \mathcal{F}_N(\mathbf{k}_A, \mathbf{k}_B) G(\mathbf{k}_B^2/Q_B^2). \tag{3}$$

The factors G describe the coupling of two gluons to a quark-antiquark pair created by the virtual photon. We compute G at lowest (one loop) order, taking the limit in which the longitudinal momentum carried by the gluons is negligible compared to the longitudinal momentum carried by the quarks. At order α_S^0, the other factor, $\mathcal{F}_N$, describes the exchange of two gluons between the $q\bar{q}$ states:

$$\mathcal{F}_N^{(0)}(\mathbf{k}_A, \mathbf{k}_B) = \frac{\pi}{N} \delta(\mathbf{k}_A - \mathbf{k}_B). \tag{4}$$

The simple pole at $N = 0$ corresponds to a cross section that is independent of s. The full function $\mathcal{F}_N$ is the
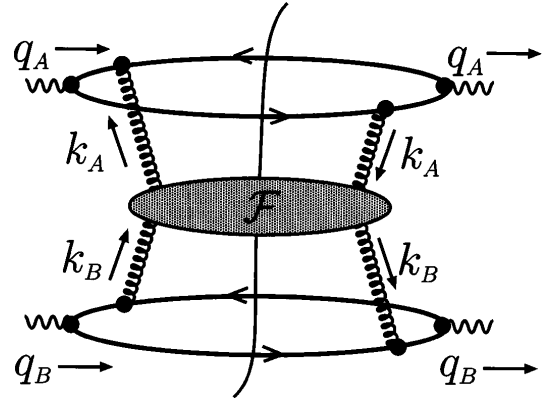

FIG. 1. Graphical representation of Eq. (3). The gluons can be attached to the quark lines in 16 different ways.

standard solution of the BFKL equation [2] that reduces to (4) at order zero.

Inserting the BFKL solution for $\mathcal{F}_N$ in Eq. (3) and working out the transverse momentum integrations gives

$$\sigma_N(Q_A^2, Q_B^2) = \frac{1}{2\pi Q_A Q_B} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i}$$
$$\times \left(\frac{Q_A^2}{Q_B^2} \right)^{\gamma - 1/2} \frac{V(\gamma) V(1 - \gamma)}{N - \bar{\alpha}_S \chi(\gamma)}, \tag{5}$$

where we have set $\bar{\alpha}_S = \alpha_S C_A / \pi$ ($C_A = 3$), and the functions χ and V are given by

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma), \tag{6}$$

$$V(\gamma) = \pi \alpha \alpha_S \left(\sum_q e_q^2 \right)$$
$$\times \frac{(1 + \gamma)(2 - \gamma)\Gamma^2(\gamma)\Gamma^2(1 - \gamma)}{(3 - 2\gamma)\Gamma(3/2 + \gamma)\Gamma(3/2 - \gamma)}. \tag{7}$$

Here Γ is the Euler Γ function, and ψ is its logarithmic derivative. While the function $\chi(\gamma)$ is a universal function from the solution to the BFKL equation, the function $V(\gamma)$ is specific to the off-shell photon probe.

Equation (5) sums the $1/N$ poles to the accuracy $(\alpha_S^2/N) \times (\alpha_S/N)^k$, for any k. By evaluating Eq. (5) to the lowest perturbative order, $k = 0$, one recovers the constant contribution $\sigma^{(0)}$ from the two-gluon exchange model for the pomeron. In general, one determines $1/N^{k+1}$ contributions to the cross section by retaining higher orders in the α_S expansion of Eq. (5), and we see that the general structure of the coefficients of the leading logarithmic series comes from both $\chi(\gamma)$ and $V(\gamma)$. The net cross section at the leading logarithmic level is obtained from Eq. (5) by taking the inverse Mellin transform (2):

$$\sigma(s, Q_A^2, Q_B^2) = \frac{1}{2\pi Q_A Q_B} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_A^2}{Q_B^2} \right)^{\gamma - 1/2}$$
$$\times \left(\frac{s}{Q^2} \right)^{\bar{\alpha}_S \chi(\gamma)} V(\gamma) V(1 - \gamma). \tag{8}$$

In the high energy limit $s \gg Q^2$, the integral (8) is dominated by the region near the saddle point of $\chi(\gamma)$ at $\gamma = 1/2$. One obtains the BFKL power dependence s^λ with $\lambda = \bar{\alpha}_S \chi(1/2) = 2.77\bar{\alpha}_S$. In Fig. 2, we show the cross section computed from Eq. (8) with $\alpha_S = 0.2$ as a function of s/Q^2.

Our primary concern in this Letter is with high energy virtual photon scattering. However, one can also use Eqs. (5) and (8) to explore small x deeply inelastic scattering [14], in which $Q_B^2 \ll Q_A^2 \ll s$. We will investigate these issues in a future paper [15].

The cross section for high energy virtual photon scattering can be measured in $e^\pm e^-$ collisions in which the outgoing leptons are tagged. The part of the $e^\pm e^-$ cross section that is contributed by transversely polarized photons is obtained by folding the $\gamma^*\gamma^*$ cross section with the flux of photons from each lepton:

$$
\sigma = \int_{\mathcal{R}} \frac{dx_A}{x_A} \frac{dx_B}{x_B} \frac{dQ_A^2}{Q_A^2} \frac{dQ_B^2}{Q_B^2}
$$
$$
\times \left(\frac{\alpha}{2\pi} \right)^2 x_A P_{\gamma/e^+}(x_A) x_B P_{\gamma/e^-}(x_B)
$$
$$
\times \sigma_{\gamma^*\gamma^*}(x_A x_B s_{ee}, Q_A^2, Q_B^2). \tag{9}
$$

Here x_A and x_B denote the fractions of the longitudinal momenta of the leptons A and B that are carried by the corresponding bremsstrahlung photons. The integration region $\mathcal{R}$ is determined by the experimental cuts. The flux factor is given by

$$
P_{\gamma/e}(x) = \frac{1 + (1-x)^2}{x}. \tag{10}
$$

In the $\gamma^*\gamma^*$ cross section, Eq. (8), one must make choices of the scale μ in α_S and the variable Q^2 that provides the scale for the logarithms of s. A reliable determination of these scales requires a next to leading order calculation, which is beyond the scope of this Letter.
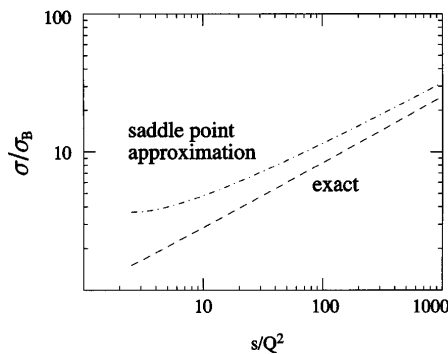

FIG. 2. The s/Q^2 dependence of the $\gamma^*\gamma^*$ total cross section, Eq. (8). We take $Q_A^2 = Q_B^2$, $\alpha_S = 0.2$, and divide σ by the Born cross section σ_B, which is independent of s. We show the results of evaluating the integral exactly and of using the saddle point approximation.

We simply choose

$$
\mu^2 = c_\mu Q_A Q_B,
$$
$$
Q^2 = c_Q Q_A Q_B, \tag{11}
$$

with $c_\mu = e^{-5/3}$ and $c_Q = 100$. Our choice for μ^2 is based on a calculation of the mean logarithmic virtuality for the gluon propagators in the two gluon exchange graph, following the prescription of [16]. The choice for Q^2 is based on an estimate of the rapidity range available for exchanged gluons. We will discuss these estimates in a later work [15]. Given the uncertainties of the leading logarithmic approximation, one should regard the numerical results that follow as being accurate only at the order of magnitude level.

Having made these scale choices, one can check Regge factorization: does the cross section decompose into a product of a function of Q_A times a function of Q_B times a function of s? Looking at Eq. (8) with $Q^2 = c_Q Q_A Q_B$, one sees that this factorization is an approximate property of the formula at large s when the γ integral is dominated by a narrow range of γ near the saddle point of $\chi(\gamma)$.

We now estimate the cross section available to study BFKL effects at $e^+ e^-$ colliders. We integrate Eq. (9) over a range $\mathcal{R}$ determined by (1) $Q_A > Q_{\min}$, $Q_B > Q_{\min}$ with $Q_{\min}^2 = 10$ GeV^2, in order to keep α_S small and (2) $x_A x_B s_{ee} > 100 Q_A Q_B$ in order that the high energy approximation be valid. We base the numerical value of this limit on a demand that the two gluon exchange graph be dominant over the (lower order) quark exchange graph when we chose $\alpha_S = 0.2$.

Performing the integrations numerically and adding a similar contribution from longitudinally polarized virtual photons, as described in Ref. [11], we obtain

$$
\sigma \simeq 1 \text{ pb} \qquad (\sqrt{s} = 200 \text{ GeV}) \tag{12}
$$

at LEP200 energies, and

$$
\sigma \simeq 4 \text{ pb} \qquad (\sqrt{s} = 500 \text{ GeV}) \tag{13}
$$

at a future next linear collider (NLC). These cross sections would give rise to about 500 events at LEP200 for a value of the luminosity $L = 500$ pb^{-1}, and about 2×10^5 events at the NLC for $L = 50$ fb^{-1}. For $Q_{\min}^2 = 100$ GeV^2, the number of events at the NLC would be about 10^4. While this looks rather marginal at LEP200, it appears that measuring off-shell photon scattering at the NLC could be a viable way of studying short distance pomeron effects.

Even with a modest luminosity, one can examine experimentally how the perturbative pomeron emerges from the soft pomeron as Q_A and Q_B are increased. For small Q_A and Q_B a simple Regge model should apply. For on-shell photons, Regge factorization gives

$$
\sigma_{\gamma\gamma} \approx \sigma_{\gamma p} \sigma_{\gamma p} / \sigma_{pp}. \tag{14}
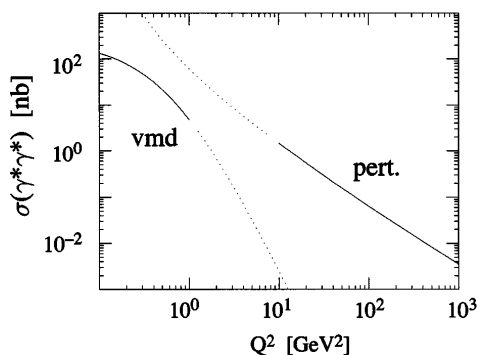$$

FIG. 3. Q^2 behavior of the vector meson dominance and perturbative cross sections in lowest order, with $Q_A^2 = Q_B^2 \equiv Q^2$.

Assuming the values $\sigma_{\gamma p} \approx 0.1$ mb and $\sigma_{pp} \approx 40$ mb, one gets $\sigma_{\gamma\gamma} \approx 250$ nb. For virtual photons with small Q_A and Q_B, the falloff of the cross section can be estimated from vector meson dominance:

$$\sigma_{\gamma^*\gamma^*} \approx \left(\frac{M_\rho^2}{M_\rho^2 + Q_A^2}\right)^2 \left(\frac{M_\rho^2}{M_\rho^2 + Q_B^2}\right)^2 \sigma_{\gamma\gamma}. \qquad (15)$$

Figure 3 shows a log-log plot of the curves corresponding to the soft and the perturbative formulas (in lowest order) for the Q^2 behavior of the cross section. In the region $Q^2 \lesssim 1$ GeV^2, one may expect the formula (15) based on vector meson dominance to apply. As one goes above this region, the cross section, instead of continuing to fall like $1/Q^8$, should begin to fall more slowly. At large photon virtualities $Q^2 \gtrsim 10$ GeV^2 the cross section should exhibit the perturbative scaling behavior in Eq. (8), $\sigma \propto 1/Q^2$ at fixed (s/Q^2).

With a large luminosity, as at a future high energy linear collider, one can explore virtual photon scattering to higher Q^2 and thus probe experimentally the effects of pomeron exchange in the region where summed perturbation theory should apply. One should be able to investigate this region in detail by varying Q_A, Q_B, and $\hat{s} = x_A x_B s_{ee}$ independently.

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